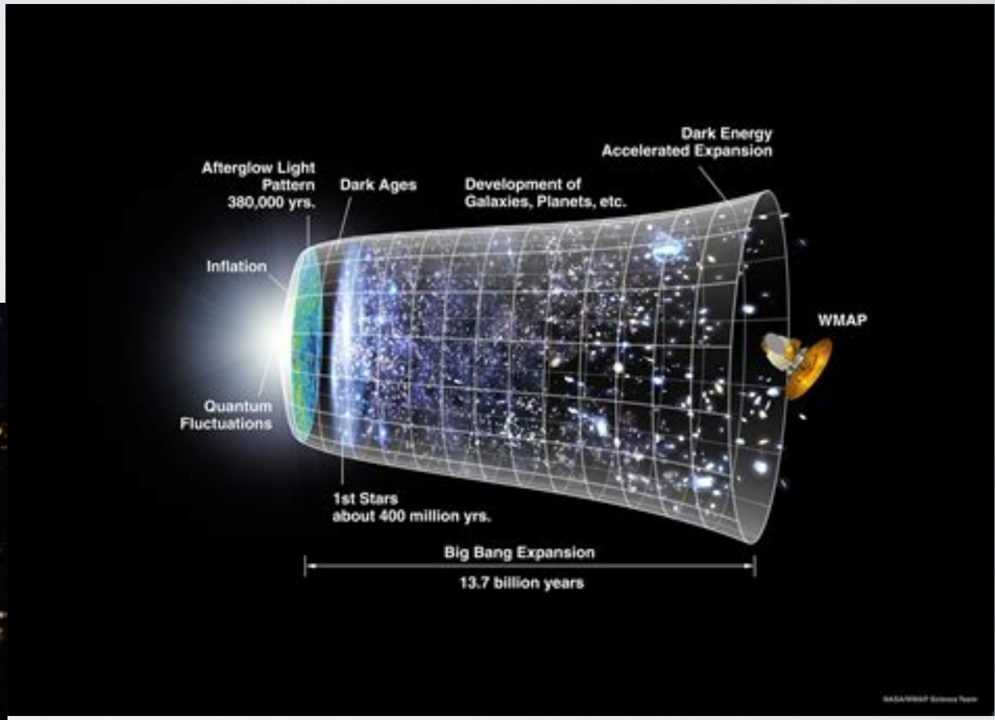
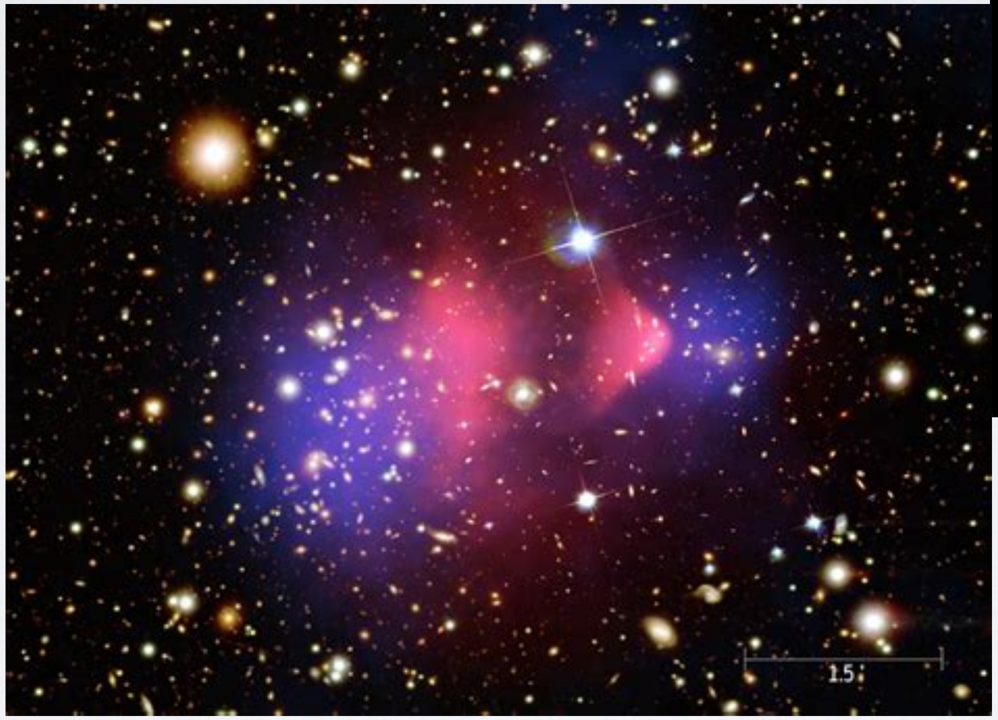
8.962 LECTURE 19

Cosmology (part II)





(Above) synopsis of evolution of "stuff" in our universe over cosmic time.

(Left) image of the "Bullet Cluster," an important datum on the nature of dark matter.

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Announcement: Your TA Lisa Drummond defends her PhD thesis, "*Gyroscopes Orbiting Gargantuan Black Holes: Spinning Secondaries in Extreme Mass Ratio Inspirals*," tomorrow at 1:30 pm (37-252). We may all be a tad distracted with respect to 8.962 issues for a little while.

Recap: The universe is homogeneous and isotropic on "large" spatial scales; we demand spacetime take a form that reflects this. Using properties of maximally symmetric spaces, we deduced that a good form of the line element is

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right]$$

The function $a(t)$ takes the value 1 right now; the parameter κ is related to $k \in [-1, 0, 1]$ by $\kappa = k/(R_0)^2$.

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$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right]$$

Run this through the Einstein field equations with a perfect fluid source; the result is known as the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H(a)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2} \qquad \text{(F1)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right) \qquad \text{(F2)}$$

Conservation of stress energy leads in addition to the condition

$$\partial_t\left(\rho a^3\right) = -P\,\partial_t\left(a^3\right)$$

This is nothing more than a cosmological form of ***dU = –P dV***.

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Useful definitions:

$$\rho_{\rm crit} = \frac{3H^2}{8\pi G}\,, \qquad \Omega = \rho/\rho_{\rm crit}$$

Using this, the 1st Friedman equation becomes

$$\Omega - 1 = \kappa/(H^2a^2) \qquad \text{or} \qquad \Omega + \Omega_{\rm c} = 1$$

with the further definition

$$\Omega_{\rm c} = -\frac{\kappa}{H^2a^2}$$

Also worth re-capping a second form of the line element:

$$ds^2 = -dt^2 + a^2(t)R_0^2\left[d\chi^2 + S_k^2(\chi)d\Omega^2\right]$$

where $S_k(\chi) = \sin(\chi)$ if $k = +1$ $(\kappa > 0)$, $S_k(\chi) = \sinh(\chi)$ if $k = -1$ $(\kappa < 0)$, and $S_k(\chi) = \chi$)if $k = 0$ $(\kappa = 0)$.

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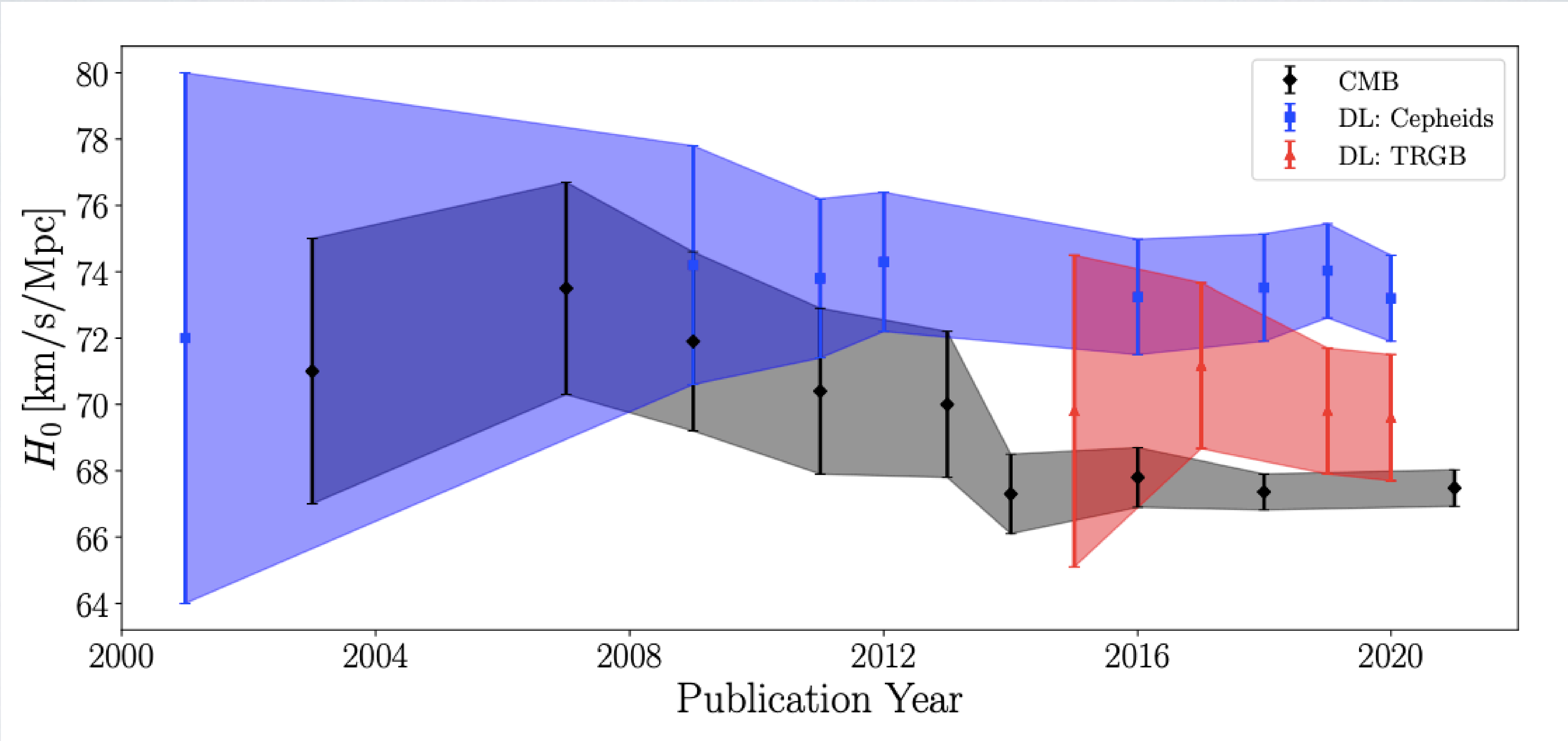
arXiv:2105.09409, Figure 1



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arXiv:2403.15526, Table 25.1

	Planck TT,TE,EE+lowE+lensing	+BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.1193 ± 0.0009
$100\,\theta_{MC}$	1.0409 ± 0.0003	1.0410 ± 0.0003
n_s	0.965 ± 0.004	0.966 ± 0.004
τ	0.054 ± 0.007	0.056 ± 0.007
$\ln(10^{10} \Delta^2_{\mathcal{R}})$	3.044 ± 0.014	3.047 ± 0.014
h	0.674 ± 0.005	0.677 ± 0.004
σ_8	0.811 ± 0.006	0.810 ± 0.006
Ω_m	0.315 ± 0.007	0.311 ± 0.006
Ω_Λ	0.685 ± 0.007	0.689 ± 0.006

If the assumption of spatial flatness is lifted, it turns out that the primary CMB on its own constrains the spatial curvature fairly weakly, due to a parameter degeneracy in the angular-diameter distance. However, inclusion of other data readily removes this degeneracy. Simply adding the *Planck* lensing measurement, and with the assumption that the dark energy is a cosmological constant, yields a 68% confidence constraint on $\Omega_{tot} \equiv \sum \Omega_i + \Omega_\Lambda = 1.011 \pm 0.006$ and further adding BAO makes it 0.9993 ± 0.0019 [2]. Results of this type are normally taken as justifying the restriction to flat cosmologies.

Note: Ω_c refers to “Cold Dark Matter” here, not spatial curvature.

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$w = w_0 + w_a(1 - a)$

from arXiv:2404.08056

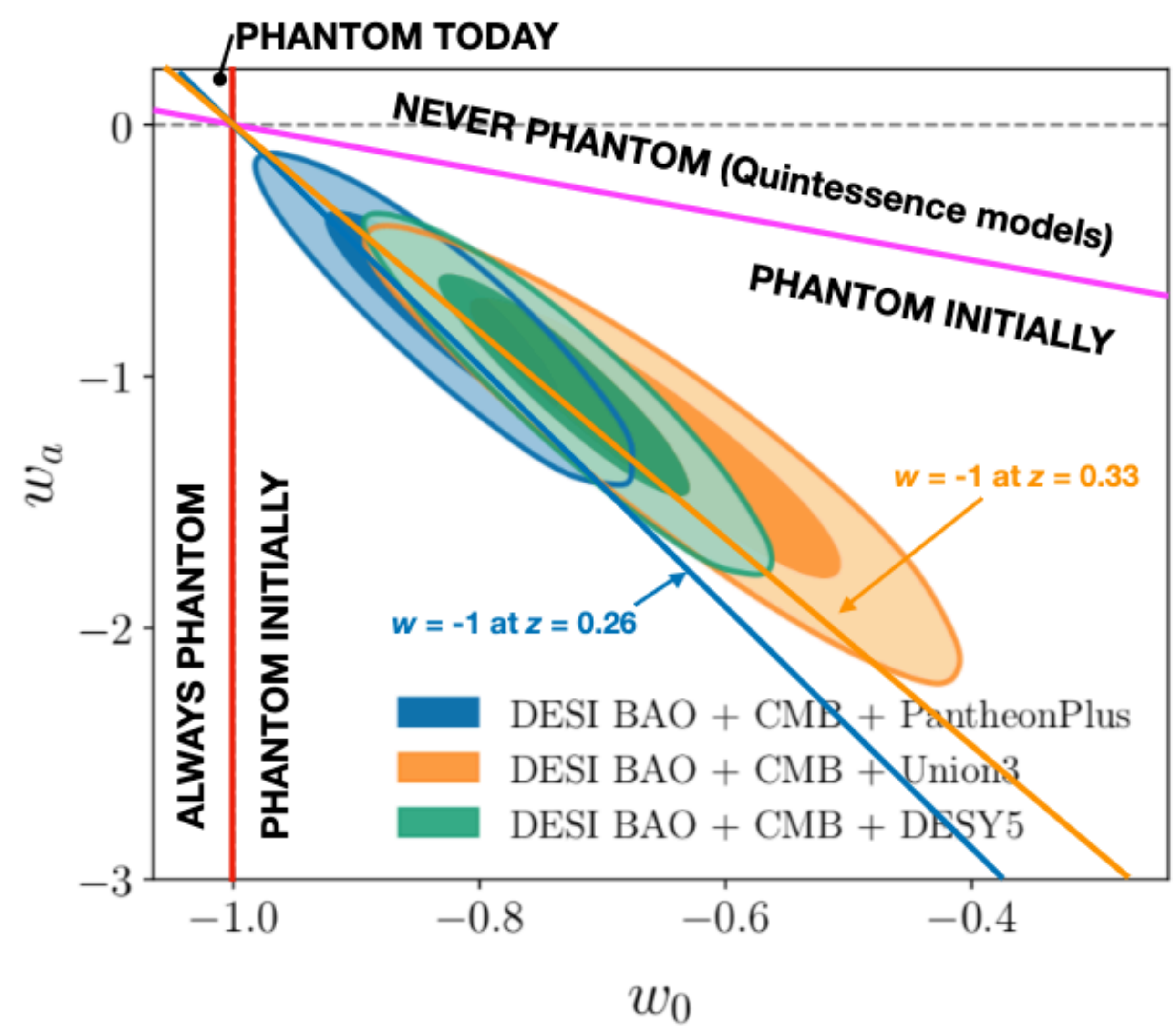

Figure 1. Observational constraints in the w_0–w_a plane from Ref. [3], combining DESI BAO and CMB constraints with three different choices of supernova sample. The magenta and red lines partition models into phantom and non-phantom behaviour at early times and today, respectively. In combination they cut the plane into four zones. The blue and orange lines mark parameter values where w crosses -1 at redshifts 0.26 and 0.33 respectively. These correspond to the pivot redshifts for the PantheonPlus and DESY5 supernova samples (blue) and Union3 (orange). This shows that all three choices have w close to -1 at the pivot scale. [Adapted from Figure 6 of Ref. [3], under Creative Commons BY 4.0 License.]