Angular dependence of radiation as a probe of Bose Condensation of exciton systems

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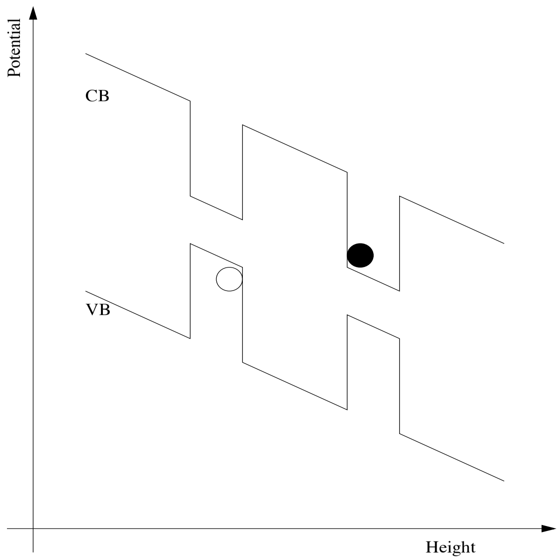
Abstract

The momentum distribution for trapped excitons is calculated; this can distinguish condensed and non-condensed distributions, while real-space profiles do not.

Contents

- Double quantum well exciton systems.
- How to identify a coherent state - differences to atomic gas.
- Zero temperature; Thomas-Fermi profile.
- Finite temperature and phase fluctuations.
- Vortices.

Double quantum well exciton systems



Electron and hole physically separated; increased exciton lifetime

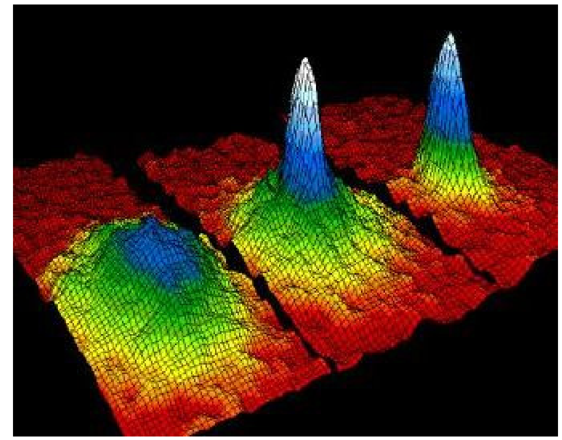
At low densities, behave as Bosons, interacting by fixed dipole moments. We model the interaction as

$$V(r) \approx \lambda\delta(r); \qquad \lambda = \frac{e^2 d}{2\epsilon_0}$$

We consider excitons in a localised trap.

Comparison to atomic gas

Atomic gas: Release from trap - measure momentum profile.
We cannot currently control traps in exciton system.



For a trap potential $U(r) = \alpha r^2$; thermal cloud $R \sim \sqrt{T/\alpha}$

Neglecting interactions, ground state of harmonic oscillator $R \sim \sqrt[4]{\hbar^2/m\alpha}$.

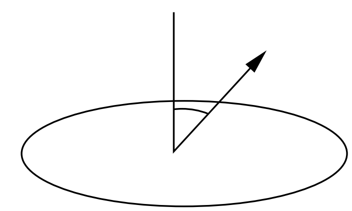
BUT, interactions cause this to spread, neglecting K.E. term $R \sim \sqrt[4]{\lambda N/\alpha}$

Interacting ground state is large, and indistinguishable from thermal state.

Angle resolved spectroscopy

Profile seen by recombination. Excitons confined in z direction, so only in-plane momentum ($\mathbf{k}_{||}$) is conserved.
Photons all at frequency $\omega_0 = k_0 c$, at angle θ to normal: $k_{||} = k_0 \sin(\theta)$ and,

$$\begin{aligned} I(\mathbf{k}_{||}) &\propto N_{ex}(\mathbf{k}_{||})\rho_{photon}(\mathbf{k}_{||}, k_0) \left| \langle f | \, \mathbf{k}_{||}.\hat{\mathbf{p}} \, | i \rangle \right|^2 \\ &\propto \left\langle \psi(\mathbf{k}_{||})^* \psi(\mathbf{k}_{||}) \right\rangle \times \left(\frac{1 + cos^2(\theta)}{cos(\theta)} \right) \end{aligned}$$



Need to calculate $\left\langle \psi(\mathbf{k}_{||})^* \psi(\mathbf{k}_{||}) \right\rangle = \sum_{\mathbf{r},\mathbf{r}'} \langle \psi(\mathbf{r})^* \psi(\mathbf{r}') \rangle \, e^{i\mathbf{k}_{||}.(\mathbf{r}-\mathbf{r}')}$
Without a condensate, broad distribution up to $k_{||} \approx \sqrt{2mk_BT}/\hbar$

Calculating the zero temperature limit

Assume all particles in identical state, using contact interaction

$$H[\Psi] = \int d^2r \Psi^*(r) \left(-\frac{\nabla^2}{2m} + U(r) - \mu \right) \Psi(r) + \lambda |\Psi(r)|^4$$

Neglecting kinetic term (large only at edge); $\Psi(r) = \sqrt{\rho(r)}$.
For a parabolic trap

$$\rho(r) = \frac{\alpha}{\lambda}(R^2 - r^2) \qquad R \sim \sqrt[4]{\lambda N/\alpha}$$

The momentum dependence has a peak of width $1/R$. If $1/R < k_0$ this produces a visible peak in angular distribution.

Finite temperature effects

Consider fluctuations on top of previous ground state, parameterise as $\psi(r) = \sqrt{\rho(r) + \pi(r)}e^{i\phi(r)}$

Low energy excitations are phase fluctuations. These destroy the coherence observed in momentum distribution (can neglect density fluctuations).

$$\begin{aligned}\left\langle \psi(\mathbf{k}_{||})^* \psi(\mathbf{k}_{||}) \right\rangle &= \int d^2\mathbf{r} d^2\mathbf{r}' \sqrt{\rho(\mathbf{r})\rho(\mathbf{r}')} \left\langle e^{i(\phi(r)-\phi(r))} \right\rangle e^{i\mathbf{k}_{||}.(\mathbf{r}-\mathbf{r}')} \\ &= \int d^2\mathbf{r} d^2\mathbf{r}' \sqrt{\rho(\mathbf{r})\rho(\mathbf{r}')} e^{-\left\langle (\phi(r)-\phi(r))^2 \right\rangle /2} e^{i\mathbf{k}_{||}.(\mathbf{r}-\mathbf{r}')}\end{aligned}$$

Need to find $G_\phi(r, r') = \langle \phi(r)\phi(r') \rangle$

Path integral calculation

Calculate expectation weighted by action:

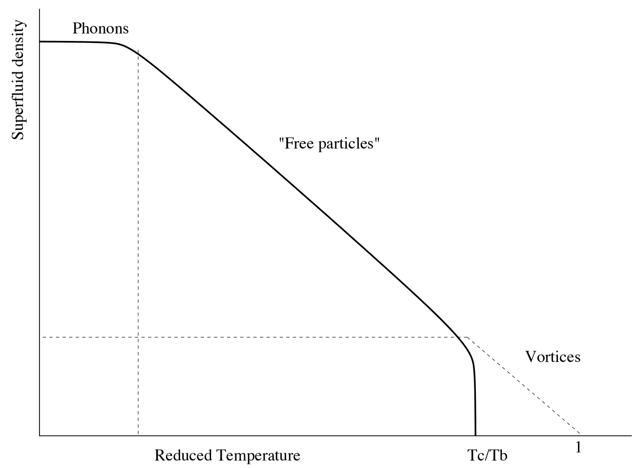
$$\langle \ldots \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi(r) \ldots e^{-S[\psi]} \qquad S[\psi] = \int_0^\beta d\tau \int d^2 r \psi(r)^* i\partial_\tau \psi(r) + \mathcal{H}(\psi)$$

We expand the action in terms of $\pi(r)$ and $\phi(r)$, and Fourier transform to give:

$$S \approx \sum_k \sum_\omega \begin{pmatrix} \pi_k & \phi_k \end{pmatrix} \begin{pmatrix} \frac{\lambda}{2} + \frac{k^2}{8m\rho} & -\omega \\ \omega & \frac{\rho k^2}{2m} \end{pmatrix} \begin{pmatrix} \pi_k \\ \phi_k \end{pmatrix}$$

Inverting this, we find low energy terms are phase fluctuations

Two dimensional effects



Density v.s. temperature for two dimension gas; phonon-like excitations with small change in density dominate at low T. The transition is controlled by vortex pair unbinding.
As density varies across trap, nature of dominant fluctuations change; edge is vortex dominated.

Calculating the phase

For a uniform density profile in two dimensions;

$$\frac{1}{2}\left\langle \left(\phi(r)-\phi(r')\right)^2 \right\rangle = \frac{m}{2\pi\beta\rho}\ln\left(\frac{|r-r'|}{\xi_T}\right)$$

where $\xi_T = \sqrt{\frac{\lambda\rho}{4m}\frac{1}{k_B T}}$ is short lengthscale cutoff. We can split the logarithm as:

$$\frac{1}{2}\left(G_\phi(r',r') + G_\phi(r,r) - 2G_\phi(r,r')\right) = -\frac{m}{2\pi\beta\rho}\left(\ln\left(\frac{\xi_T}{R}\right) - \ln\left(\frac{|r-r'|}{R}\right)\right)$$

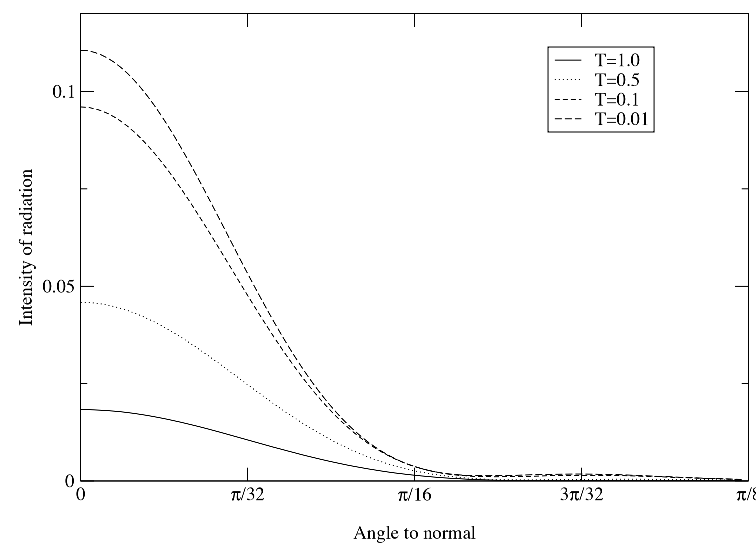
Sharp peaks in momentum are due to far separated points, for which the first term above dominates.

Variable density profile

If $\rho(r)$ varies, then $G_\phi(r,r')$ satisfies

$$-\frac{\beta}{m}\nabla\left(\rho(r)\nabla G_\phi(\mathbf{r},\mathbf{r}')\right)=\delta^2(\mathbf{r}-\mathbf{r}')$$

Can solve exactly for various profiles; in all cases logarithmic divergence as $\mathbf{r}\to\mathbf{r}'$. Including only $G_\phi(r,r)$ gives:



$$N_{ex}(\mathbf{k}_{||})=\left|\int d^2\mathbf{r}\sqrt{\rho(\mathbf{r})}\left(\frac{\lambda\rho}{m(k_BTR)^2}\right)^{mk_BT/2\pi\rho(\mathbf{r})}e^{i\mathbf{k}_{||}.\mathbf{r}}\right|^2$$

Vortices

When phase is coherent across the system, we can have vortices

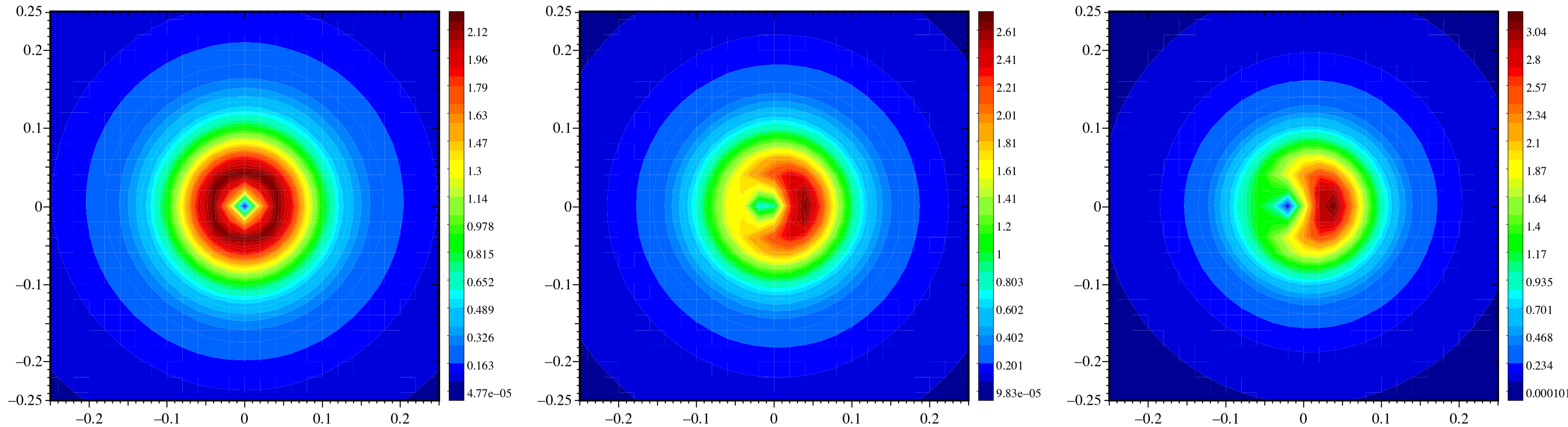
$$\psi(r) \approx \rho(r) \exp\left(i \tan^{-1}(y - y_0/x - x_0)\right)$$

We consider various configurations of vortices (at $T = 0$),

$$N_{ex}(\mathbf{k}_{||}) = \left| \int d^2\mathbf{r} \sqrt{\rho(\mathbf{r})} \exp\left(i \sum_j \tan^{-1}(y - y_j/x - x_j) \right) e^{i\mathbf{k}_{||}.\mathbf{r}} \right|^2$$

Moving a vortex

As we move a vortex in the y direction, the momentum distribution changes as follows.



(Note that now the angular distribution depends on azimuthal as well as polar angle).

Vortex arrays

We can also consider triangular arrays of vortices (19 vortices in a Gaussian density profile).