Post-Newtonian Λ term

$$\Lambda^{\alpha\beta} = -h^{\mu\nu}\partial^2_{\mu\nu}h^{\alpha\beta} + \partial_\mu h^{\alpha\nu}\partial_\nu h^{\beta\mu} + \frac{1}{2}g^{\alpha\beta}g_{\mu\nu}\partial_\lambda h^{\mu\tau}\partial_\tau h^{\nu\lambda}$$
$$- g^{\alpha\mu}g_{\nu\tau}\partial_\lambda h^{\beta\tau}\partial_\mu h^{\nu\lambda} - g^{\beta\mu}g_{\nu\tau}\partial_\lambda h^{\alpha\tau}\partial_\mu h^{\nu\lambda} + g_{\mu\nu}g^{\lambda\tau}\partial_\lambda h^{\alpha\mu}\partial_\tau h^{\beta\nu}$$
$$+ \frac{1}{8}\left(2g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu}\right)\left(2g_{\lambda\tau}g_{\epsilon\pi} - g_{\tau\epsilon}g_{\lambda\pi}\right)\partial_\mu h^{\lambda\pi}\partial_\nu h^{\tau\epsilon}\,.$$

Iterative solution of field eqn

$$h^{\alpha\beta} = \sum_{n=1}^{\infty} G^n \, h_n^{\alpha\beta}$$

Field equation holds at every *n* ... only terms with $m < n$ will act as sources at order G^n!

Consider solution in exterior, where $T^{\alpha\beta} = 0$:

$$\Box h_1^{\alpha\beta} = 0$$

Gives us the linearized theory solution we worked out a few weeks ago.

Iterative solution of field eqn

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Consider solution in exterior, where $T^{\alpha\beta} = 0$:

$$\Box h_2^{\alpha\beta} = N^{\alpha\beta}(h_1, h_1)$$

$$\Box h_3^{\alpha\beta} = M^{\alpha\beta}(h_1, h_1, h_1) + N^{\alpha\beta}(h_1, h_2) + N^{\alpha\beta}(h_2, h_1)$$

$$\Box h_4^{\alpha\beta} = L^{\alpha\beta}(h_1, h_1, h_1, h_1) + M^{\alpha\beta}(h_2, h_1, h_1) + M^{\alpha\beta}(h_1, h_2, h_1) + M^{\alpha\beta}(h_1, h_1, h_2) + N^{\alpha\beta}(h_2, h_2) + N^{\alpha\beta}(h_1, h_3) + N^{\alpha\beta}(h_3, h_1)$$

Example result

On an earlier pset, we derived the flux of energy carried by GWs from a circular binary:

$$\mathcal{F} = \frac{32 G^4}{5} \frac{\mu^2 M^3}{R^5}$$

$$\mu = m_1 m_2 / M \ , \quad M = m_1 + m_2$$

$$R = \text{separation of masses}$$

Rewrite in terms of an observable:

$$\Omega = \sqrt{\frac{GM}{R^3}}$$

$$x \equiv (GM\Omega)^{1/3} \ , \quad \nu = \mu/M$$

Leading order form we derived becomes

$$\mathcal{F} = \frac{32}{5} \frac{1}{G} \nu^2 x^5$$

Example result

Examine how this changes when we iterate in the post-Newtonian expansion:

$$\mathcal{F} = \frac{32}{5}\frac{1}{G}\nu^2 x^5$$

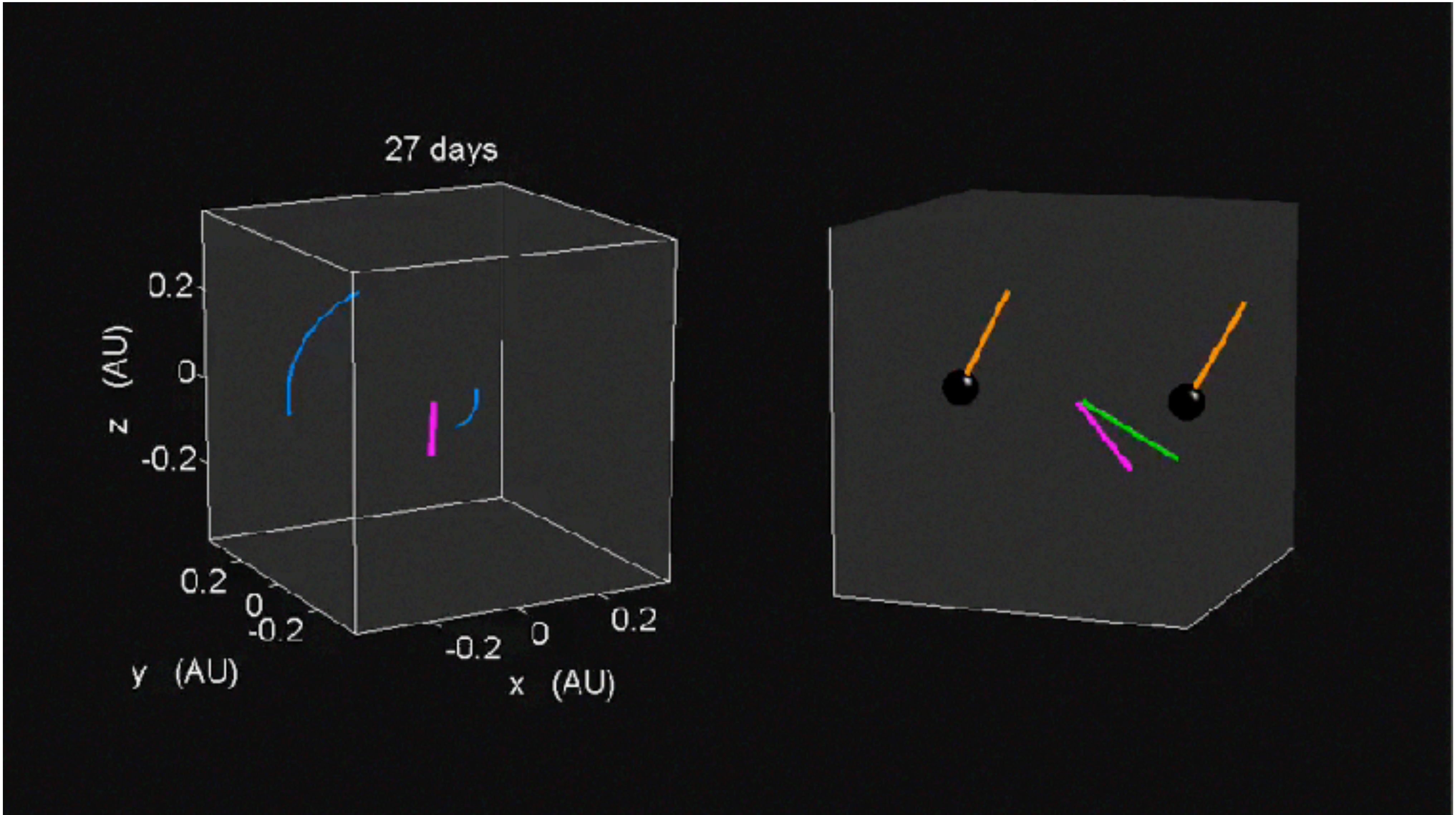
Example result

Examine how this changes when we iterate in the post-Newtonian expansion:

$$\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2}$$
$$+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \Bigg[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_{\mathrm{E}} - \frac{856}{105}\ln(16\,x)$$
$$+ \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\Bigg] x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\}\,.$$

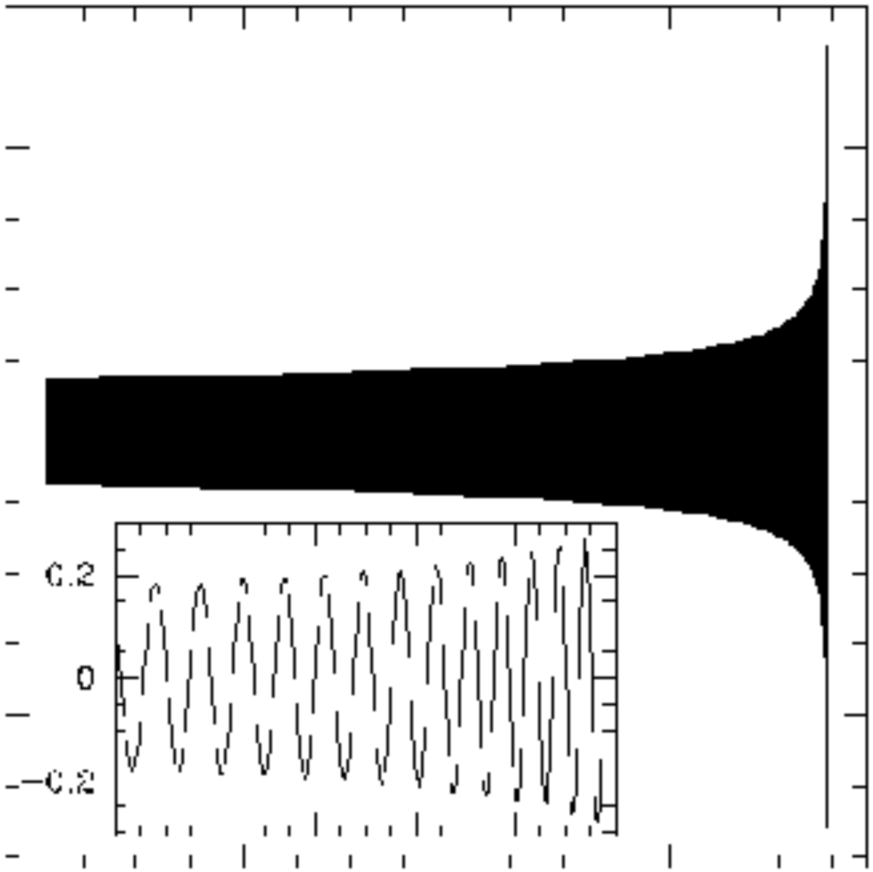
New physics

Each body's spin enters dynamics … consistent with idea that all forms of energy gravitate.



Dynamics directly imprint GWs

Post-Newtonian descriptions directly "flavor" models are used to measure GWs today.



Example 1:
Two non-spinning black holes.

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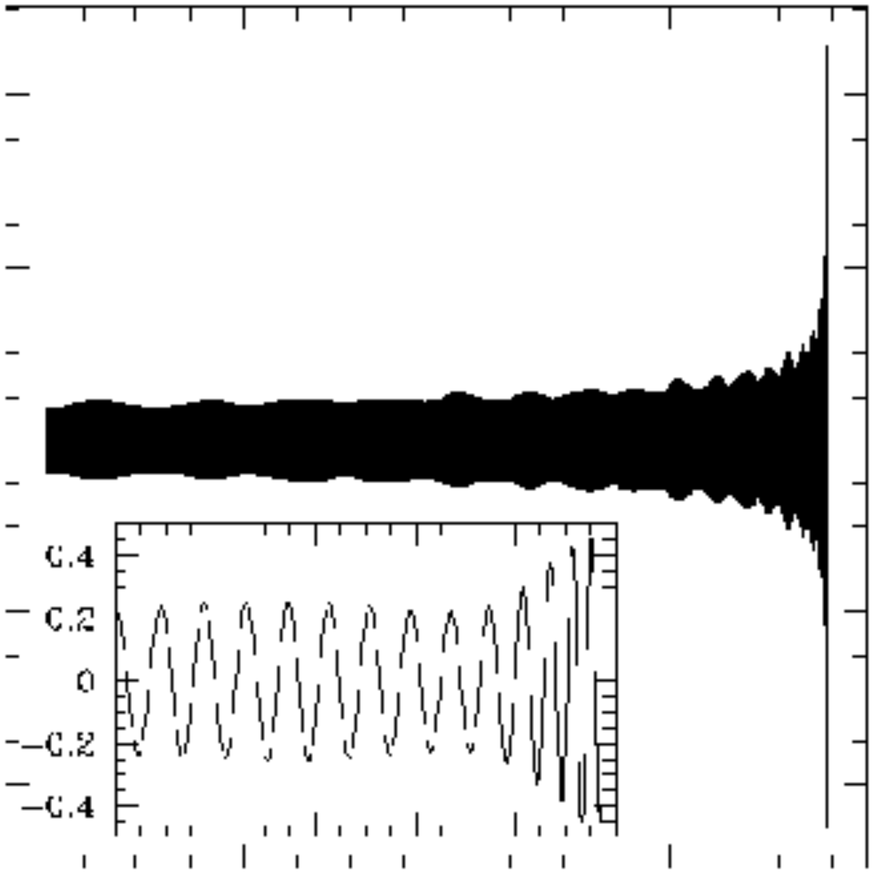

Example 2:
Two *rapidly* spinning black holes.