Inside Quantum Devices

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Outline

Introduction

- General view of a solid-state quantum computer.
- Trapped electrons in semiconductors.

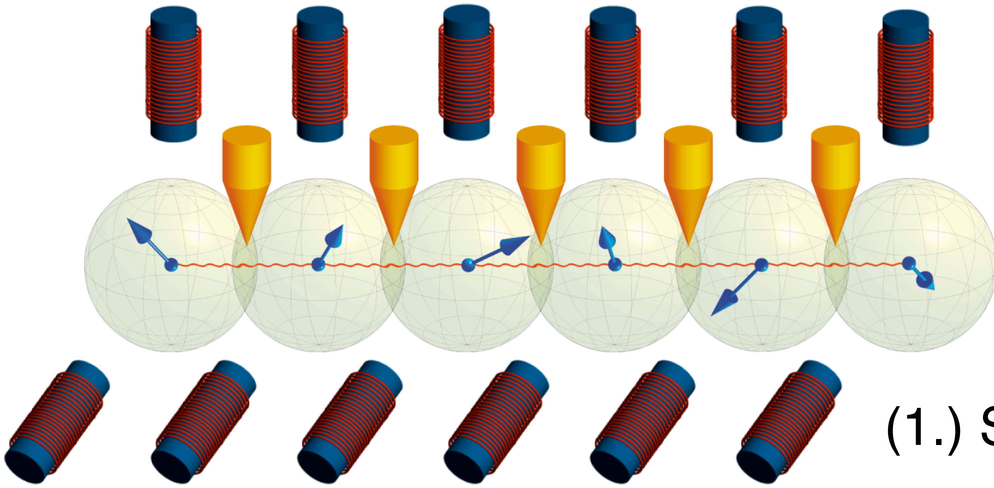
Quantum devices

- Selective spin decoherence.
- Spin valve.

Quantum processors

- Na in Si.
- Electrons in SAWs.

Requirements for a practical quantum computer



(1.) Scalable physical system with
well characterised qubits.

(2.) A universal set of quantum logic gates

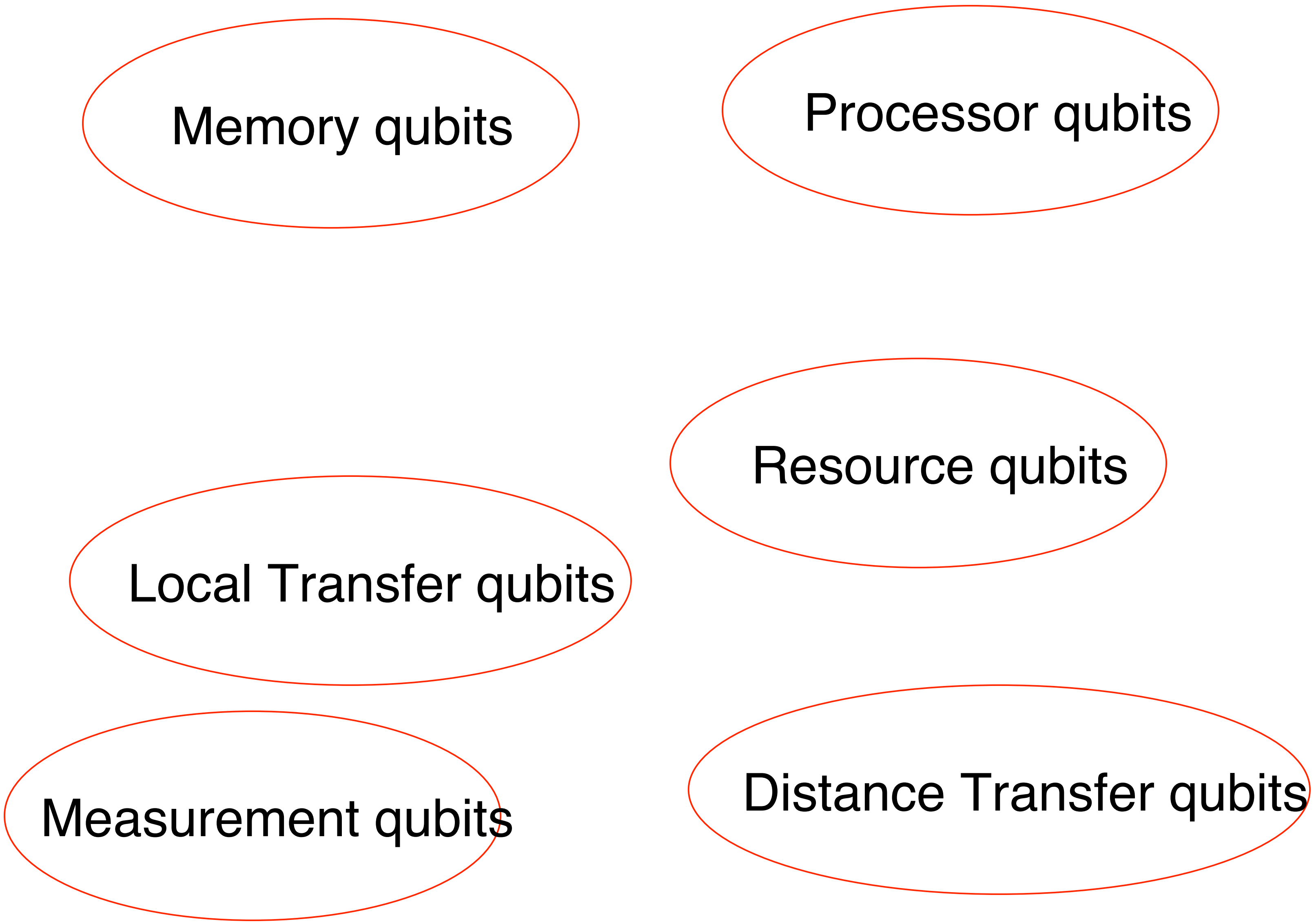
(3.) The ability to initialise each qubit.

(4.) The ability to measure each qubit.

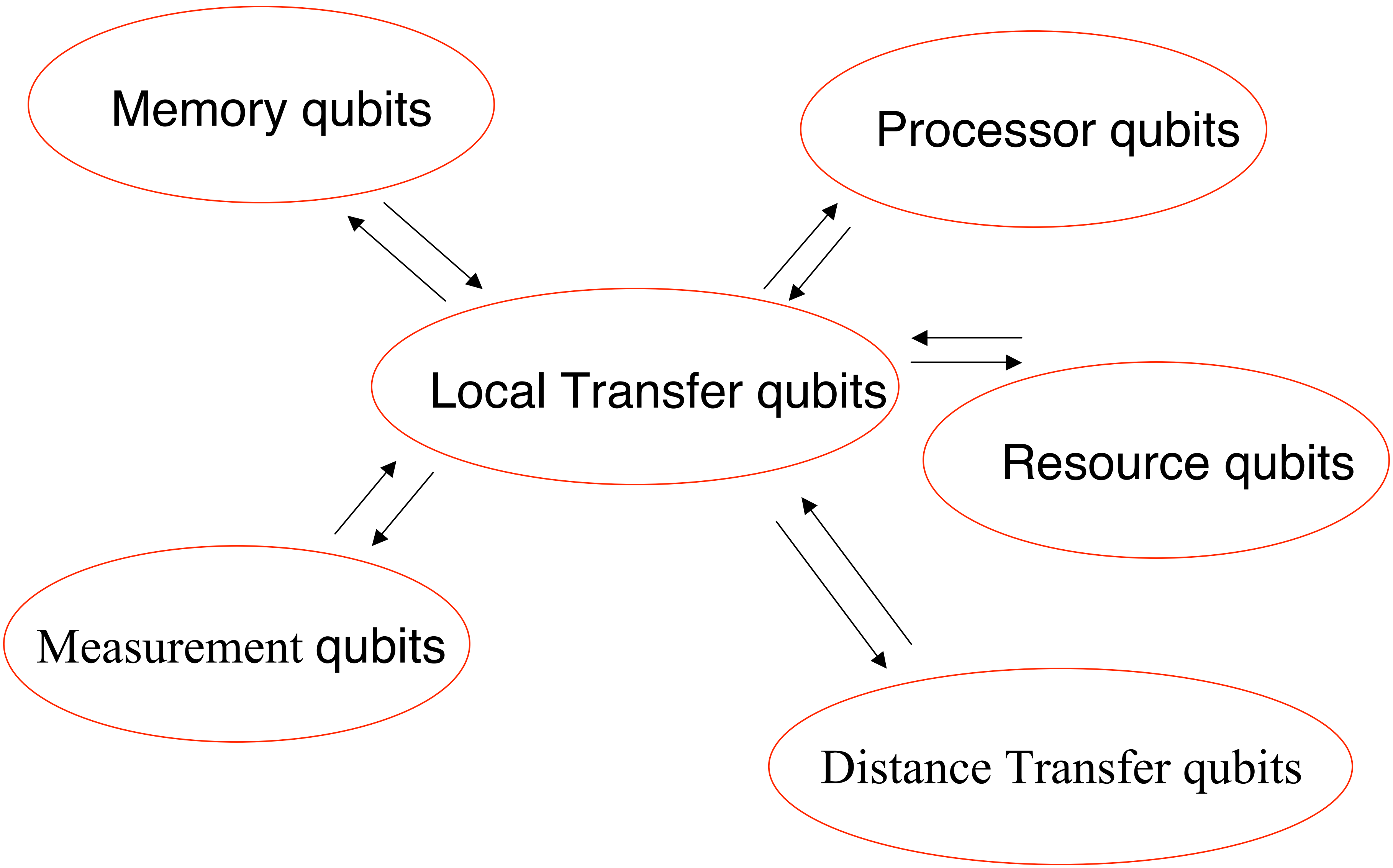
(5.) Decoherence times much longer
than each logic gate operation time.

D. P. DiVincenzo quant-ph/0002077

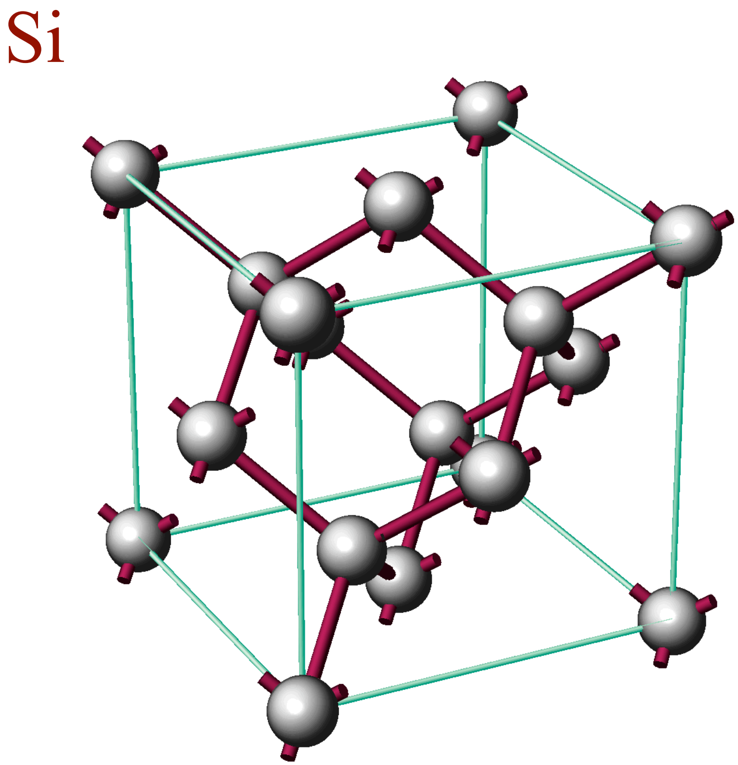
Quantum Computer



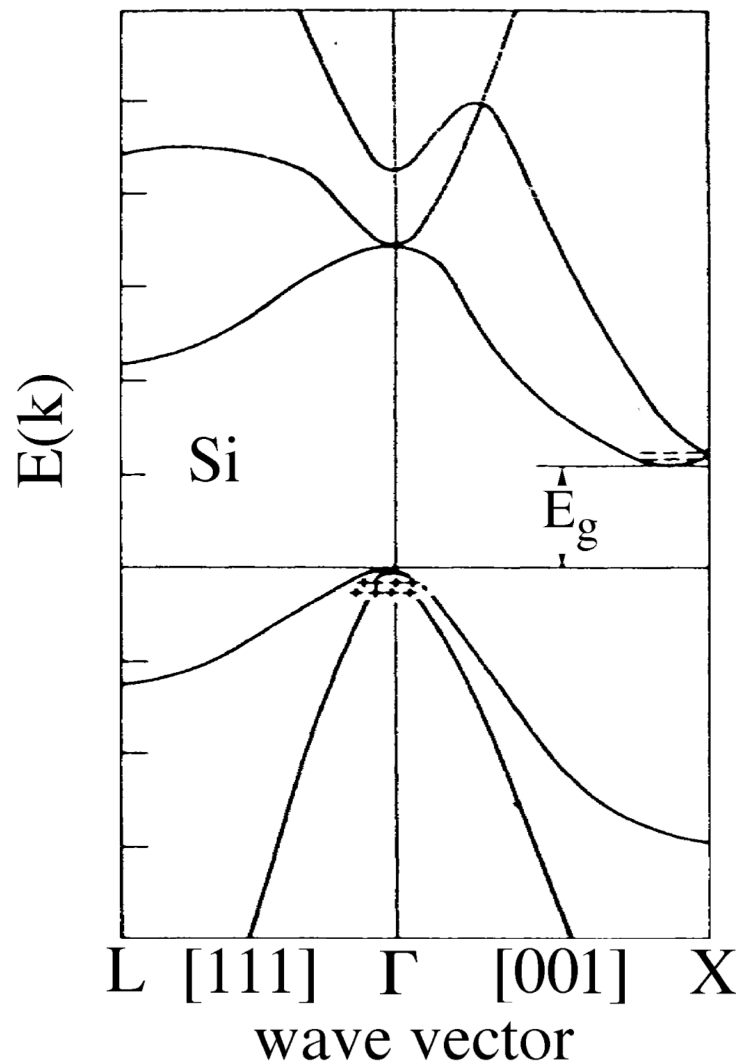
Quantum Computer



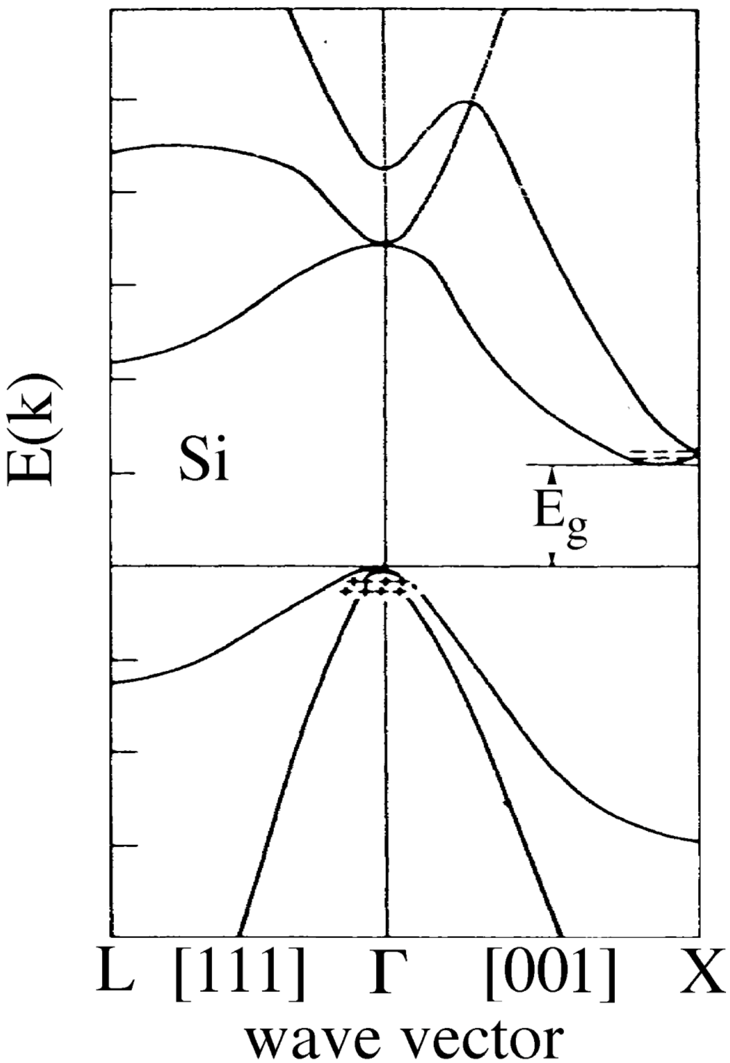
Electrons in semiconductors

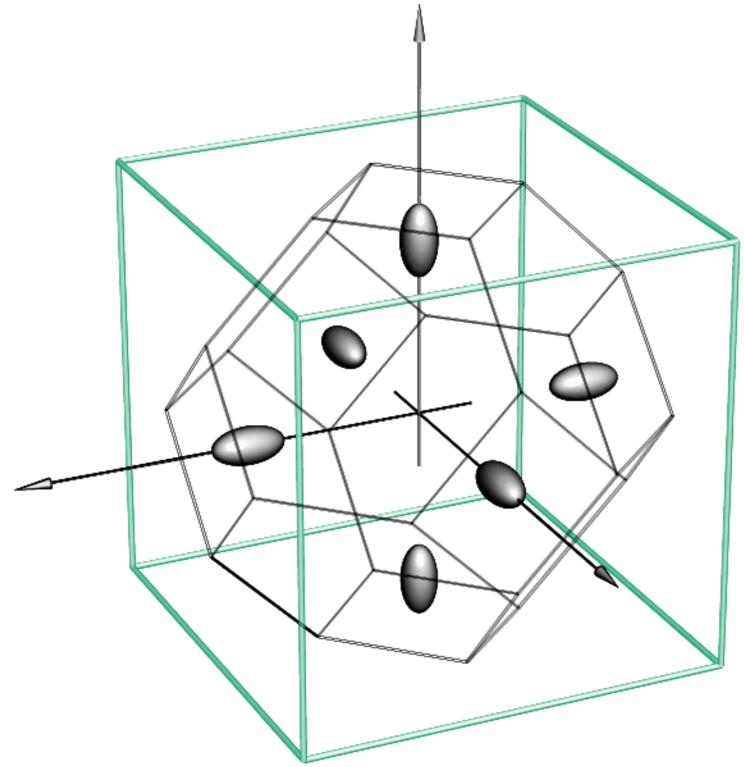


$$|\psi\rangle \sim e^{ik.r}$$



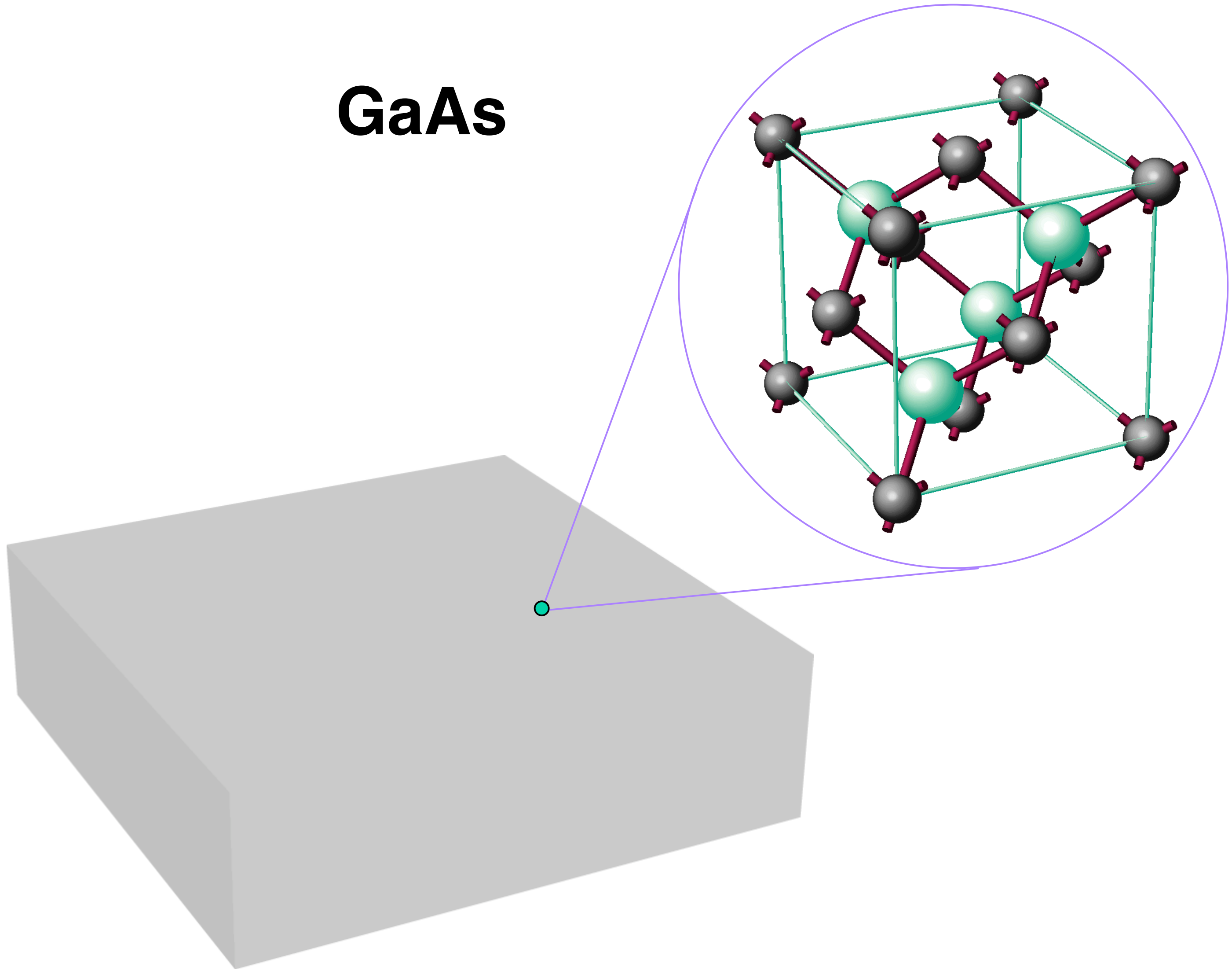
Electrons in semiconductors



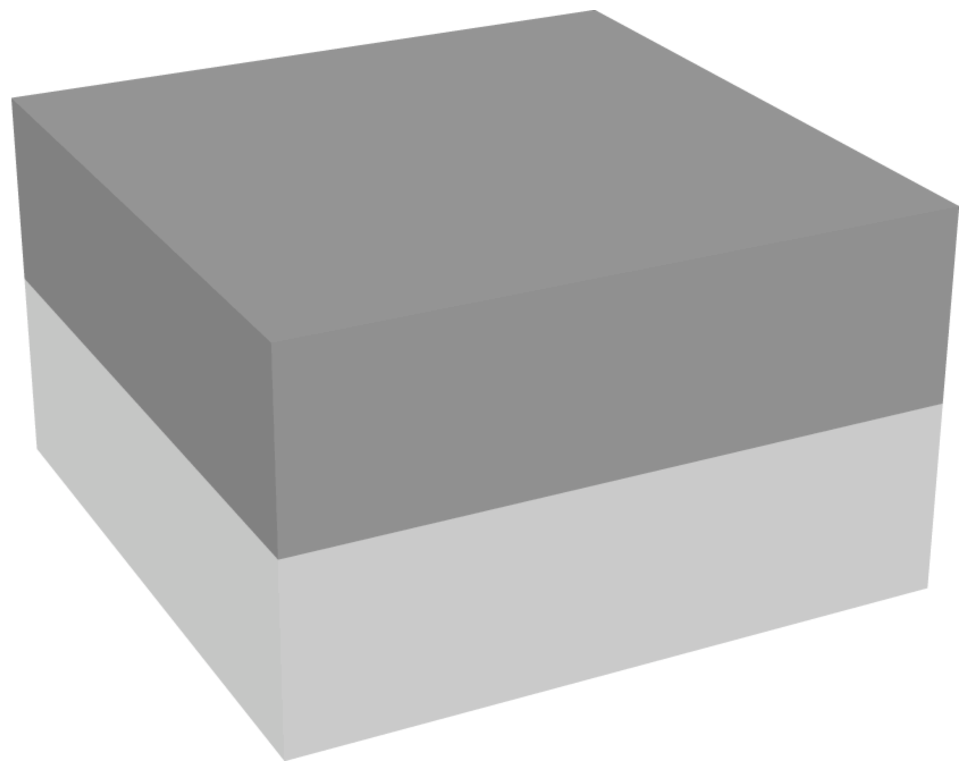


$$E_k = \sum_{i,j} \frac{\hbar^2}{2} \left(m^{*-1}\right)_{i,j} k_i k_j$$

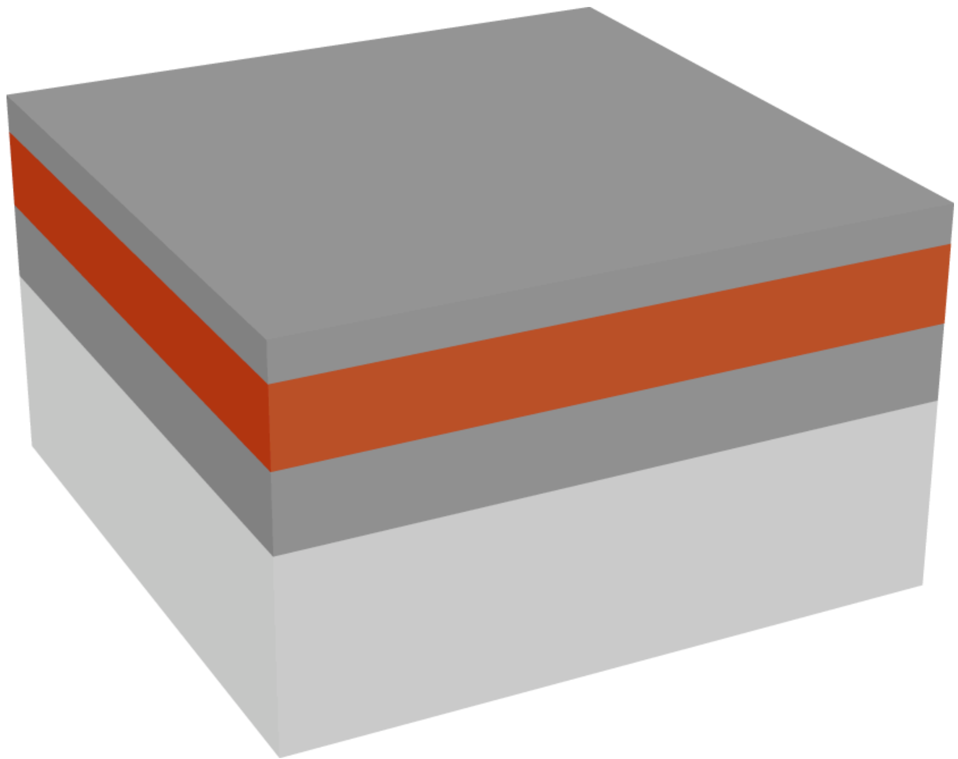
GaAs



GaAs – AlGaAs heterostructure



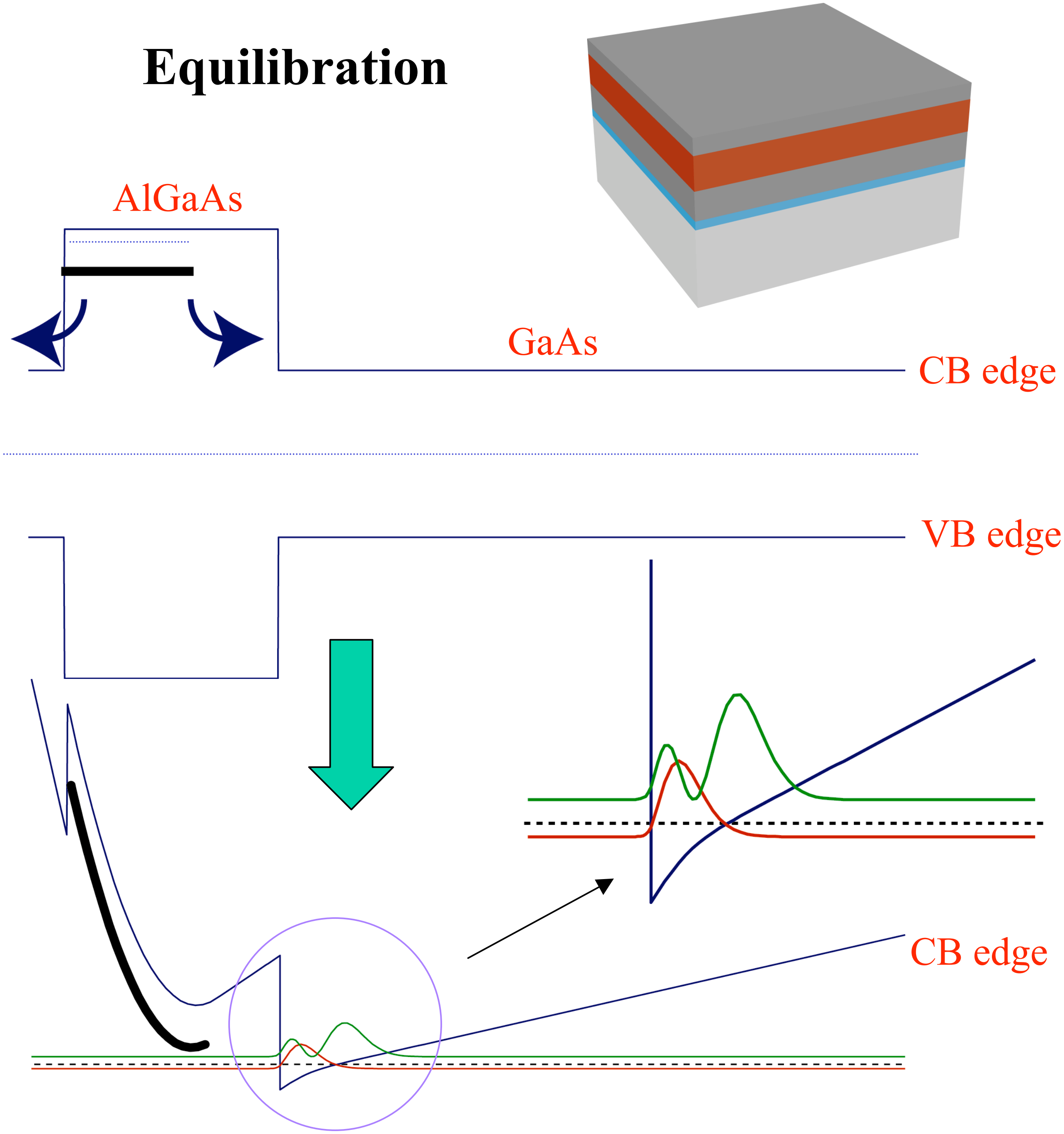
Modulation doping



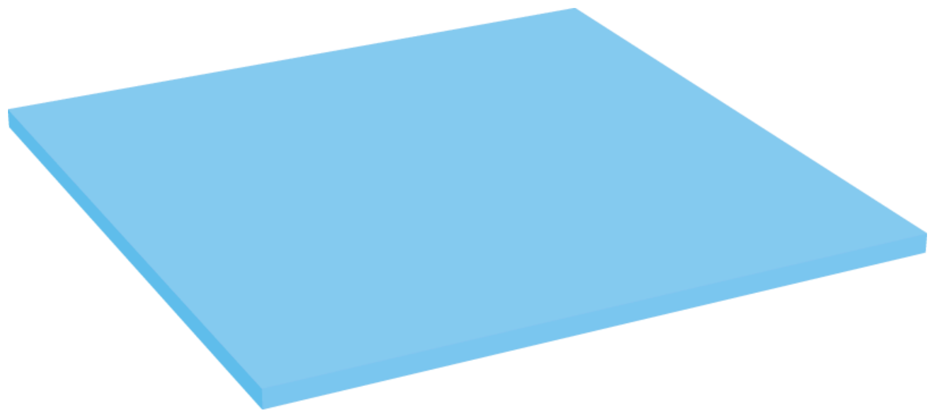
Equilibration

GaAs-AlGaAs heterostructure

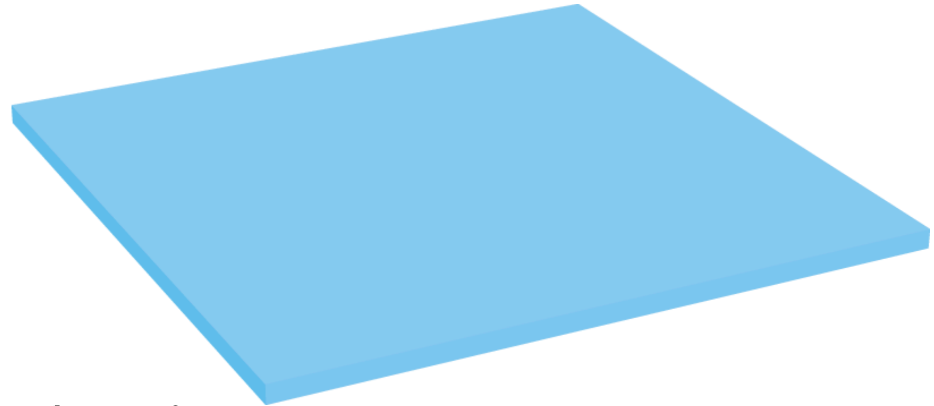
Conduction band edge after equilibration



The two-dimensional electron system



Quasi-particles

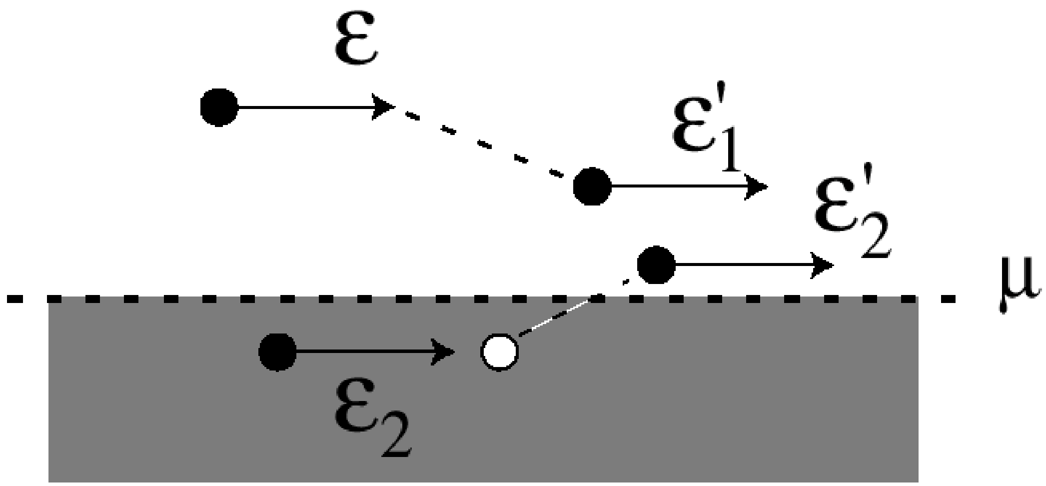


$$H = \sum_{i=1}^{N} \left(\frac{(p_i + eA_i)}{2m^*} + V_i(r_i) \right) + \sum_{i \neq j = 1}^{N} V(r_i, r_j)$$

$$\begin{aligned} R_i &= R_i(r_1, ..., r_N, p_1, ..., p_N) \\ P_i &= P_i(r_1, ..., r_N, p_1, ..., p_N) \\ A'_i &= A_i + \nabla \phi_i \end{aligned}$$

$$H = \sum_{i=1}^{N} \left(\frac{(p_i + e\alpha_i)}{2m^*} + V_i(R_i) \right) + \sum_{i \neq j = 1}^{N} V(R_i, R_j, P_i, P_j)$$

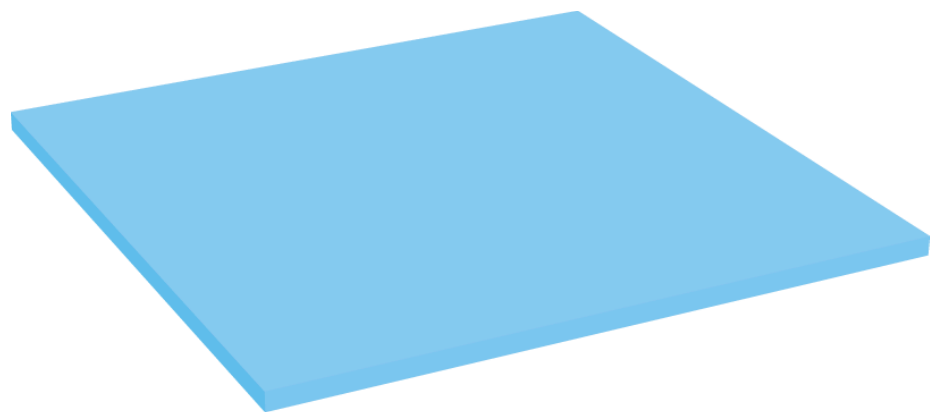
Quasi-Particle life time



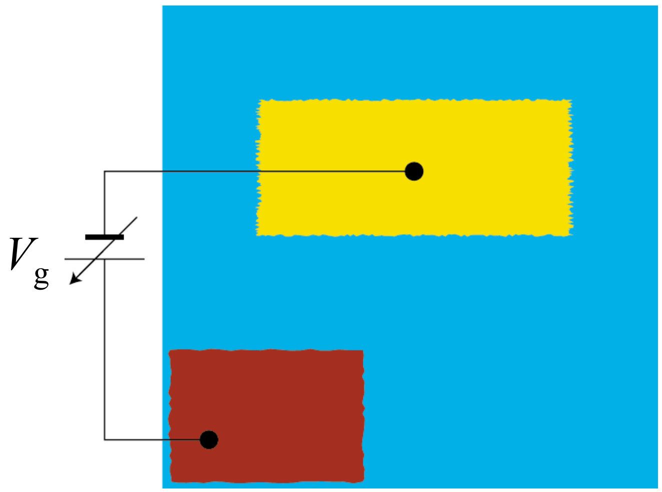
$$\frac{1}{\tau_\epsilon} \propto \int \delta(\epsilon+\epsilon_2-\epsilon_1'-\epsilon_2') \left\{\rho_2(\epsilon_2) f_2^0 d\epsilon_2\right\} \left\{\rho_1(\epsilon_1')(1-f_{1'}^0) d\epsilon_{1'}\right\} \left\{\rho_2(\epsilon_{2'})(1-f_{2'}^0) d\epsilon_{2'}\right\}$$

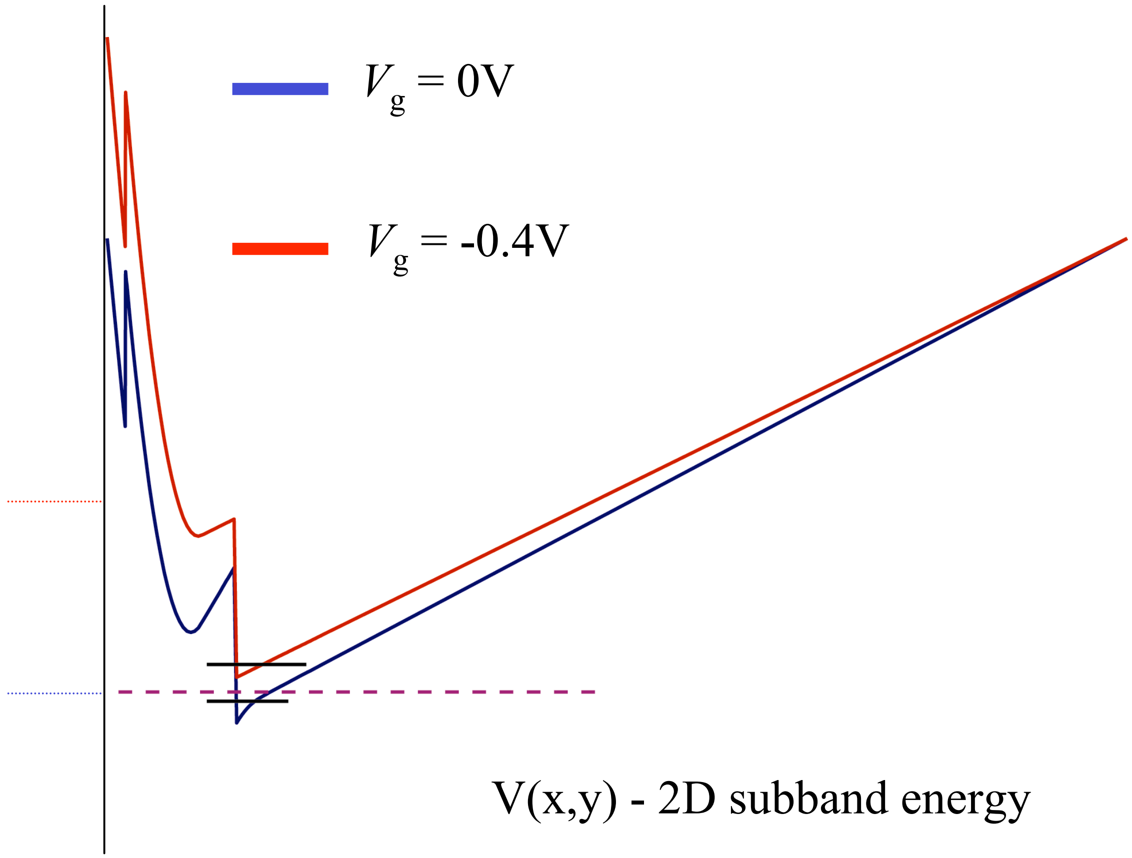
$$\propto \quad \rho^3(\mu) \frac{(\pi k_B T)^2 + (\epsilon-\mu)^2}{1+\exp\left(\frac{\epsilon-\mu}{k_B T}\right)}$$

The two-dimensional quasi-particle gas



Electrostatic surface-gates

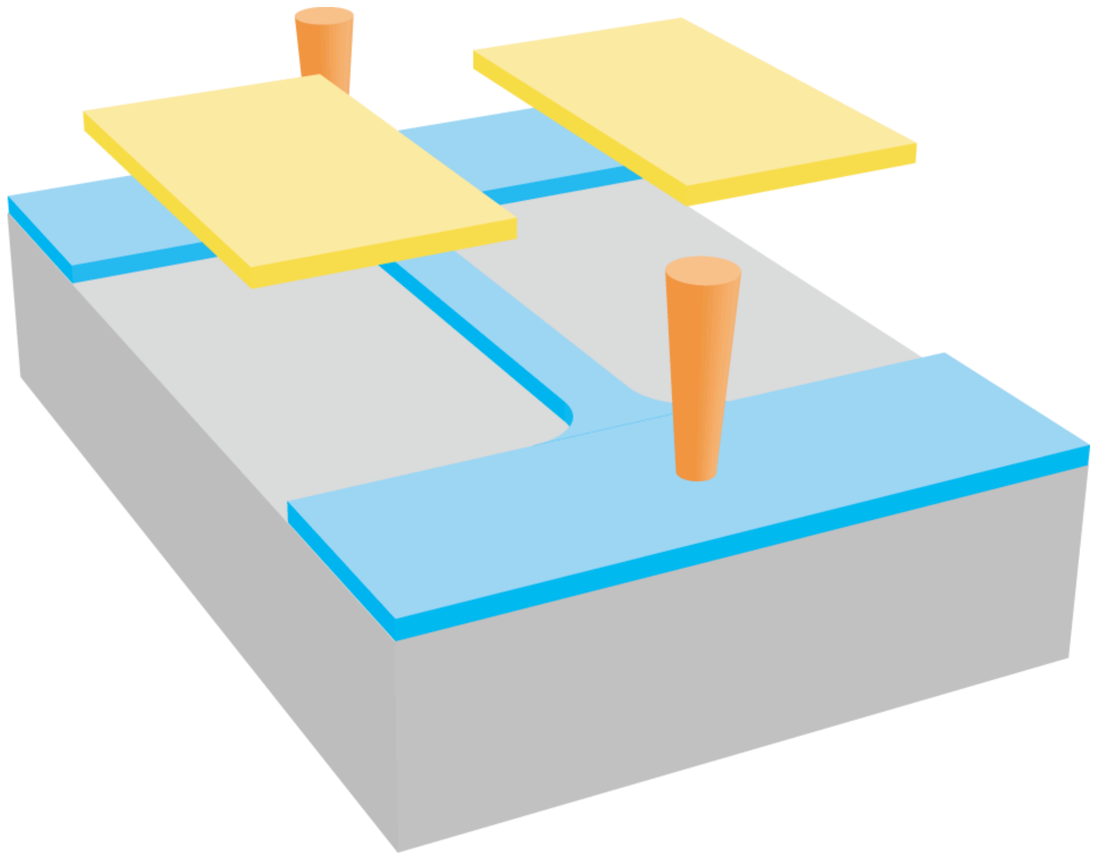




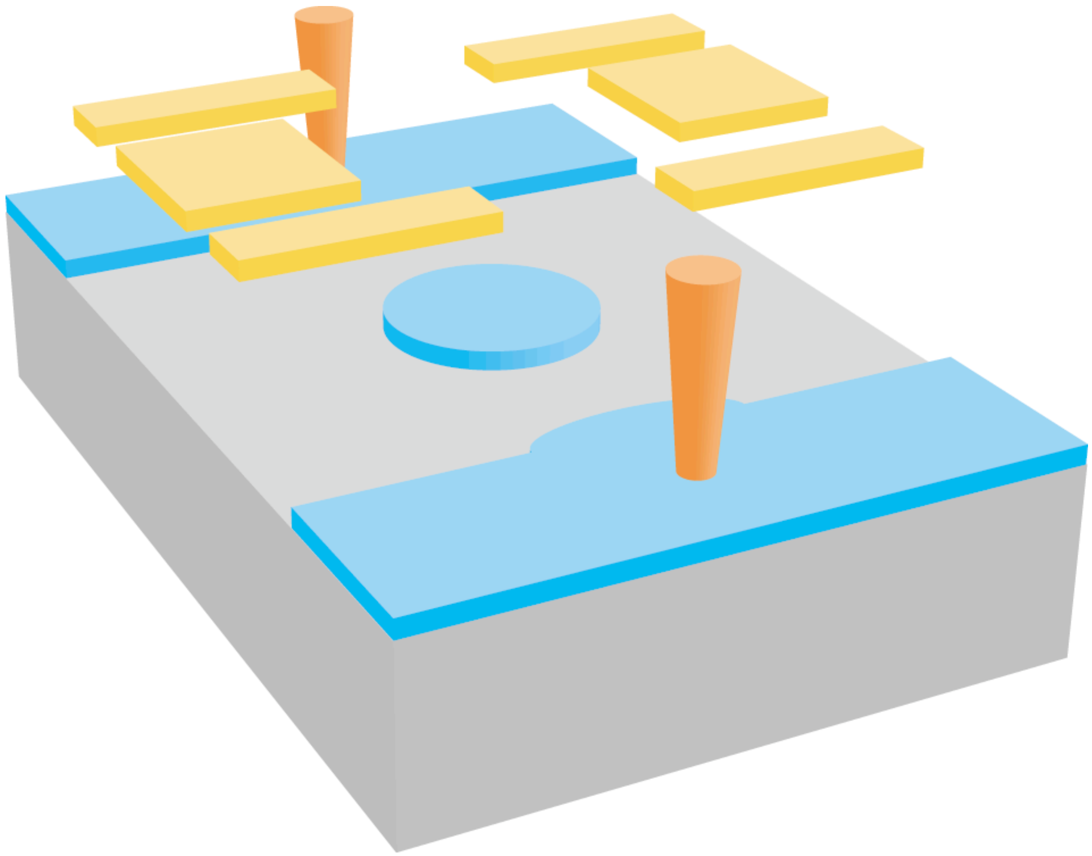
2D Schroedinger equation in the effective mass approximation

$$-\frac{\hbar^2}{2m^*}\nabla^2\Psi + V(x,y)\Psi = E\Psi$$

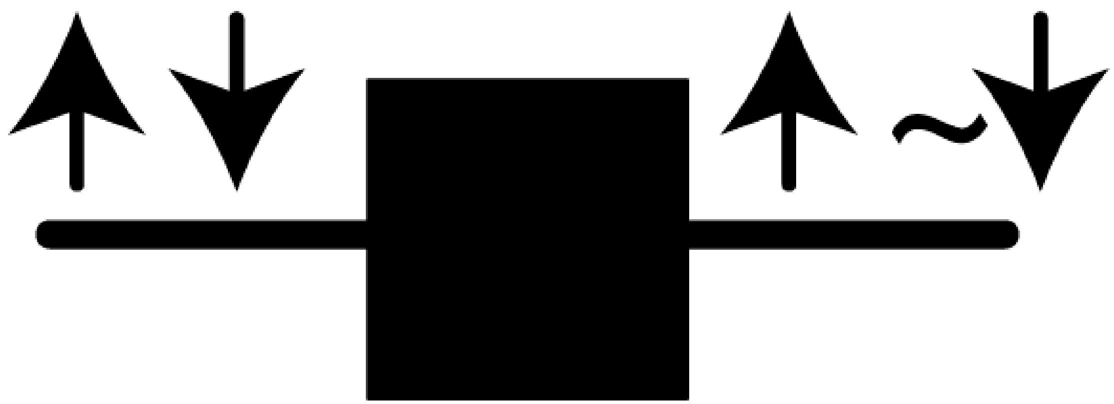
The quantum wire



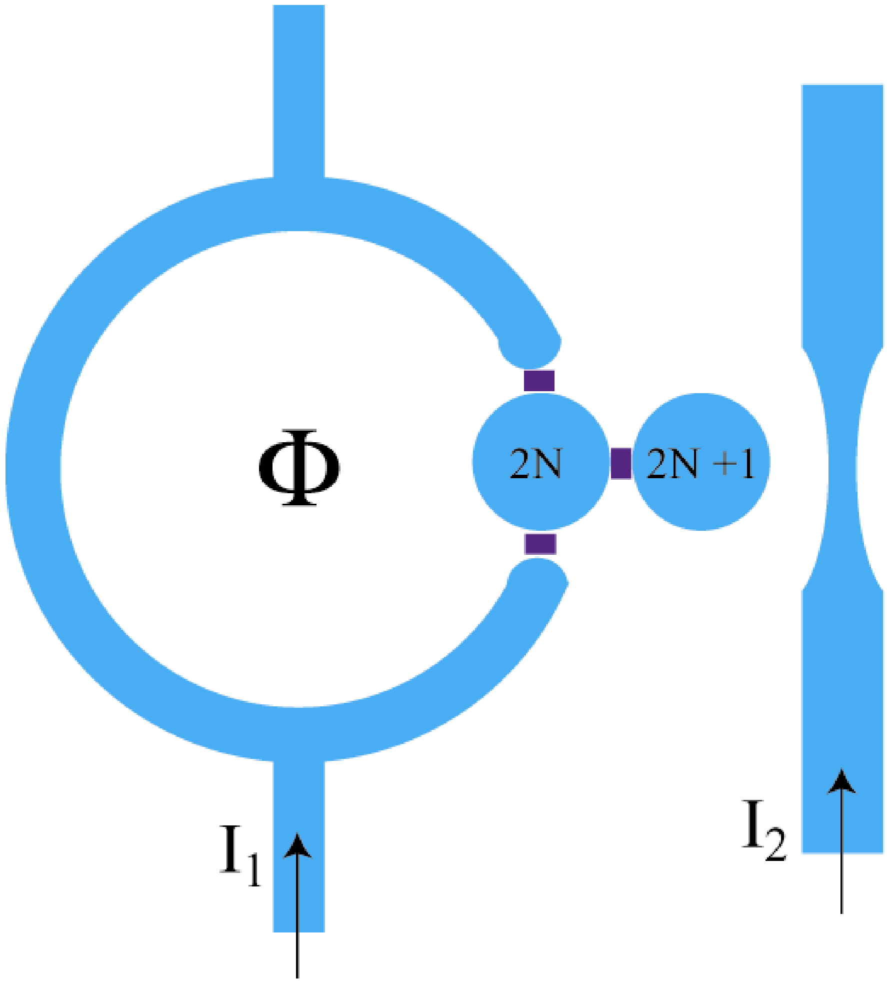
The quantum dot



Device I: Selective spin decoherence



Spin-Decoherence from non-invasive detection: the idea.

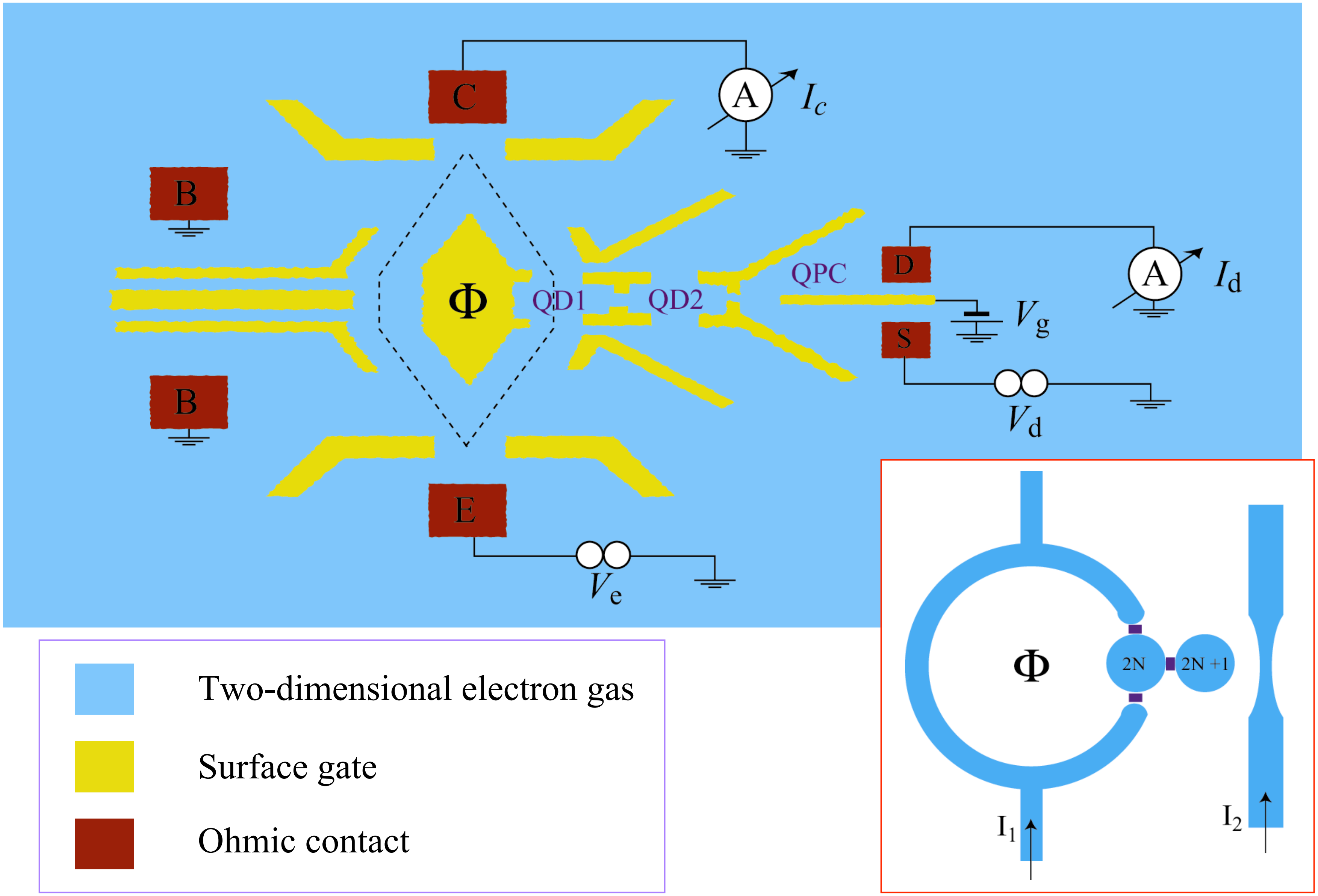


Outline

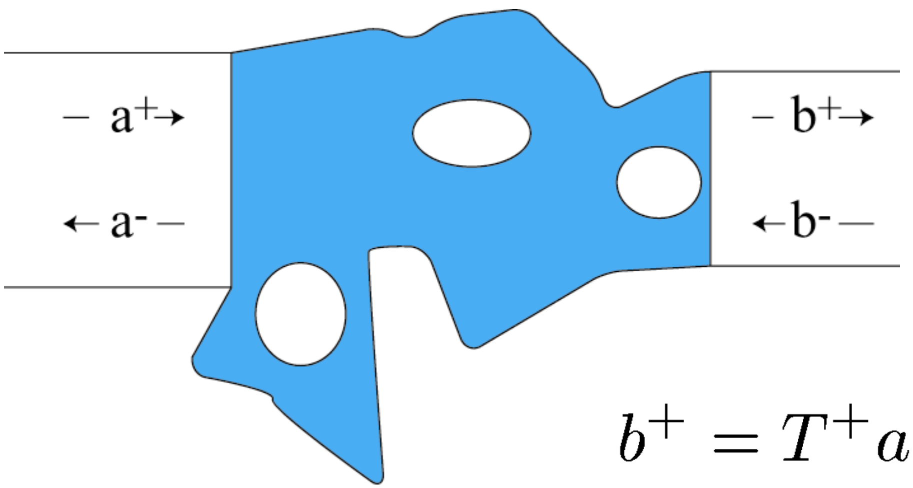
- Experimental implementation.
- Landauer conductance formalism.
- Quantised ballistic conductance.
- Aharonov-Bohm effect.
- Quantum dots.
- Non-invasive detection.
- Orbital decoherence from non-invasive detection.
- Double quantum dots.
- Spin-decoherence from non-invasive detection.

Based on the which-path detector in: **E. Buks *et al* 391 872 Nature (1998)**

Spin-Decoherence from non-invasive detection: the devic



Conductance: Landauer formalism



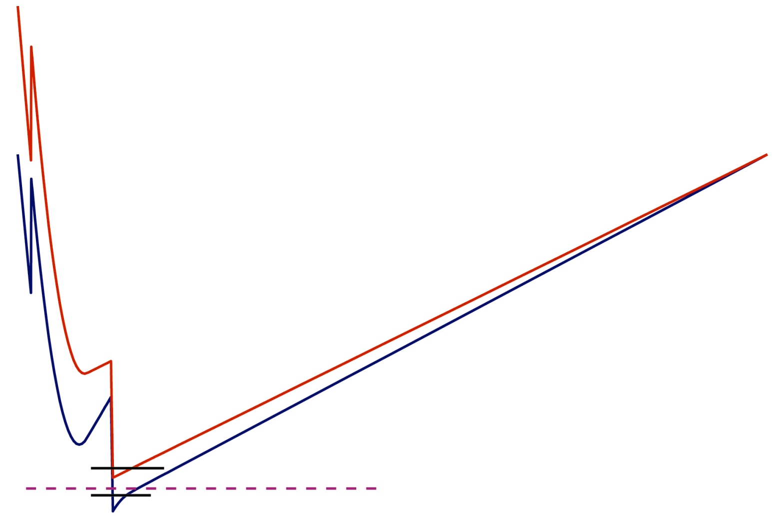
$$b^+ = T^+ a^+ + R^+ b^-$$
$$a^- = R^- a^+ + T^- b^-$$

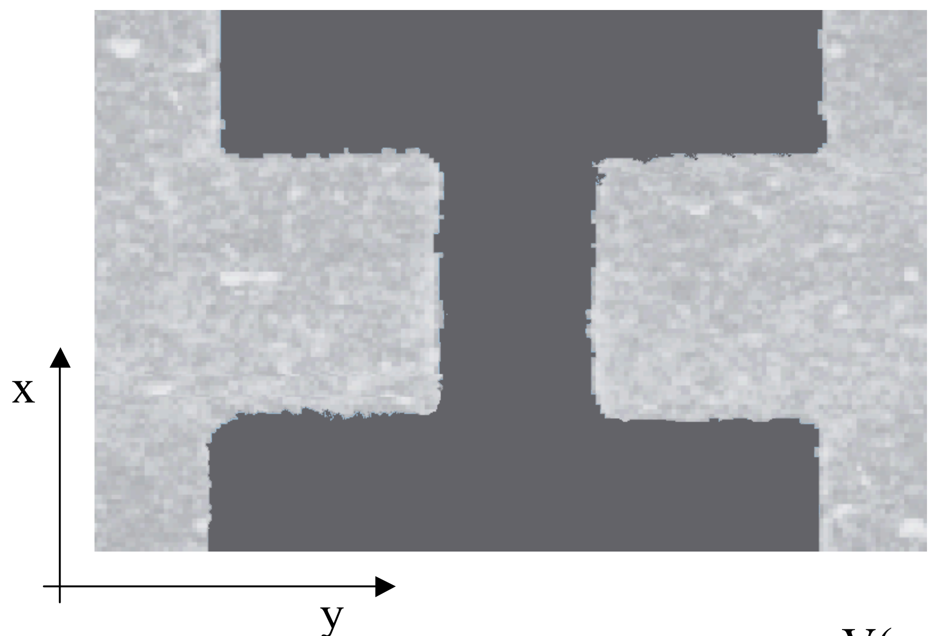
Equilibrium/ Linear response/ Low temperature.

$$G = \frac{e^2}{h} \sum_{i,j=1}^{N} \sum_{\sigma,\mu=\pm} \left| T^+_{i\sigma,j\mu} \right|^2$$

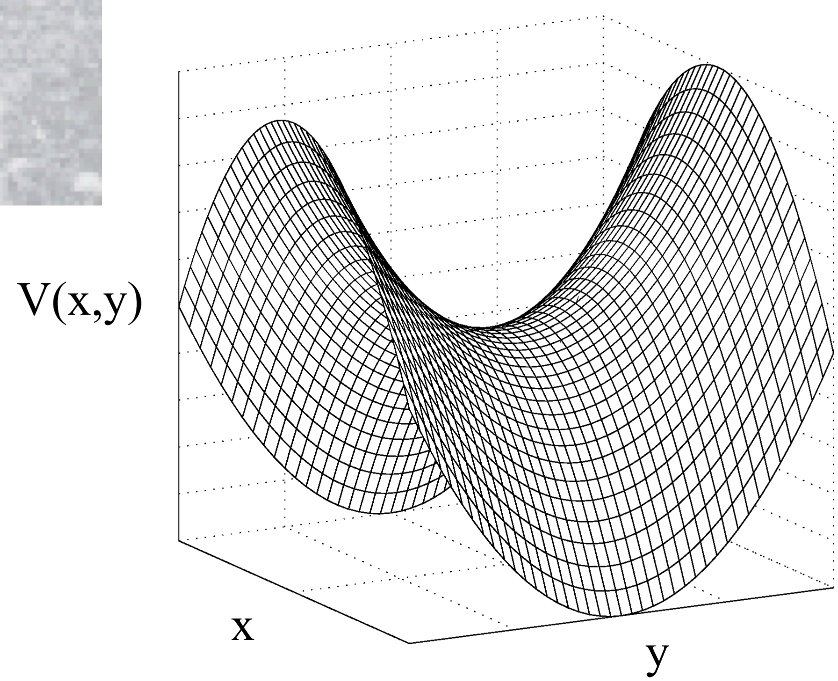
Transmission coefficients are evaluated at the device chemical potential

Quasi-1D systems.

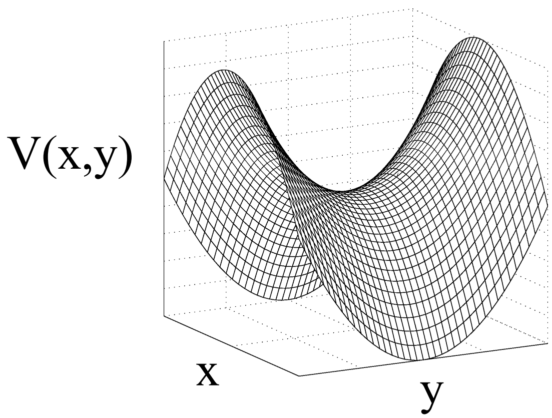


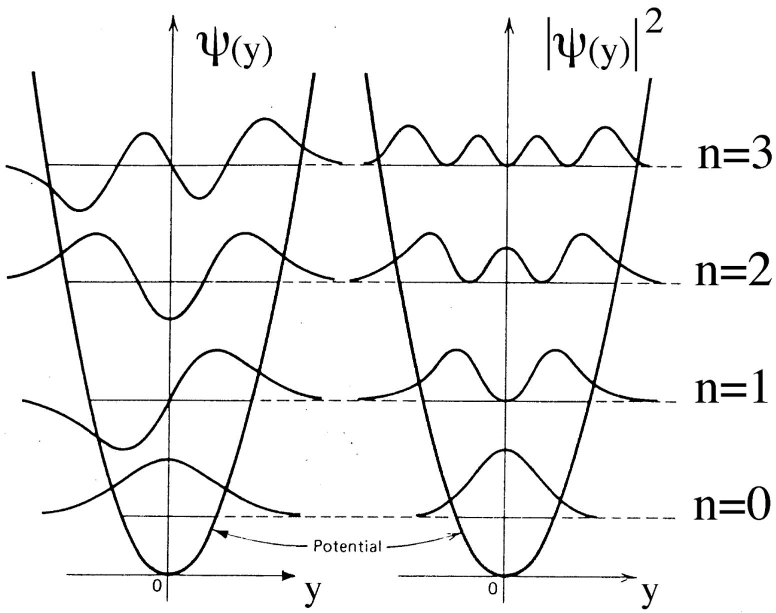


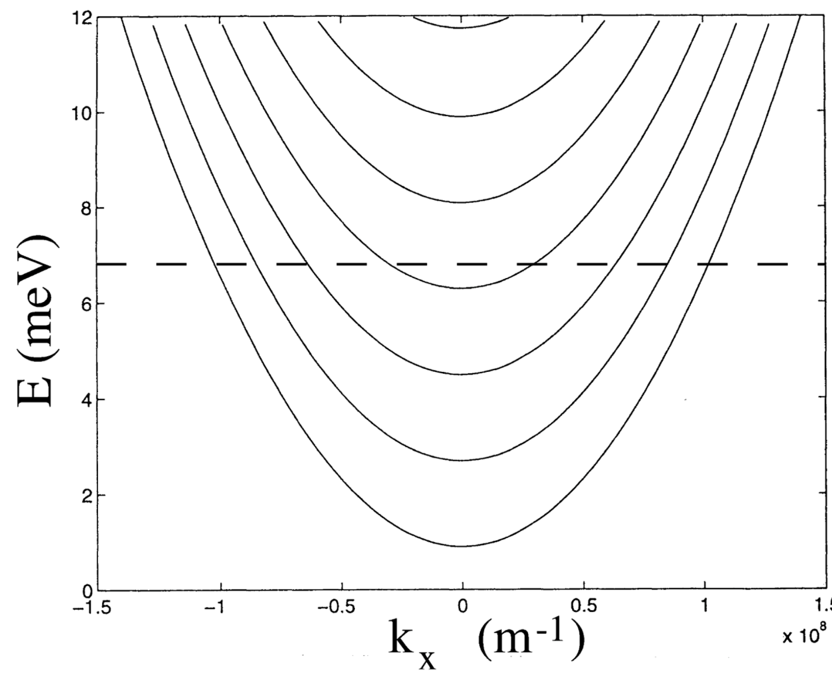
SEM image of a typical surface-gate pattern for a 1D system



Quantum states in quasi-1D systems.



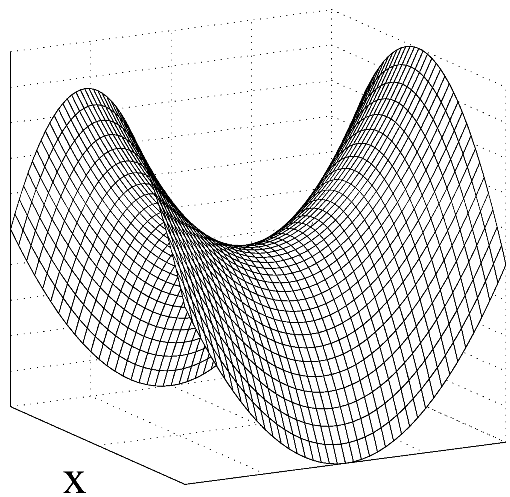




Dispersion:

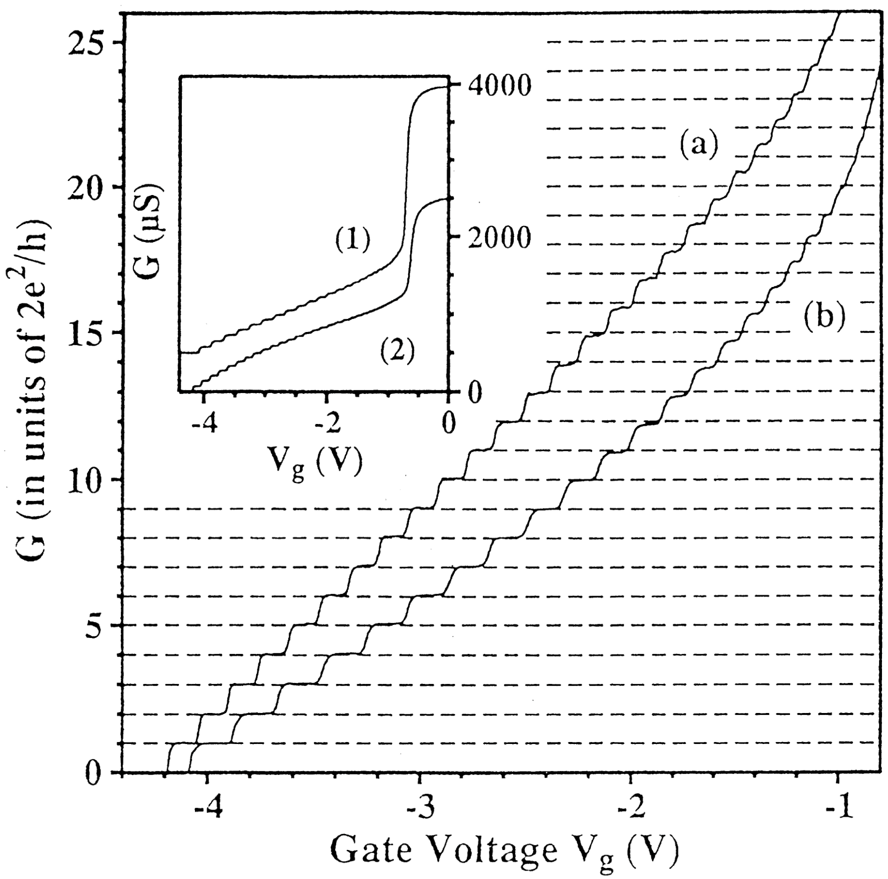
$$E_i = \frac{\hbar^2 k^2}{2m^*} + E_{i,0}$$

Ballistic quantisation



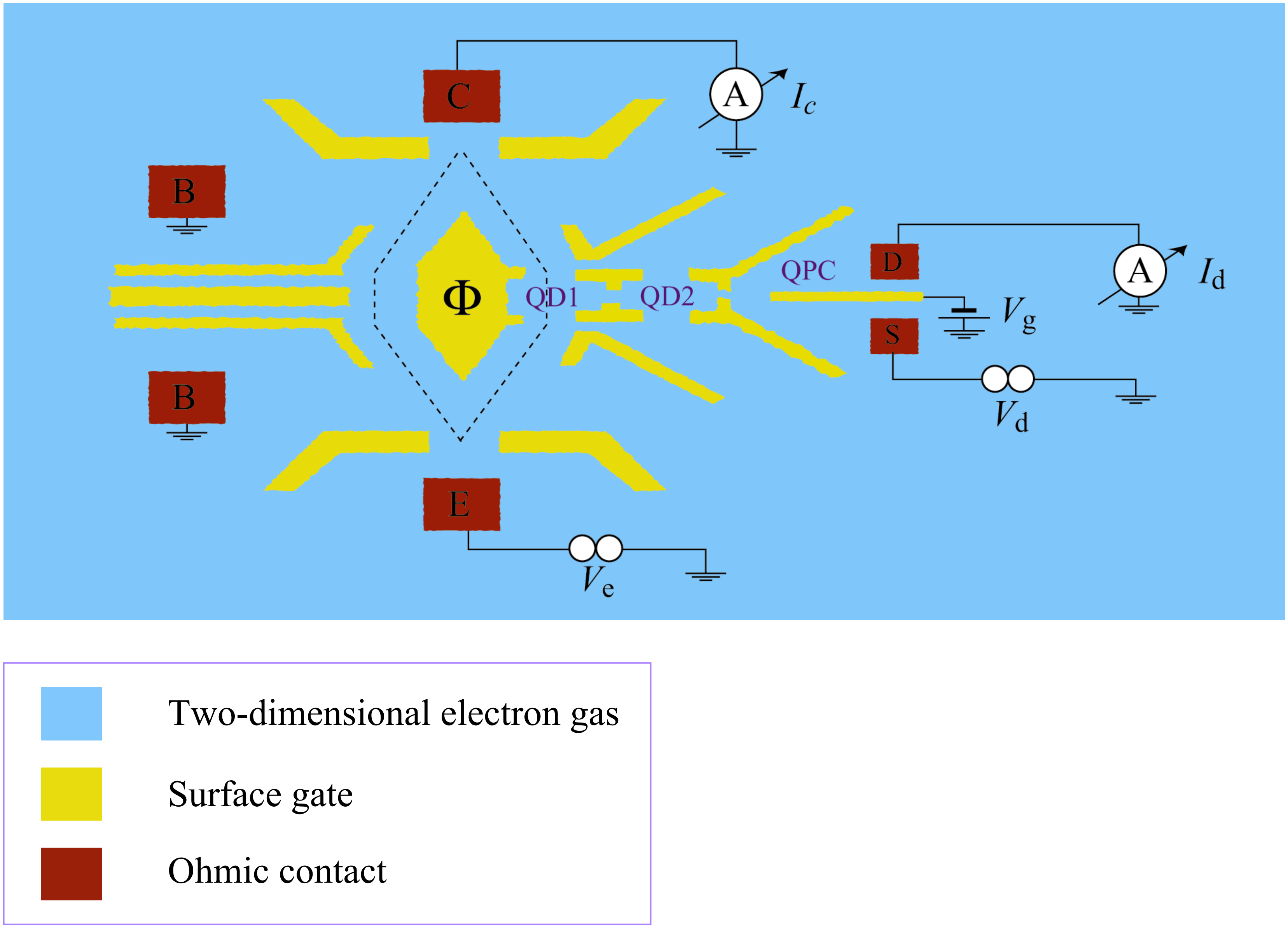
$$T^{+}_{i\sigma,j\mu} = \delta_{i,j}\delta_{\sigma,\mu}\frac{1}{1+\exp\left(-\pi\epsilon_i\right)}$$

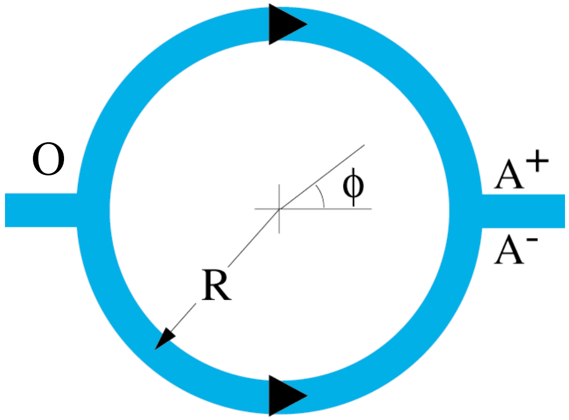
$$G = 2\frac{e^2}{h}\sum_{i=1}^{N}\left|\frac{1}{1+\exp\left(-\pi\epsilon_i\right)}\right|^2$$



K. J. Thomas *et al* Appl. Phys. Lett. 67 109 (1995).
M. Buttiker Phys. Rev. B 41 7906 (1990).

Detector circuit: Aharonov-Bohm effect.





Aharonov-Bohm effect: Theory

Conductance

$$t_{OA} \sim t_{OA^+} + t_{OA^-} \sim e^{i\phi_{A^+}}\eta_+ + e^{i\phi_{A^-}}\eta_-$$

$$G = \frac{e^2}{h}|t_{OA}|^2 \sim 2 + \eta_-^\dagger \eta_+ e^{i(\phi_{A^+}-\phi_{A^-})} + \eta_+^\dagger \eta_- e^{-i(\phi_{A^+}-\phi_{A^-})}$$

Spin part

Orbital part

WKB approximation

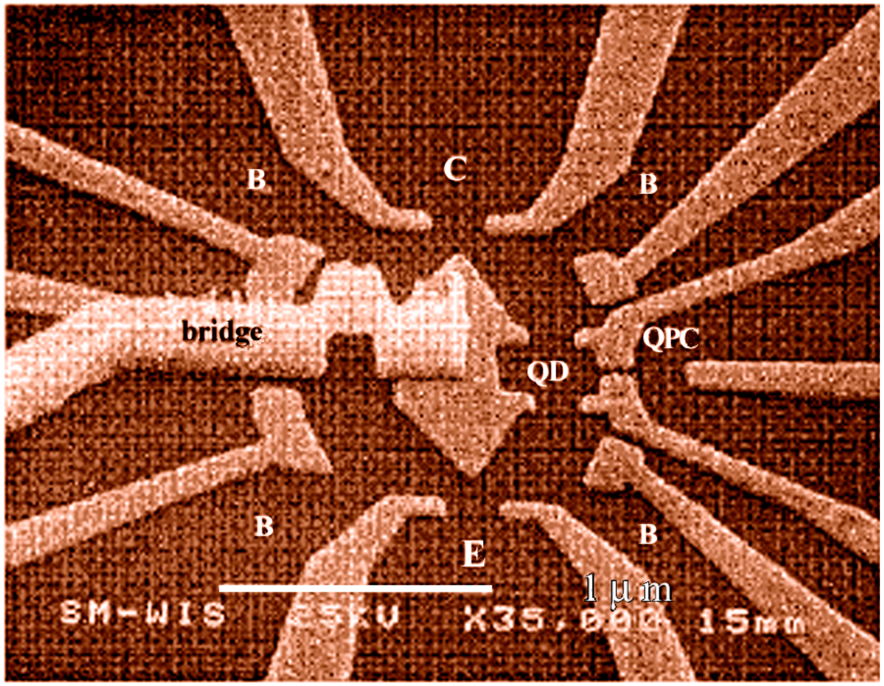
$$\phi_{A^+} = \int_\pi^0 k.dl = \int_\pi^0 \frac{(p+eA)}{\hbar}.dl = \frac{2\pi pR}{\hbar} + \pi\frac{e}{h}BS$$

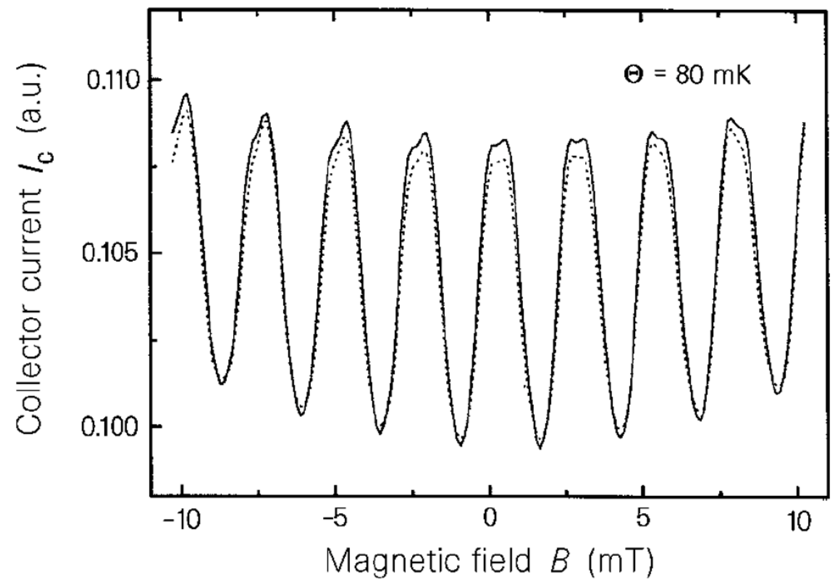
$$\phi_{A^-} = \int_{-\pi}^0 k.dl = \int_{-\pi}^0 \frac{(p+eA)}{\hbar}.dl = \frac{2\pi pR}{\hbar} - \pi\frac{e}{h}BS$$

$$\phi_{A^+} - \phi_{A^-} = 2\pi\frac{e}{h}BS$$

$$G \sim 1 + \cos(\xi_+ - \xi_-)\cos\left(2\pi\frac{e}{h}BS - (\alpha_+ - \alpha_-)\right)$$

Aharonov-Bohm: Experiment



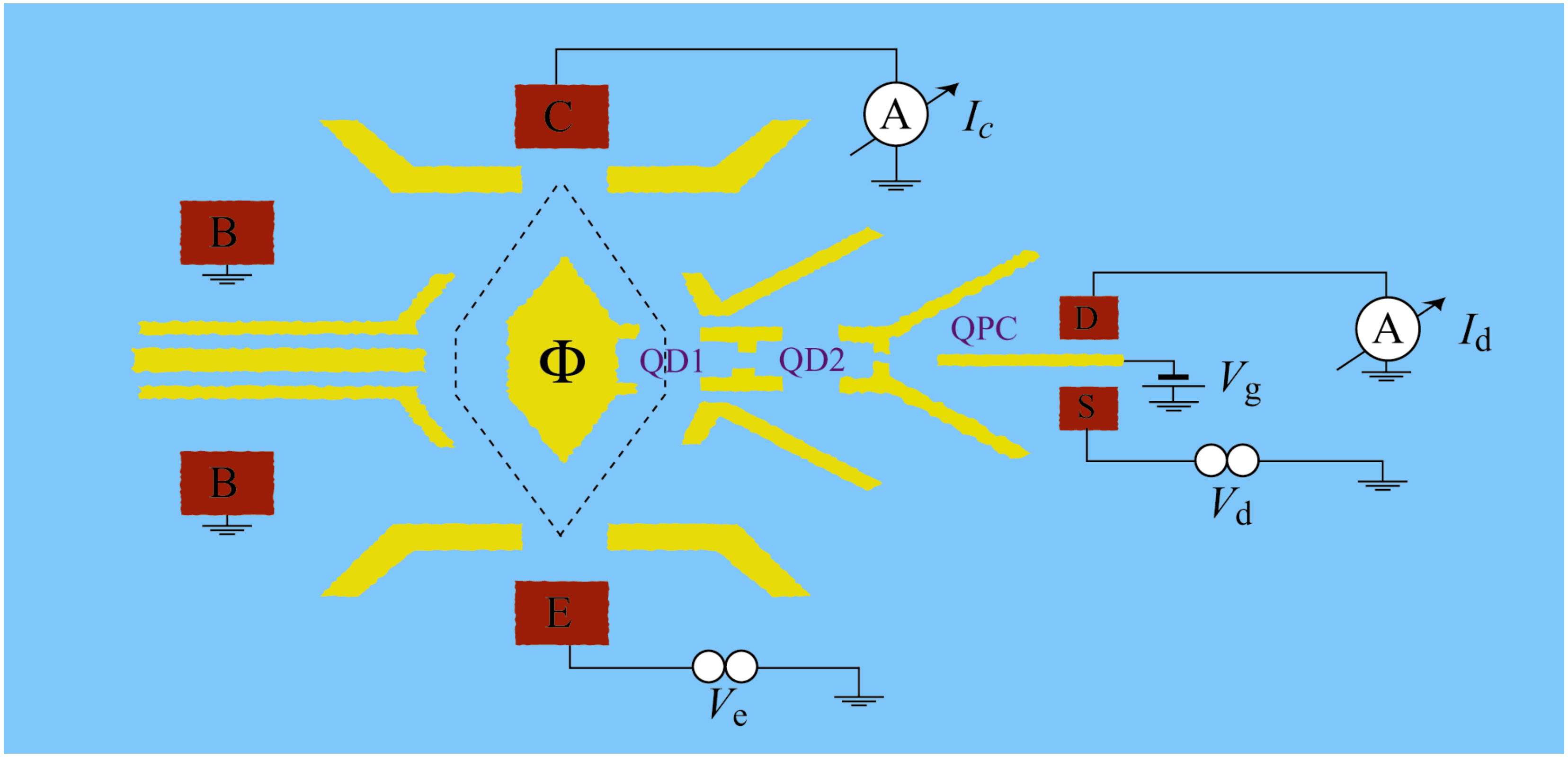


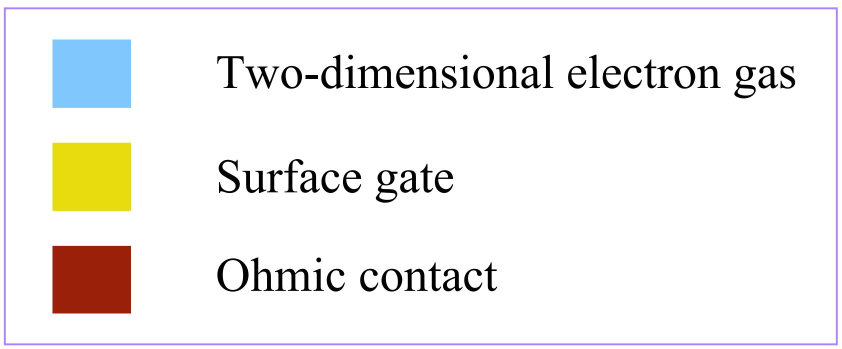
$$G \sim 1 + \cos\left(\xi_+ - \xi_-\right) \cos\left(2\pi \frac{e}{h} BS - (\alpha_+ - \alpha_-)\right)$$

R. Webb et al Phys. Rev. Lett. 54 2696 (1985).

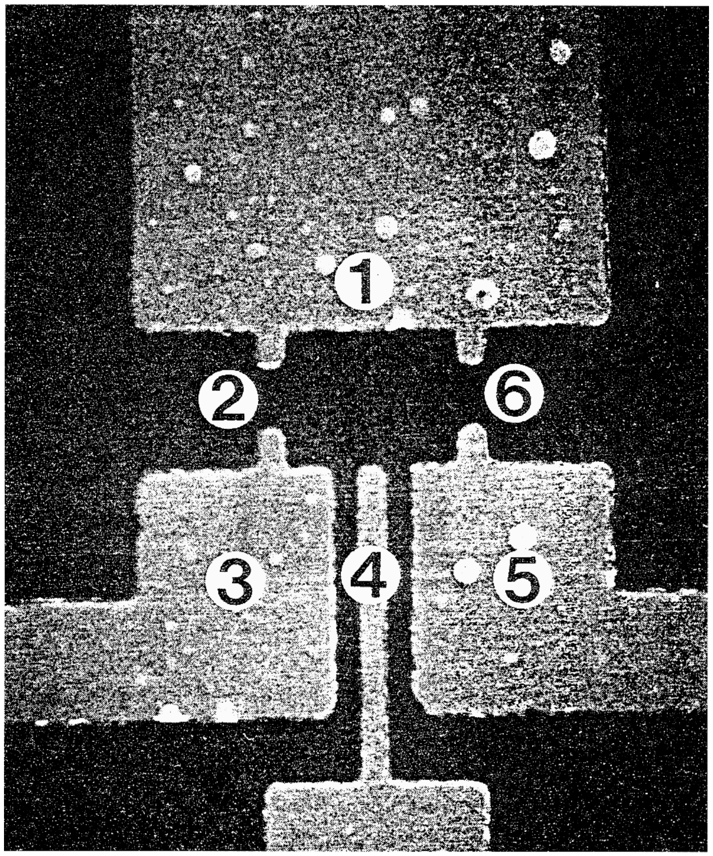
E. Buks et al 391 872 Nature (1998)

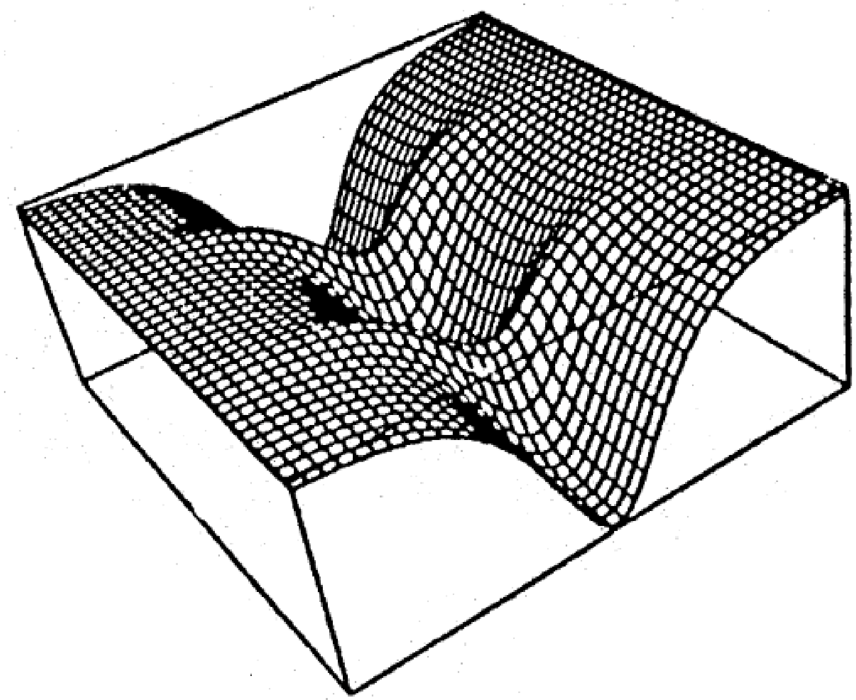
Detector circuit: quantum dots.





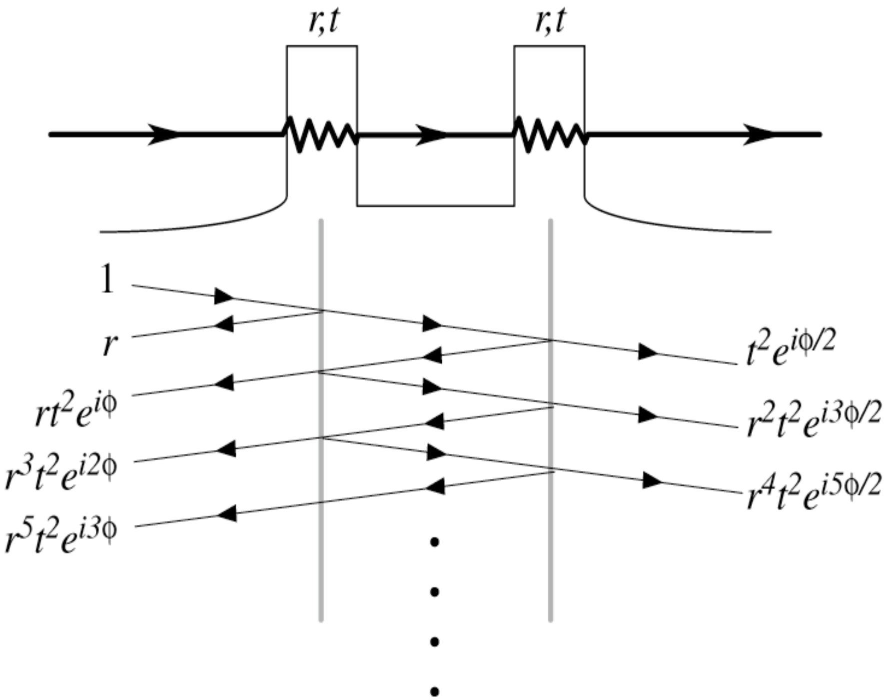
Quantum Dots

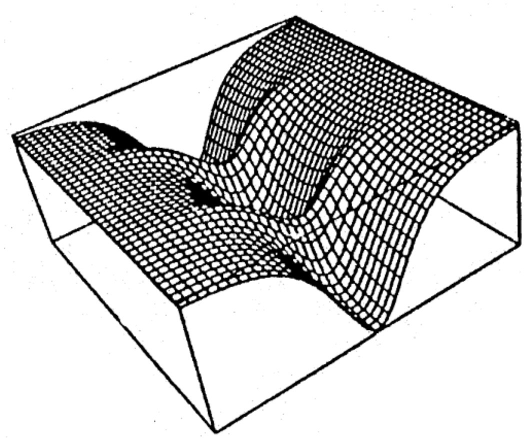




P.L McEuen *et al* Phys. Rev. Lett. 66 1926 (1991).

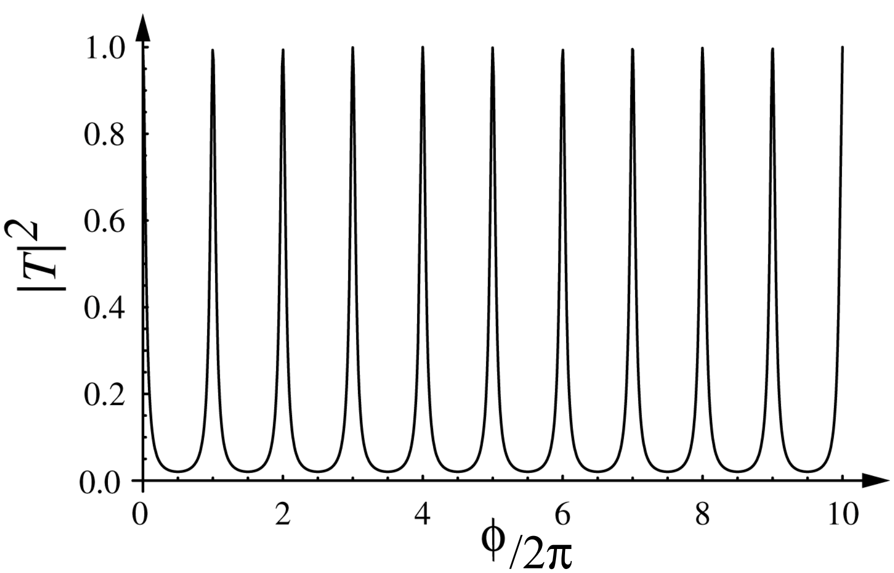
Resonant tunnelling: Fabrey-Perot





Landauer conductance

$$G = \frac{e^2}{h}\left|T\right|^2 = \frac{e^2}{h}\frac{t^4}{1 + r^2 - 2r^2\cos\left(\phi\right)}$$



Coulomb Blockade

External charge

$$Q_{ext} = \sum_i Q_i$$

Electrostatic potential of electrons in dot

$$\phi(Q) = \frac{Q}{C} + \frac{Q_{ext}}{C}$$

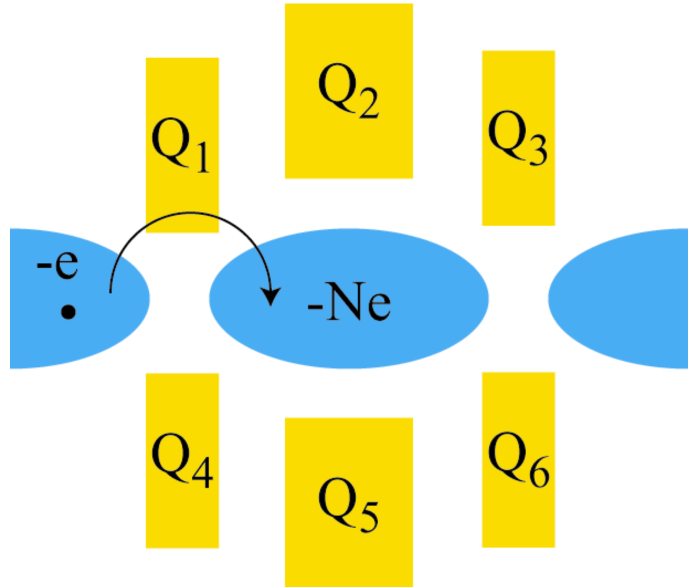
Potential energy of electrons in dot:

$$U(Q) = \int_0^Q \phi(Q)dQ = \frac{Q^2}{2C} + Q\frac{Q_{ext}}{C}$$

For

$$Q = -Ne$$

$$U(N) = \frac{(Ne - Q_{ext})^2}{2C} - \frac{Q_{ext}^2}{2C}$$



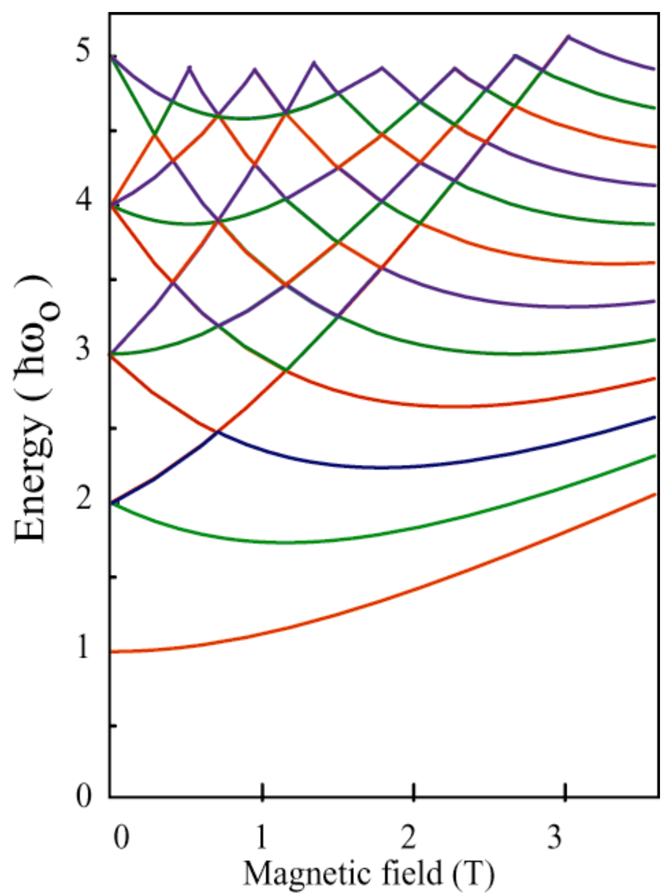
Energy to add one electron:

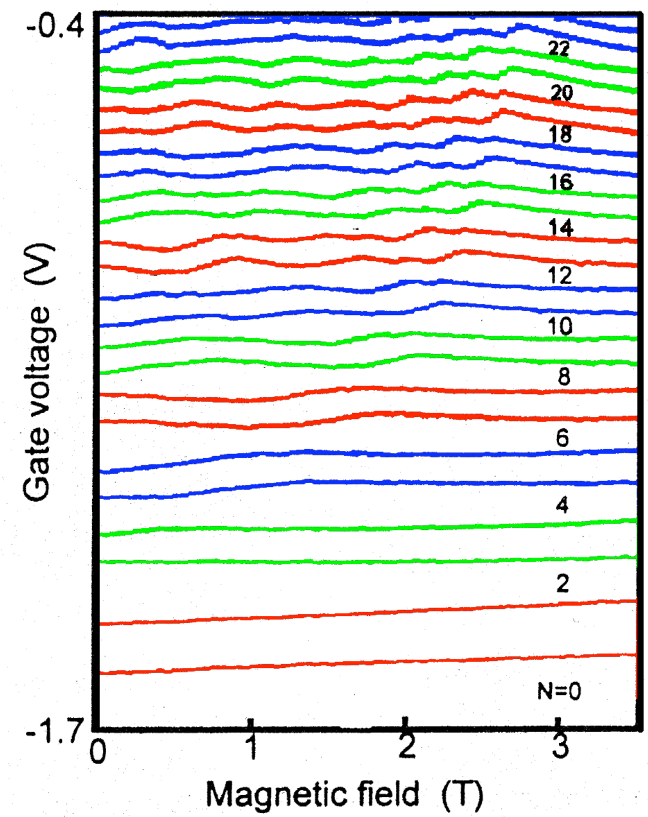
$$U(N+1) - U(N) = \frac{e^2 - 2e(Ne - Q_{ext})}{2C}$$

$$Q_{ext} = Ne \qquad \Delta U = \frac{e^2}{2C}$$

$$Q_{ext} = (N + \frac{1}{2})e \qquad \Delta U = 0$$

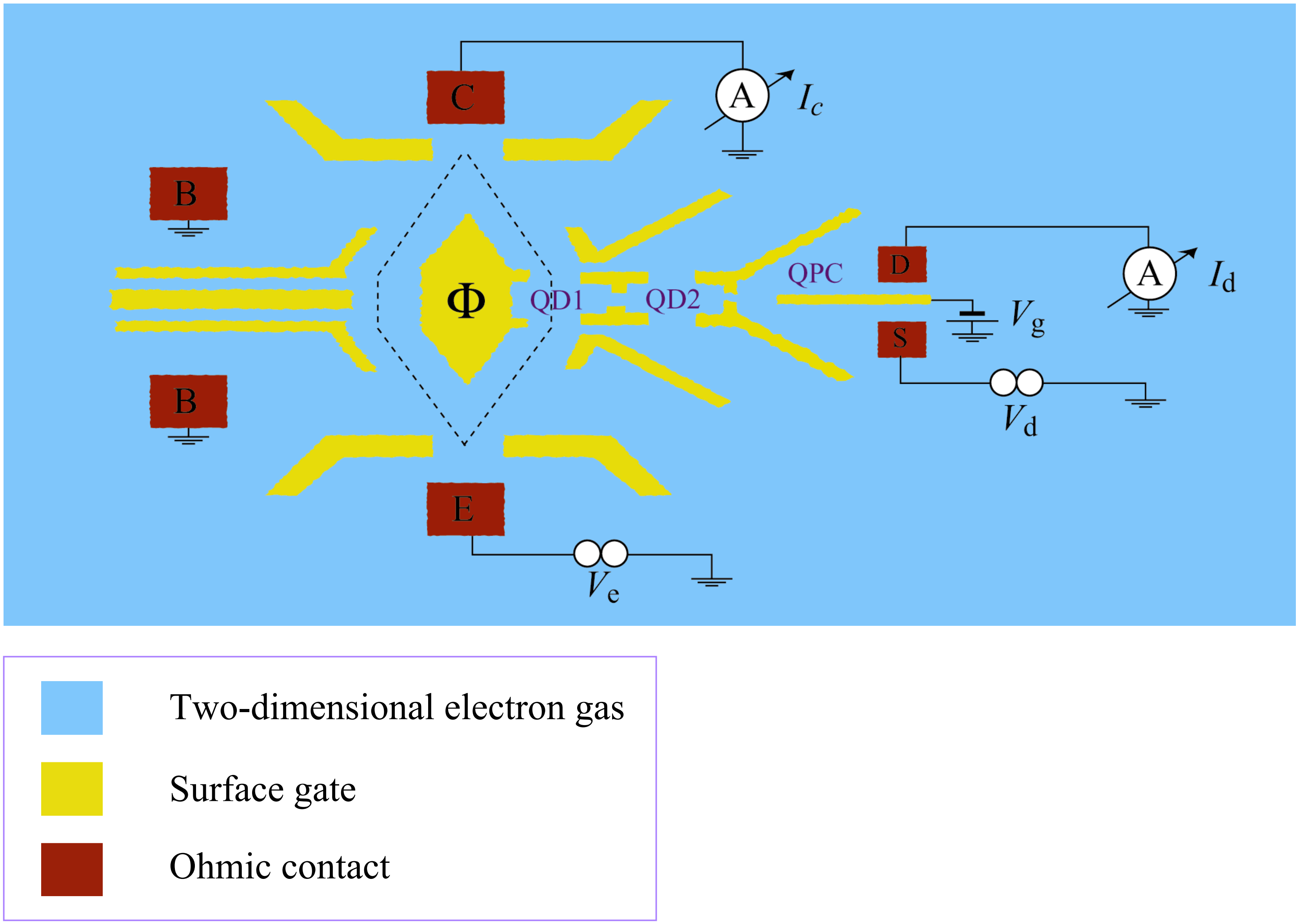
Quantum-dot transmission spectroscopy



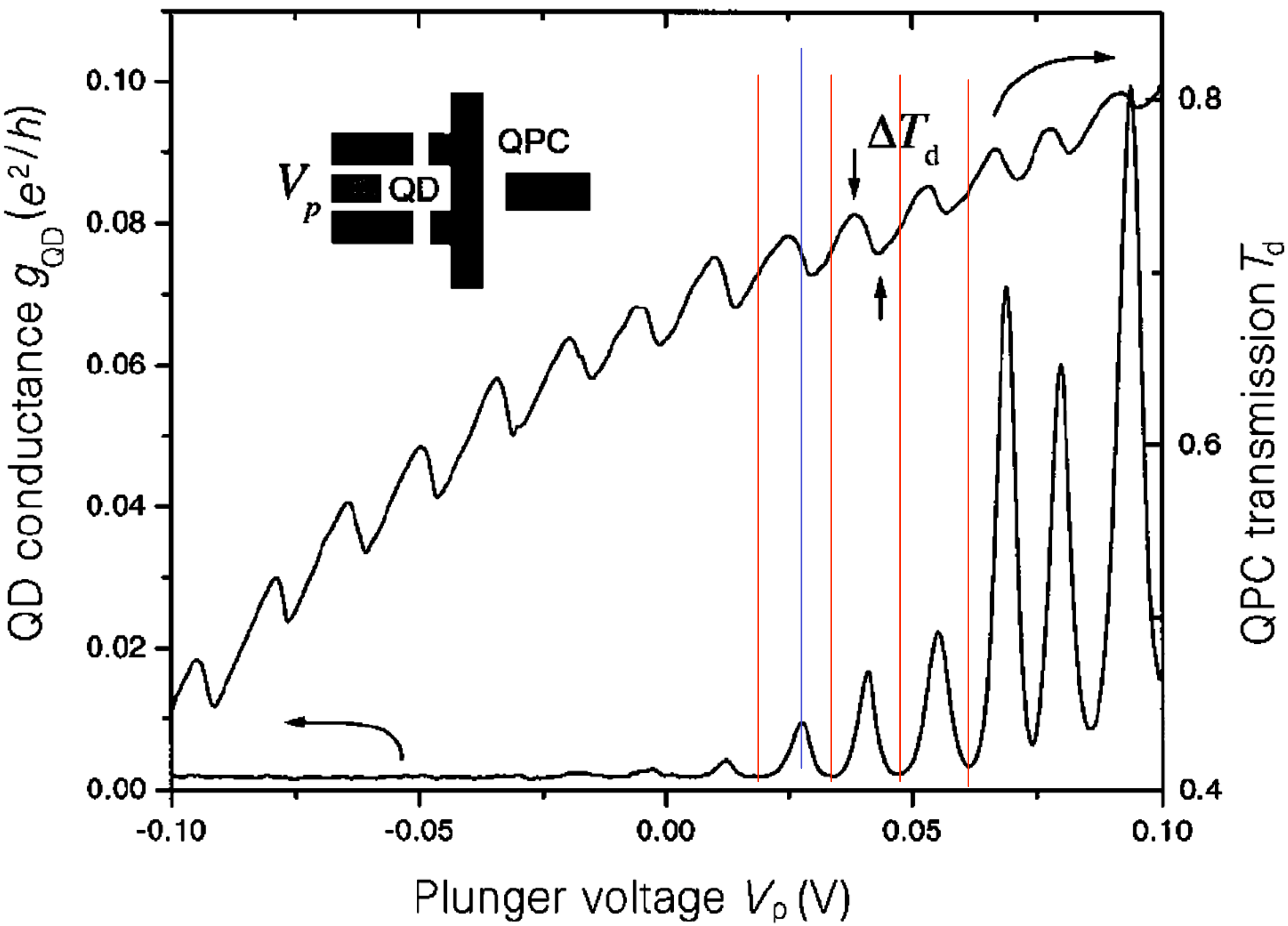


L.P. Kouwenhoven *et al* Science (1997).

Detector circuit: non-invasive quantum detection

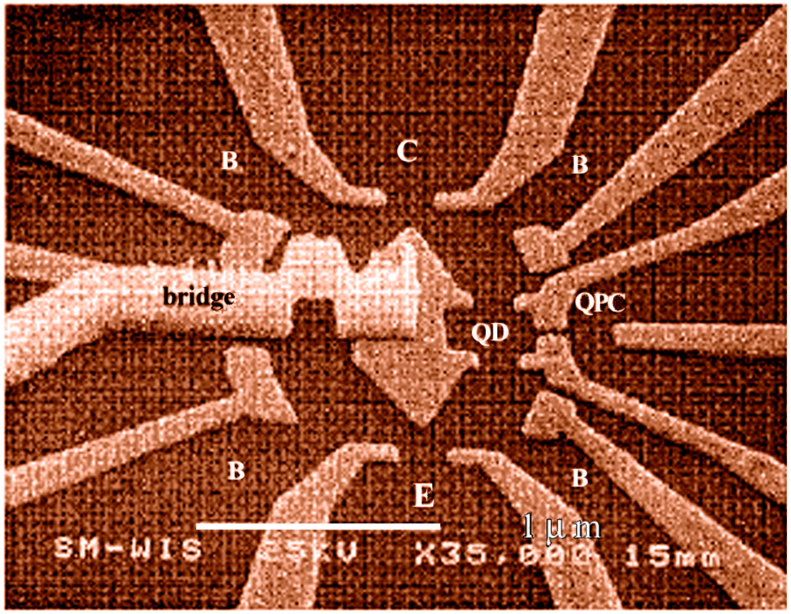


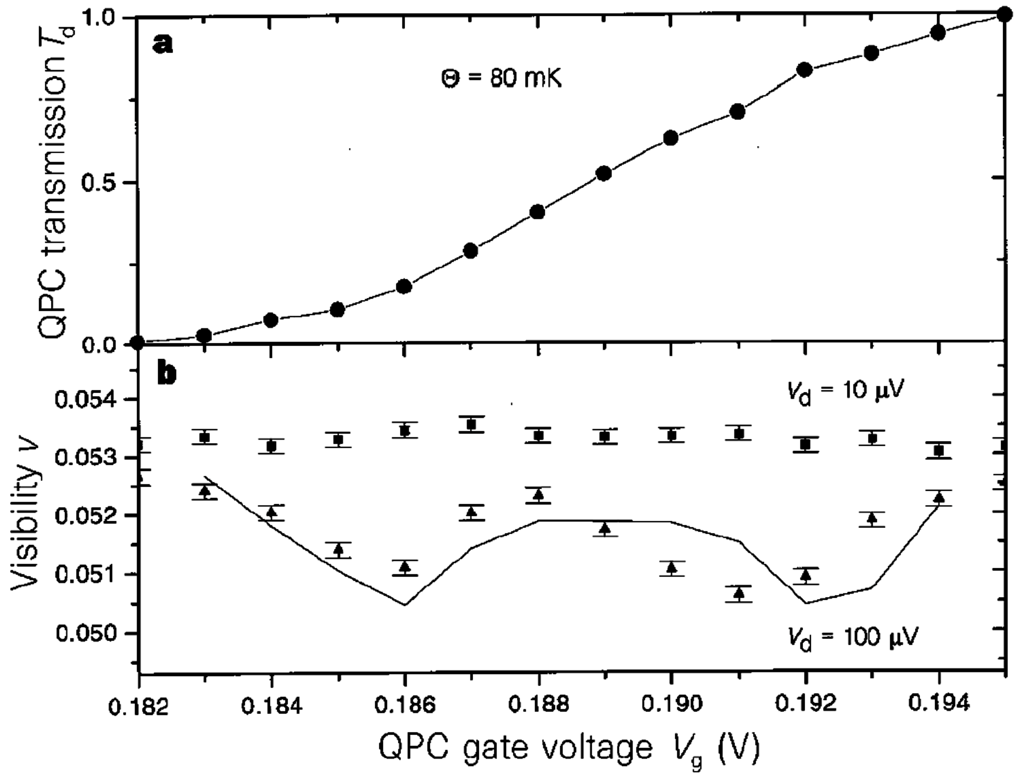
Non-invasive quantum detection

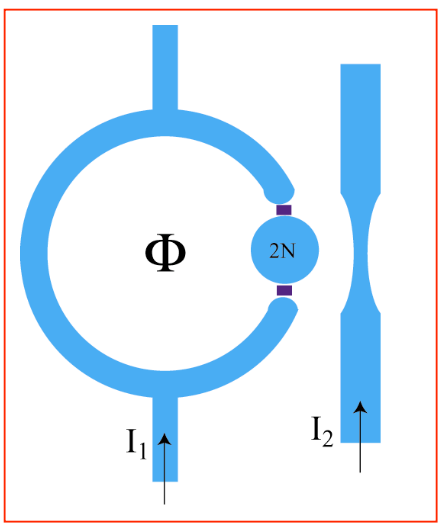


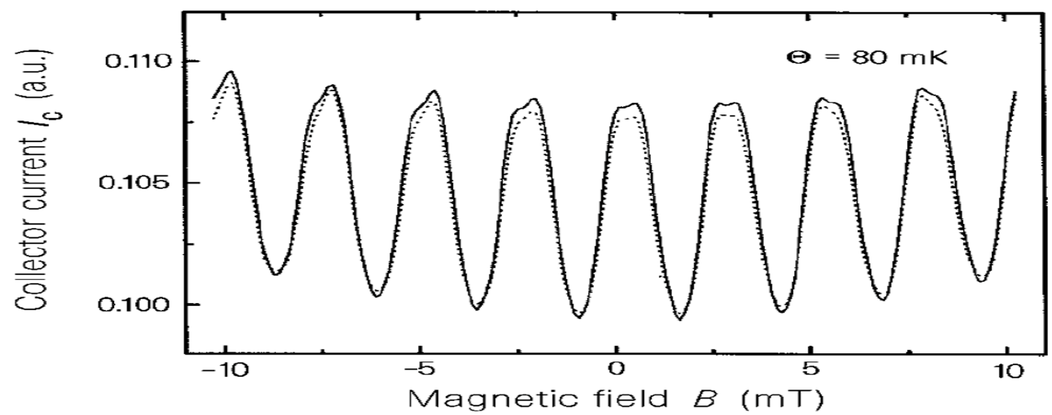
M. Field *et al* Phys. Rev. Lett. 70 1311 (1993).
E. Buks *et al* 391 872 Nature (1998)

Orbital decoherence from non-invasive quantum detection.



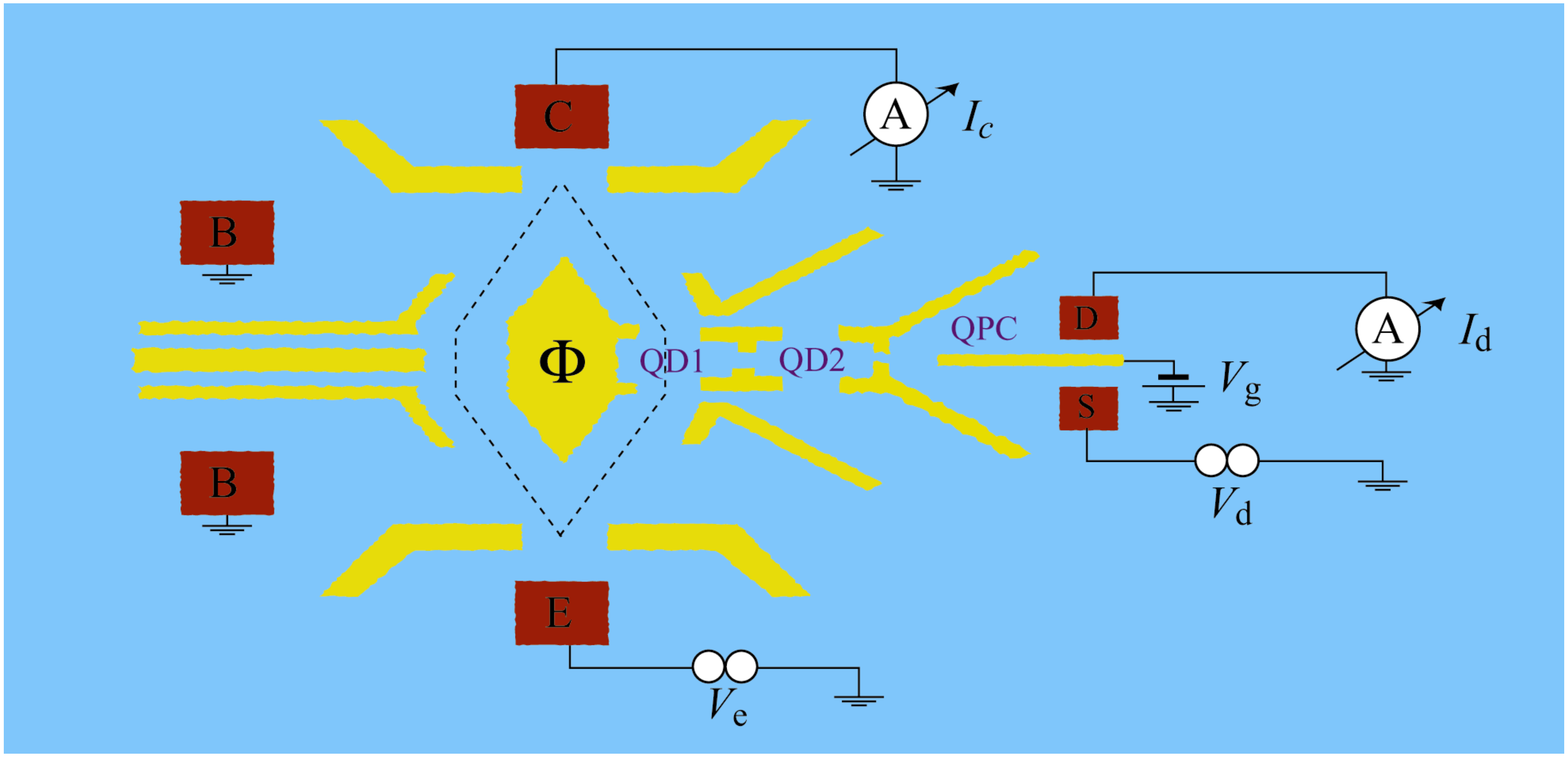


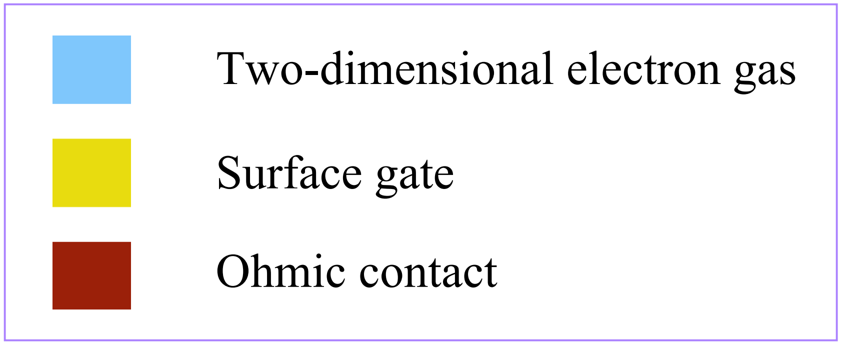




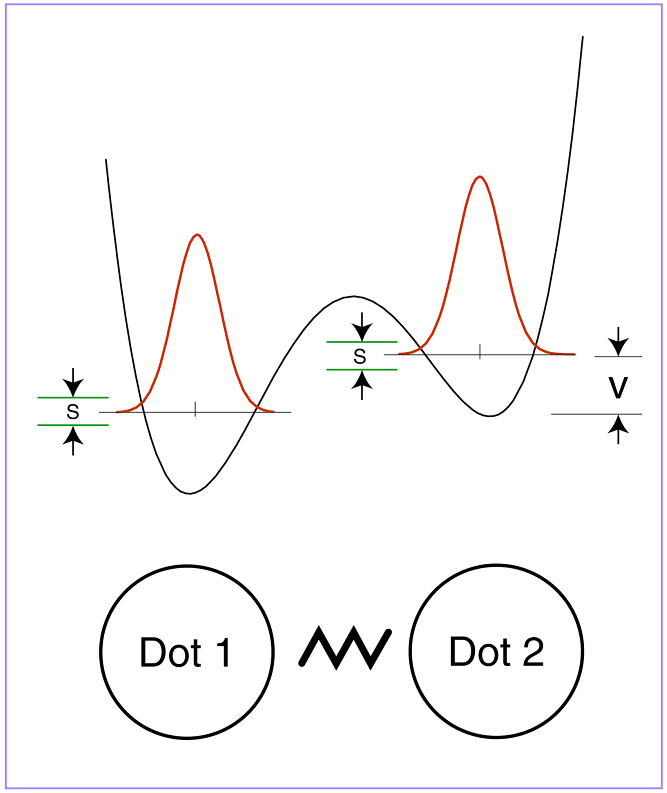
E. Buks *et al* 391 872 Nature (1998)

Detector circuit: double quantum dot.





Double quantum dot



Hubbard Model:

$$H = \sum_{\sigma=\uparrow,\downarrow} \left(\frac{1}{2}(\sigma S - V) c^\dagger_{1,\sigma} c_{1,\sigma} + \frac{1}{2}(\sigma S + V) c^\dagger_{2,\sigma} c_{2,\sigma} + \nu \left(c^\dagger_{1,\sigma} c_{2,\sigma} + c^\dagger_{2,\sigma} c_{1,\sigma} \right) \right) + U \left(n_{1,\uparrow} n_{1,\downarrow} + n_{2,\uparrow} n_{2,\downarrow} \right)$$

Double quantum-dot eigen states

Basis states

$$|T_{+1}\rangle = c^\dagger_{1,\uparrow} c^\dagger_{2,\uparrow}|0\rangle$$

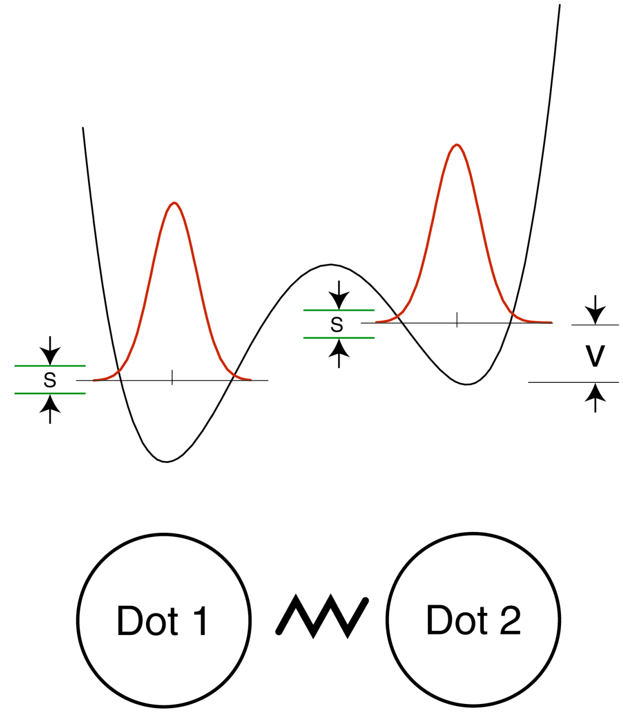
$$|T_{-1}\rangle = c^\dagger_{1,\downarrow} c^\dagger_{2,\downarrow}|0\rangle$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(c^\dagger_{1,\uparrow} c^\dagger_{2,\downarrow} + c^\dagger_{1,\downarrow} c^\dagger_{2,\uparrow}\right)|0\rangle$$

$$|S\rangle = \frac{1}{\sqrt{2}}\left(c^\dagger_{1,\uparrow} c^\dagger_{2,\downarrow} - c^\dagger_{1,\downarrow} c^\dagger_{2,\uparrow}\right)|0\rangle$$

$$|D_1\rangle = c^\dagger_{1,\uparrow} c^\dagger_{1,\downarrow}|0\rangle$$

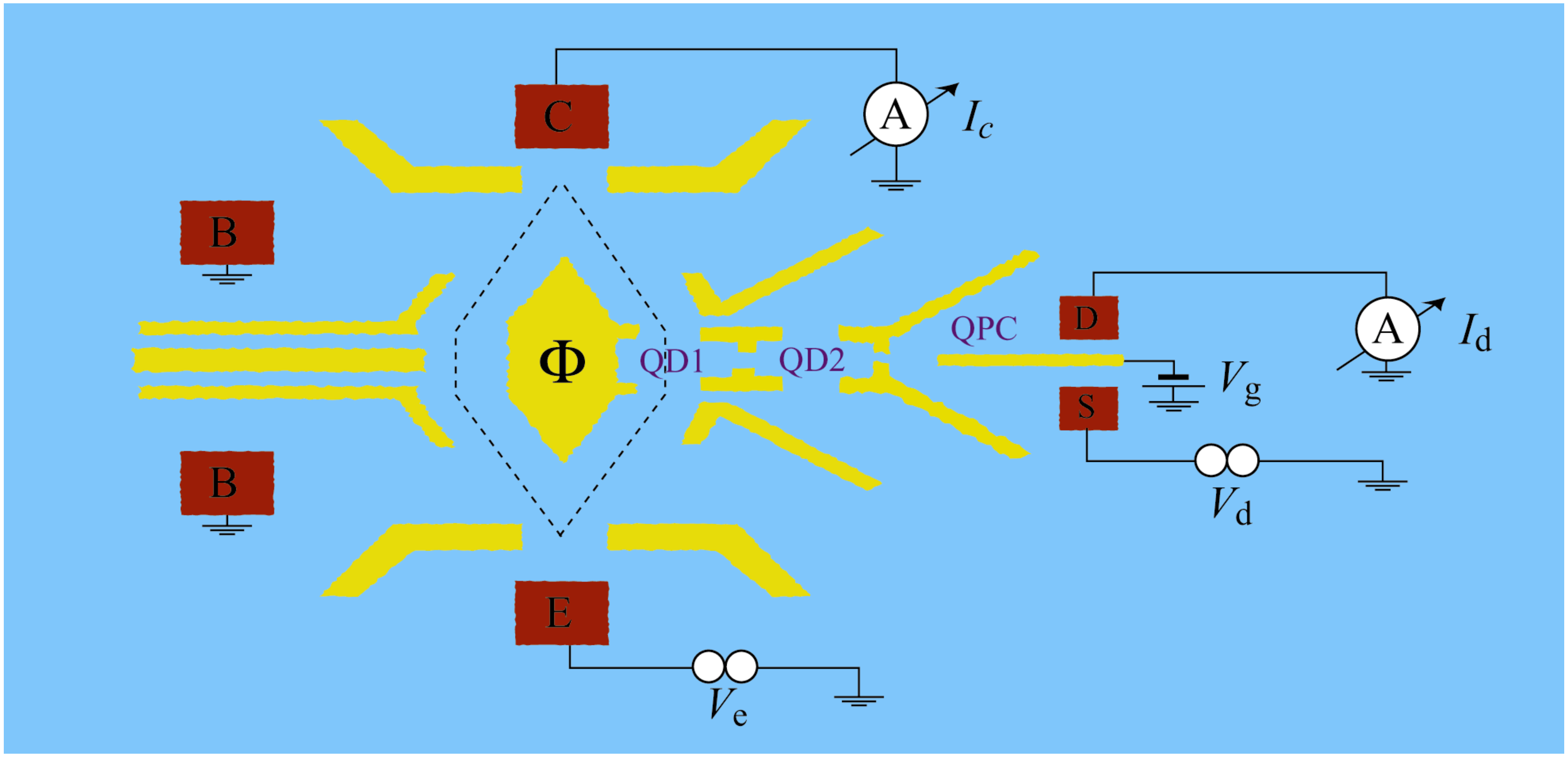
$$|D_2\rangle = c^\dagger_{2,\uparrow} c^\dagger_{2,\downarrow}|0\rangle$$

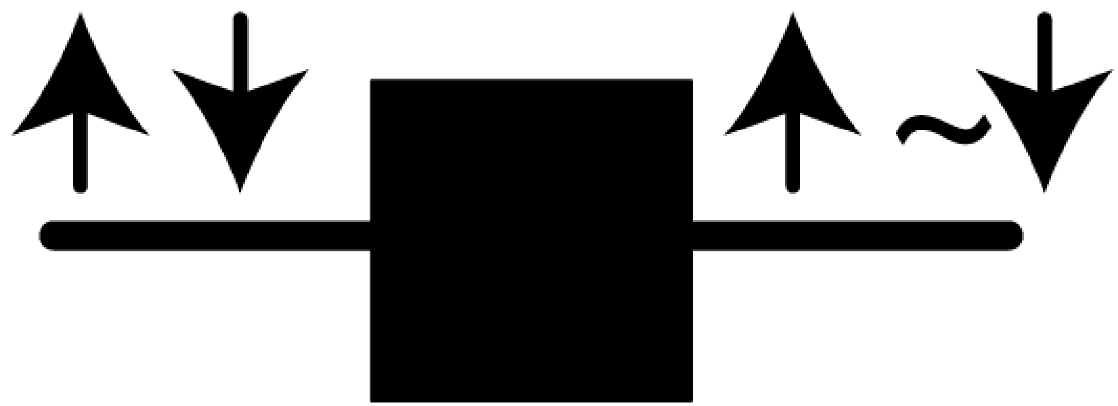


Hamiltonian

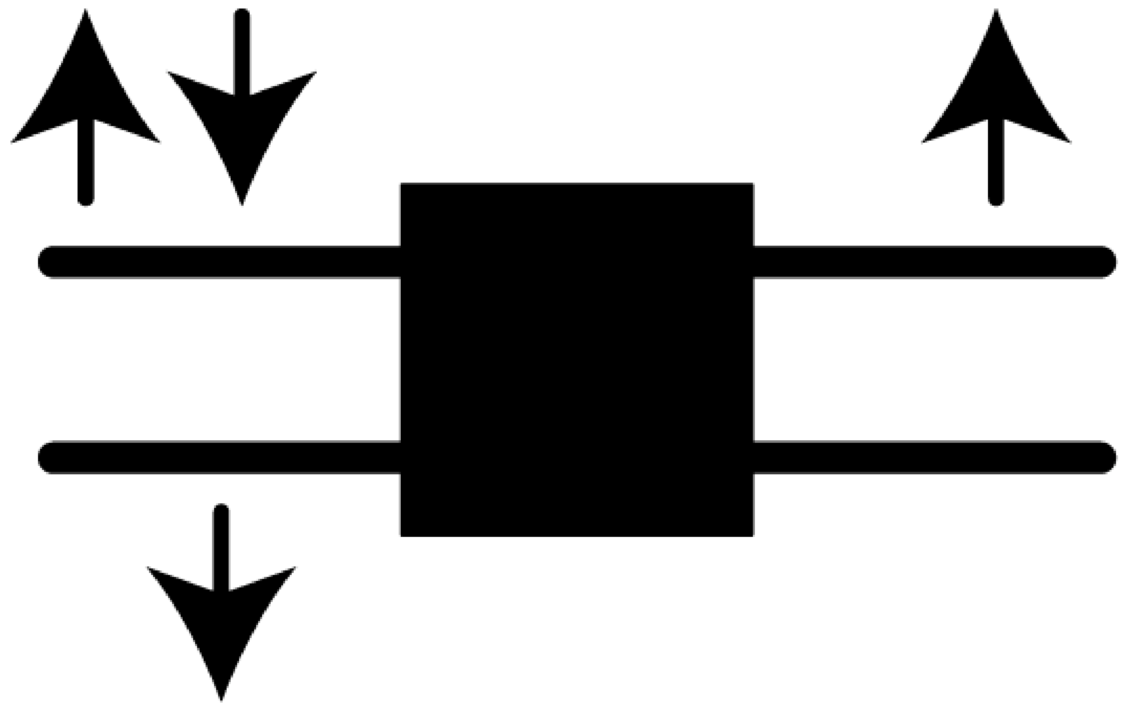
$$\begin{bmatrix} S & 0 & 0 & 0 & 0 & 0 \\ 0 & -S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{2}\nu & -\sqrt{2}\nu \\ 0 & 0 & 0 & -\sqrt{2}\nu & U+V & 0 \\ 0 & 0 & 0 & -\sqrt{2}\nu & 0 & U-V \end{bmatrix} \begin{bmatrix} T_{+1} \\ T_{-1} \\ T_0 \\ S \\ D_1 \\ D_2 \end{bmatrix} = i\hbar\frac{\partial}{\partial t} \begin{bmatrix} T_{+1} \\ T_{-1} \\ T_0 \\ S \\ D_1 \\ D_2 \end{bmatrix}$$

Spin-decoherence from non-invasive quantum measurement

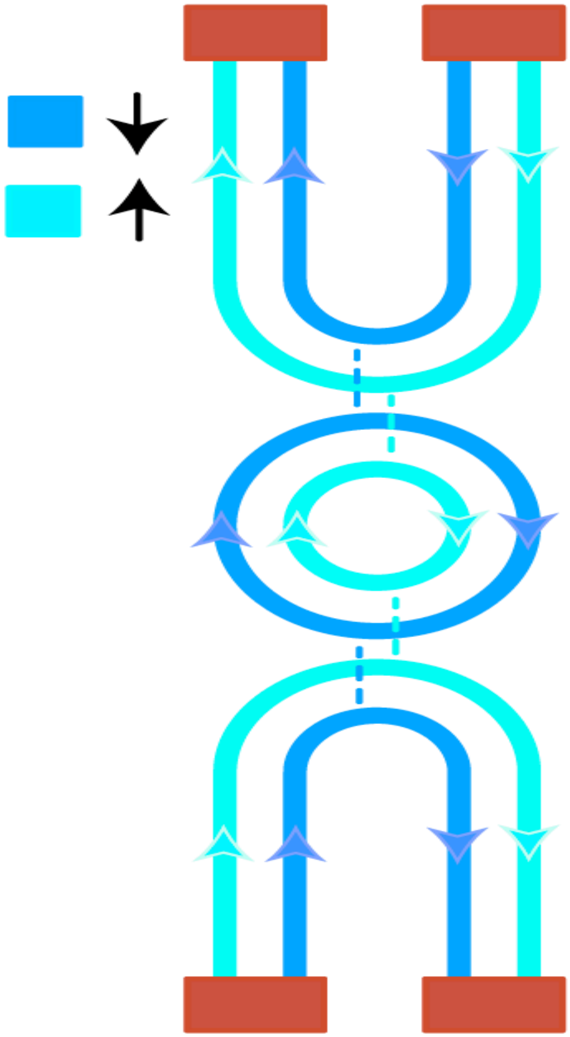




Device II: Spin Valve



Spin valve.

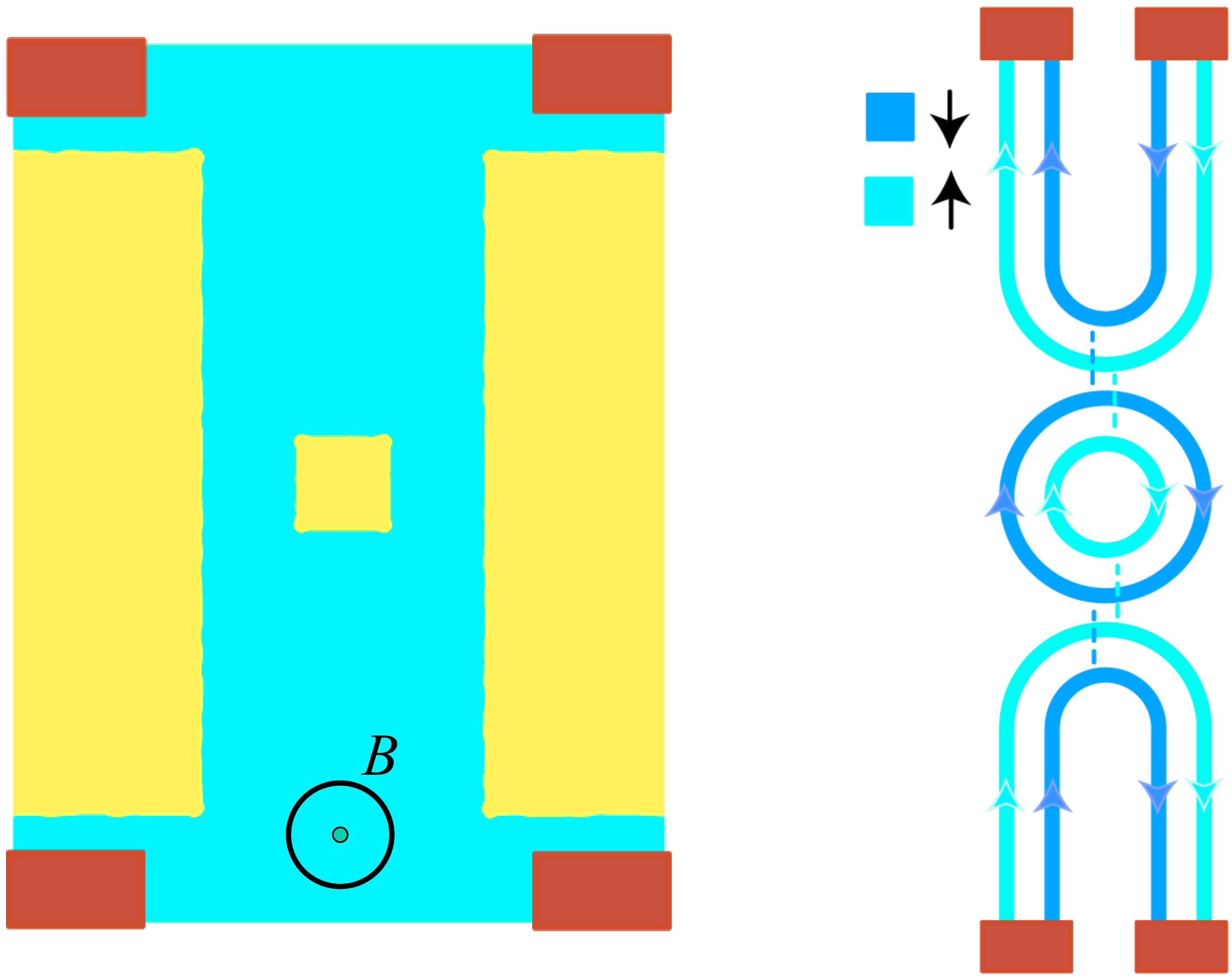


Outline

- Experimental implementation.
- Edge states in 1D channels.
- Edge states around anti-dots.
- Resonant tunnelling through anti-dots.
- Selfconsistent edge states.
- Selfconsistent edge states around an anti-dot.
- Fractional quantum Hall effect.
- Quasi-particles in the vicinity of an Anti-dot.
- Experimental results.

Based on conductance studies of Anti-dots: **D. Mace *et al* Phys. Rev. B**

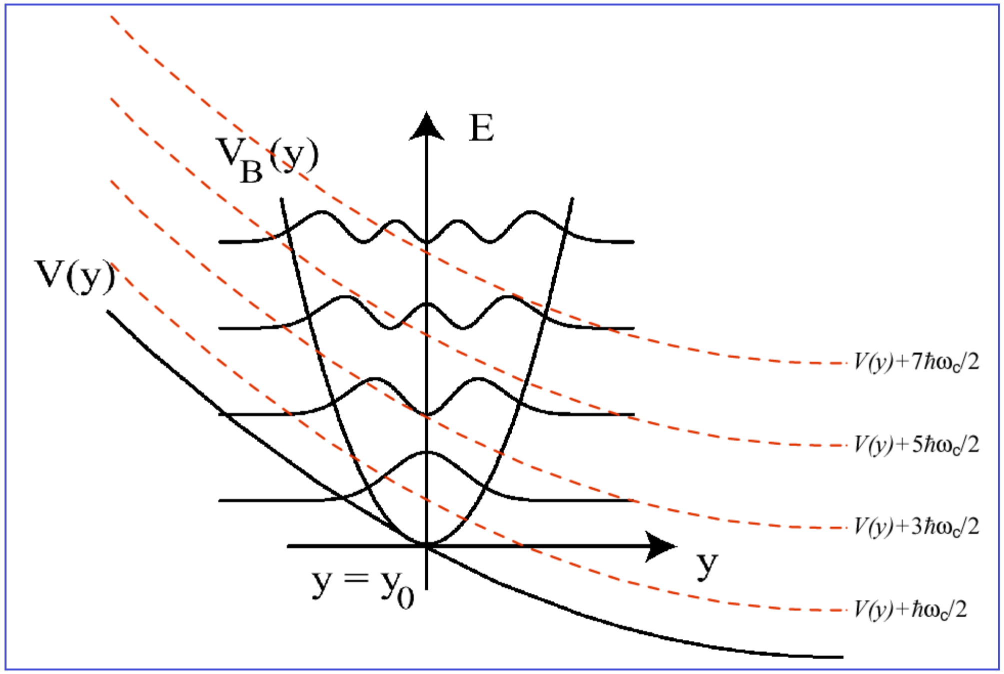
Edge-state spin valve: the device.



Edge states in a quasi-one-dimensional channel

$$\left(\frac{1}{2m^*}(p+eA)+V(y)\right)\Psi = E\Psi$$

$$A = -By\hat{x}$$



$$\Psi_{k_x,n} = \exp\left(ik_x x\right)\phi_{k_x,n}\left(y\right)$$

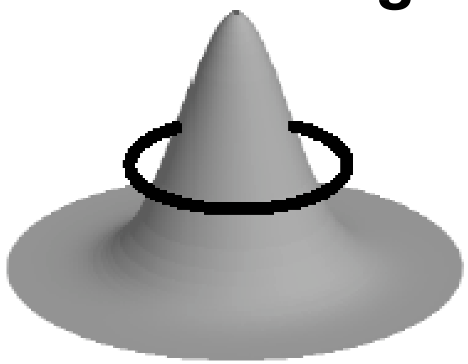
$$\left(\frac{p_y^2}{2m^*}+\frac{1}{2m^*}(p_x-eBy)^2+V(y)\right)\Psi = E\Psi$$

$$V_B(y) = \frac{1}{2}m^*\omega_c^2(y-y_0)^2$$

$$y_0 = \frac{\hbar k_x}{eB}$$

$$E_{y_0,n} = \left(n+\frac{1}{2}\right)\hbar\omega_c + V(y_0)$$

Edge states around a quantum anti-dot



$$\Delta\phi = \oint \frac{p+eA}{\hbar}.dl = \frac{2\pi rp}{\hbar} + \frac{e}{\hbar}\int \nabla \wedge A.dS = \frac{2\pi rp}{\hbar} + \frac{e}{\hbar}\int B.dS$$

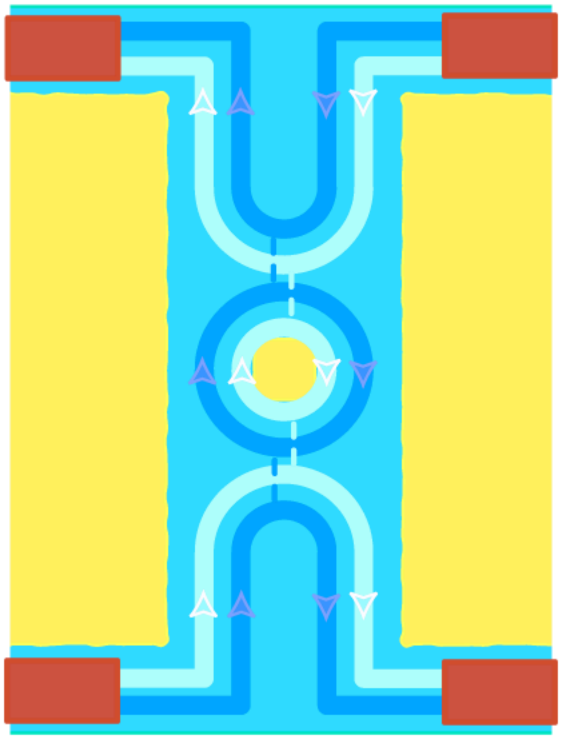
$$\Delta\phi = \frac{2\pi rp}{\hbar} + 2\pi\frac{e}{\hbar}BS$$

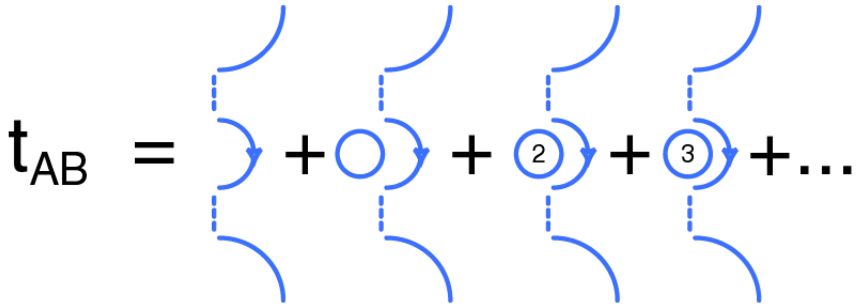
$$2\pi\frac{e}{h}BS = 2m\pi \Rightarrow BS = m\frac{h}{e}$$

$$E_{n,m} = (n+1/2)\hbar\omega_c \pm \frac{1}{2}g\mu_B B + V(r_m)$$

$$B\pi r_m^2 = m\frac{h}{e}$$

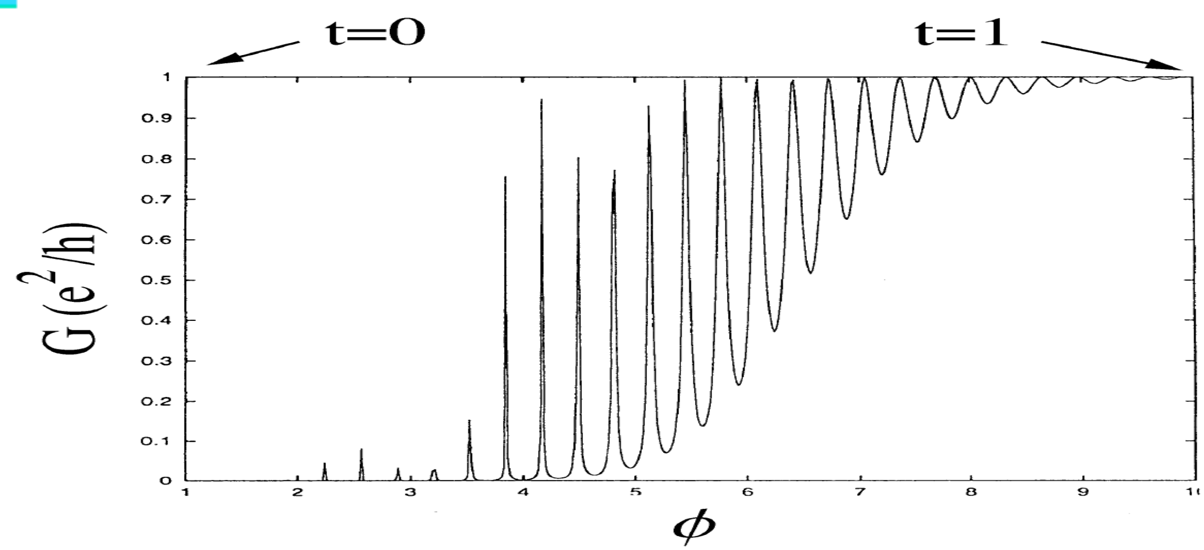
Resonant transmission through an Anti-Dc





$$t_{AB} = t^2 e^{i\phi/2}\left(1 + r^2 e^{i\phi} + (r^2 e^{i\phi})^2 + (r^2 e^{i\phi})^3 + ...\right)$$

$$= \frac{t^2 e^{i\phi/2}}{1 - r^2 e^{i\phi}}$$

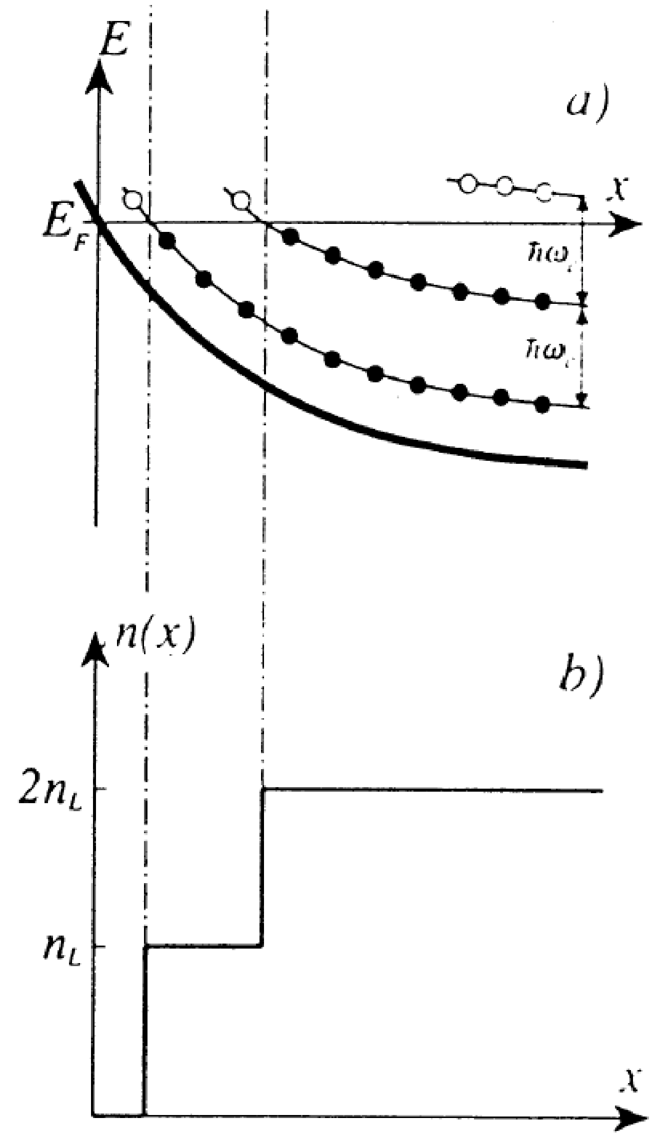


Selfconsistent effects modify the edge potential

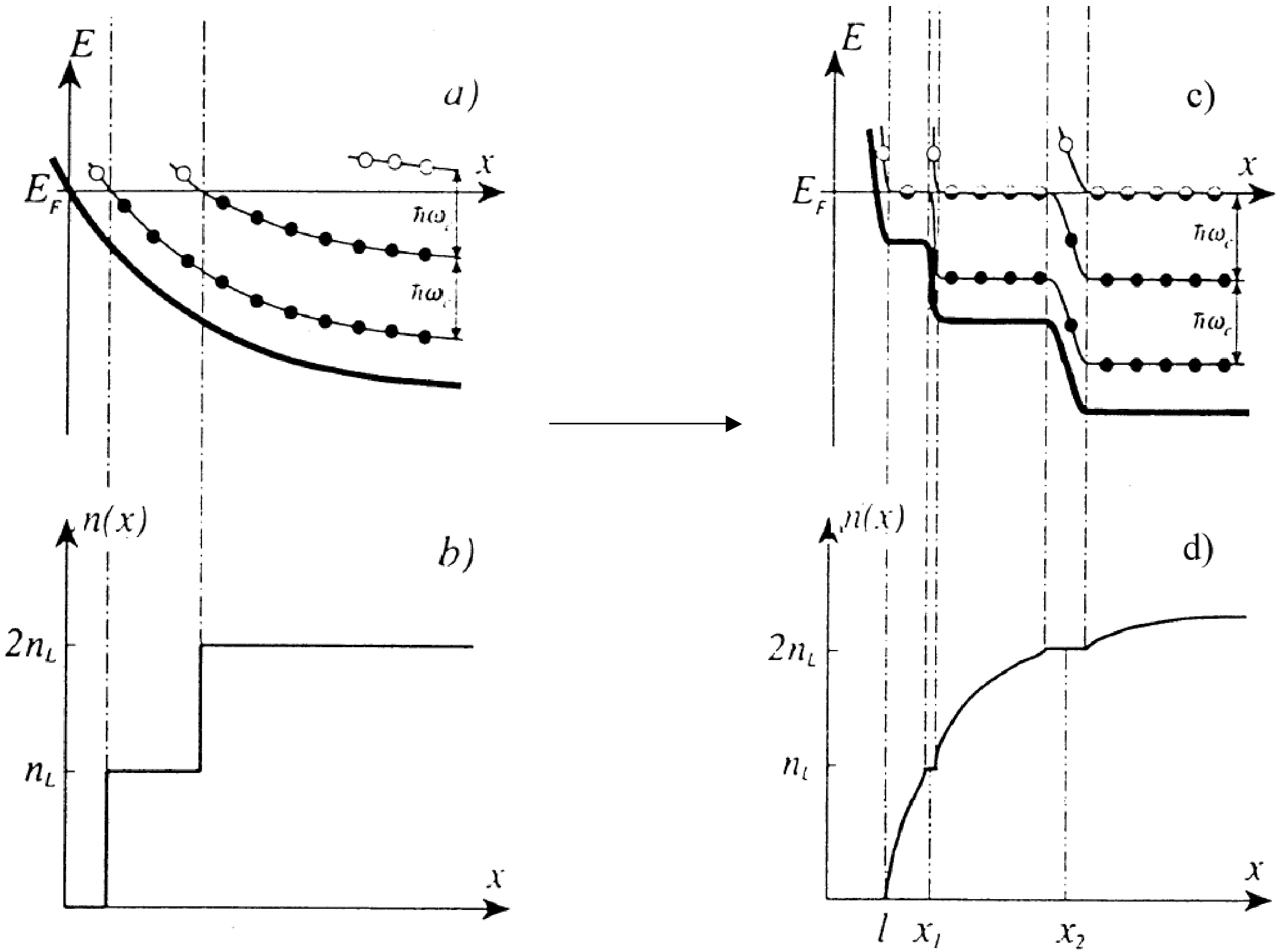
Density of states in a Landau level

$$n_{2D} = \nu \frac{eB}{h}$$

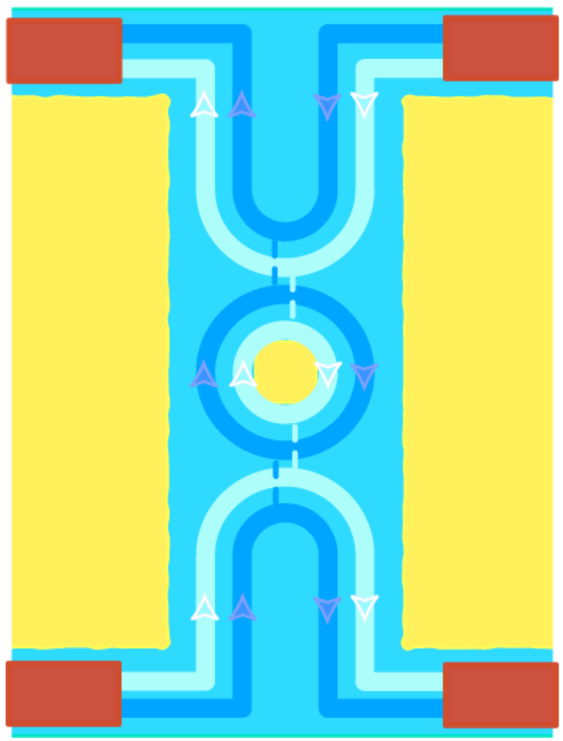
ν - filling factor

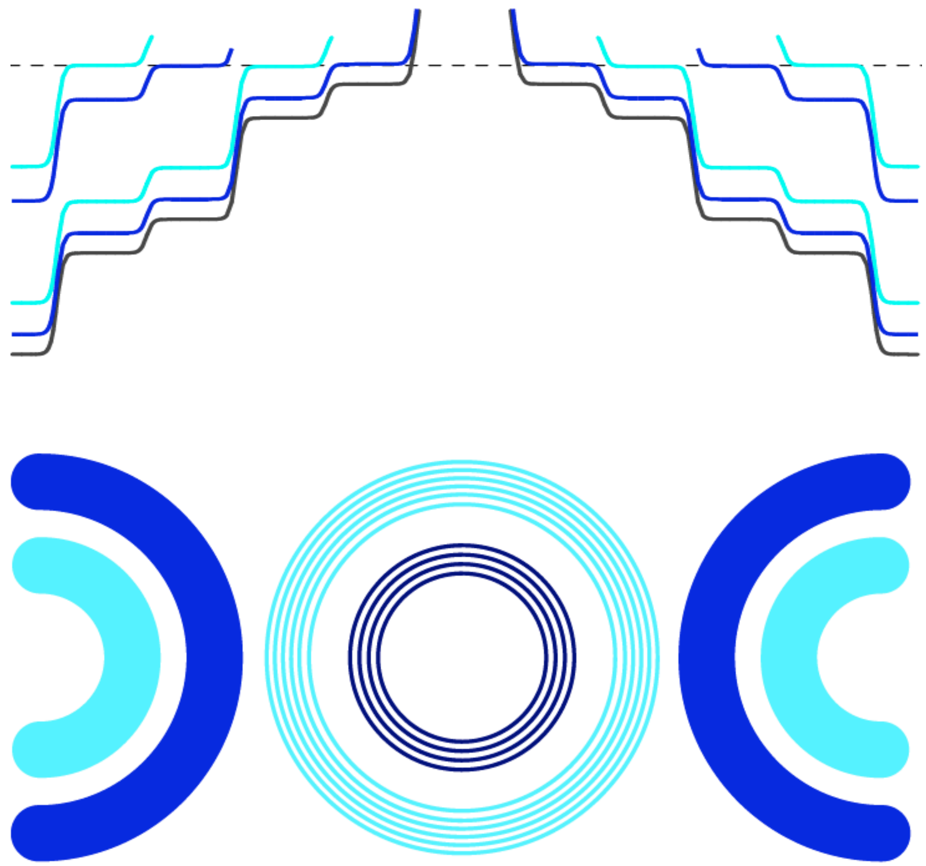


Selfconsistent effects modify the edge potential



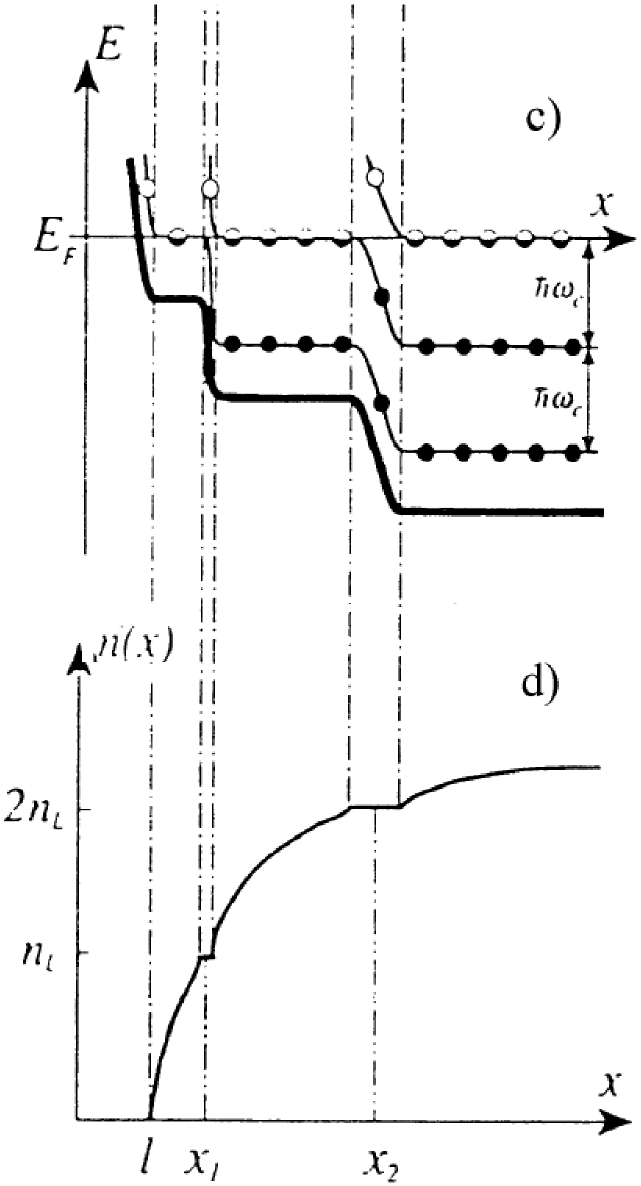
Selfconsistent edge states on an anti-dot





Fractional filling of edge states

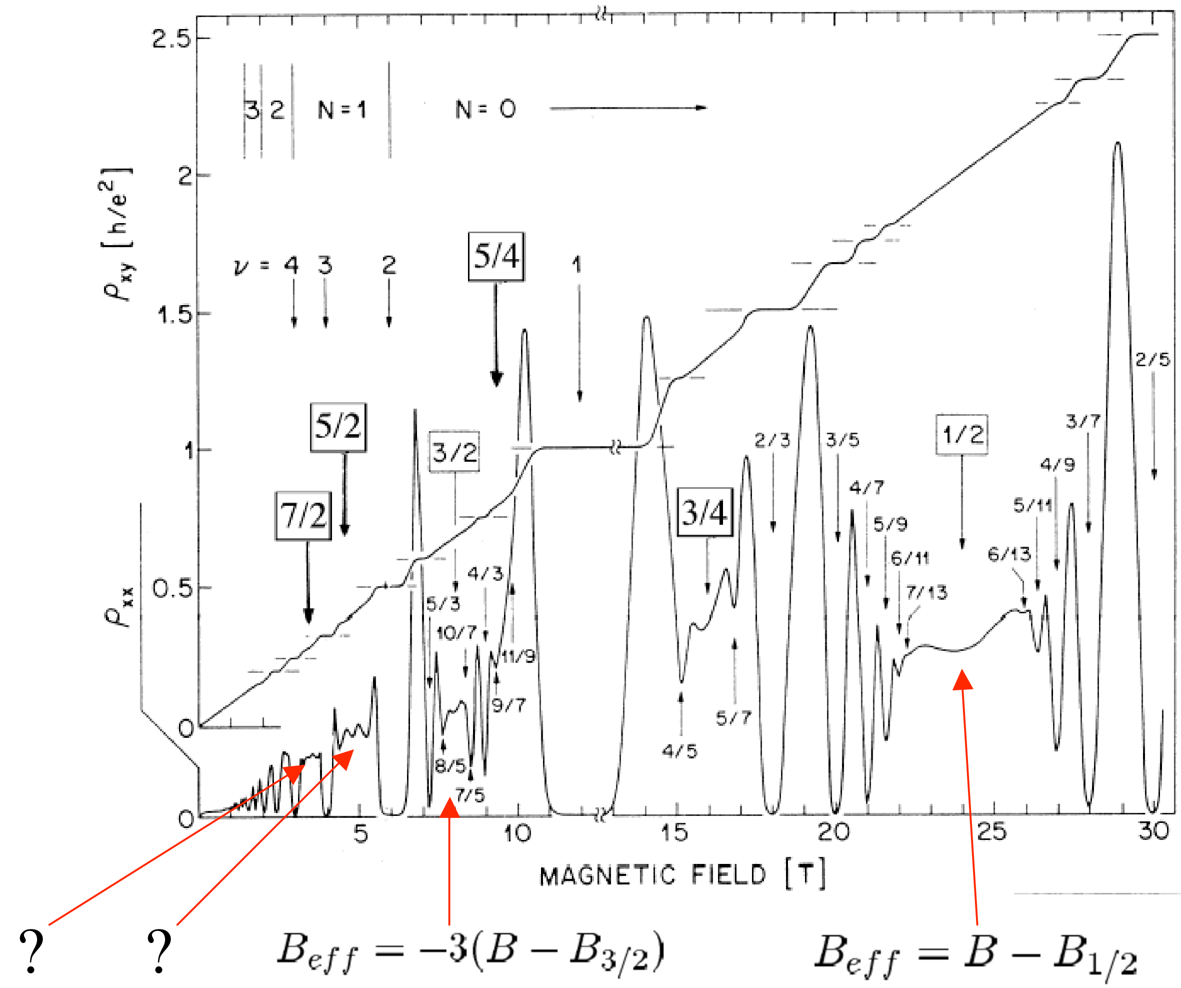
$$n_{2D} = \nu \frac{eB}{h}$$



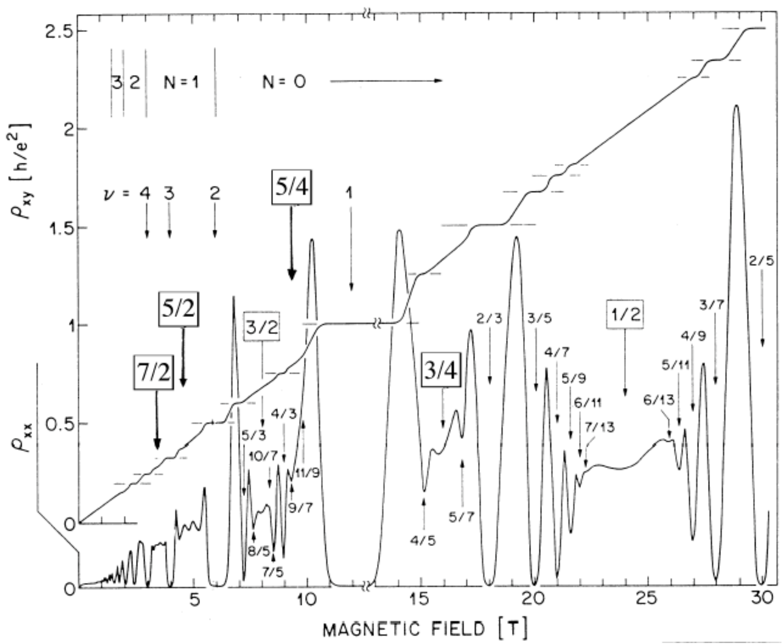
New quasi-particles at high magnetic field: the Fractional Quantum Hall effect

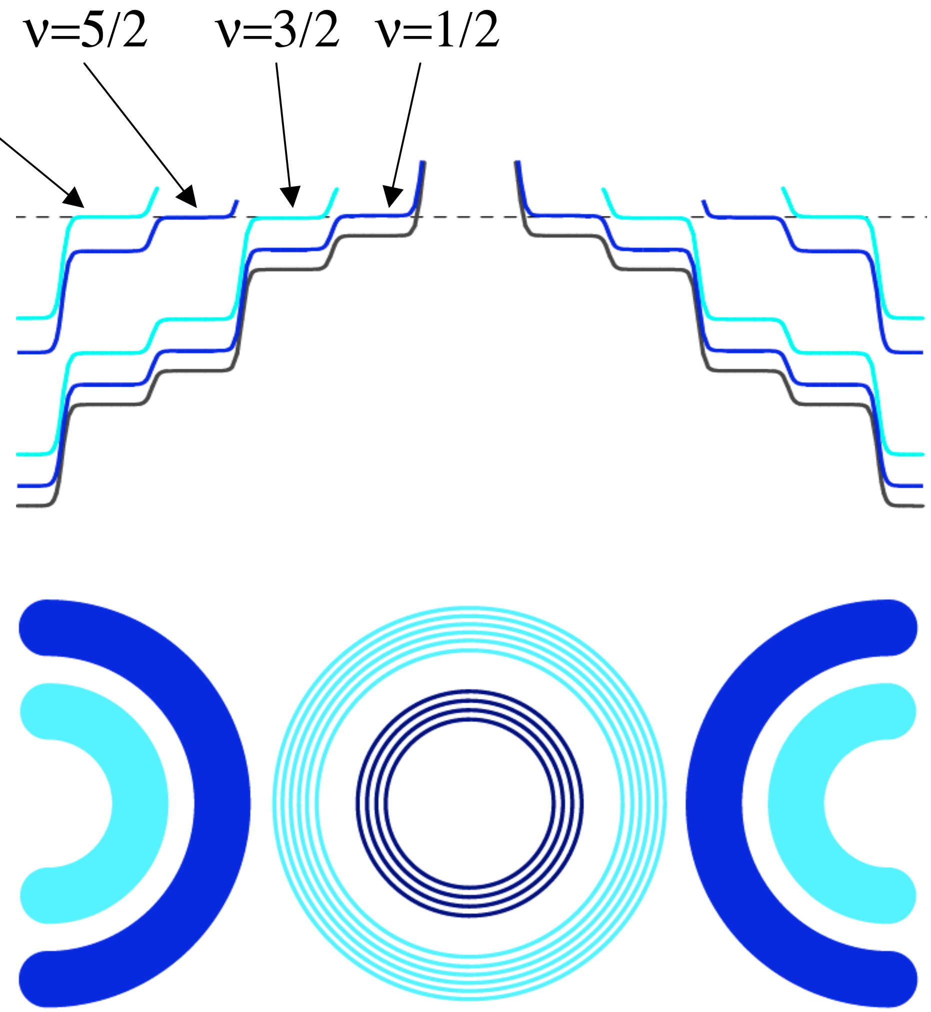
$$H = \sum_{i=1}^{N} \left(\frac{(p_i + e\alpha_i)}{2m^*} + V_i(R_i) \right) + \sum_{i \neq j=1}^{N} V(R_i, R_j, P_i, P_j)$$

$$n_{2D} = \nu \frac{eB}{h}$$

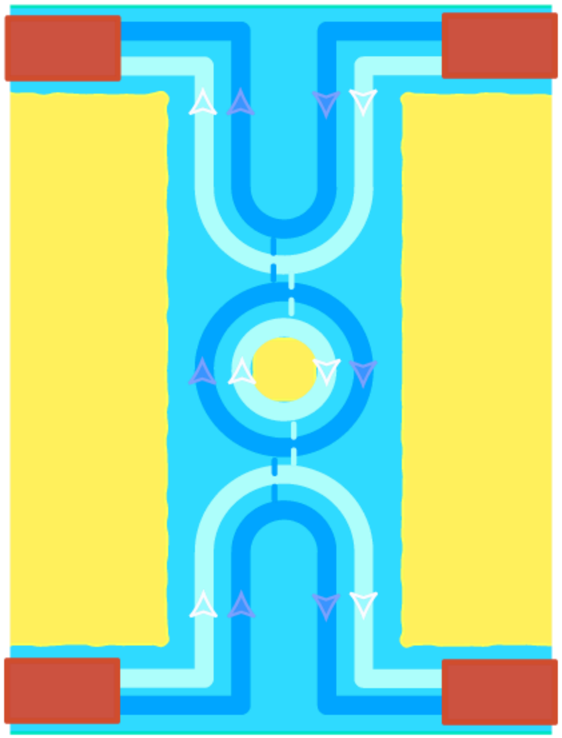


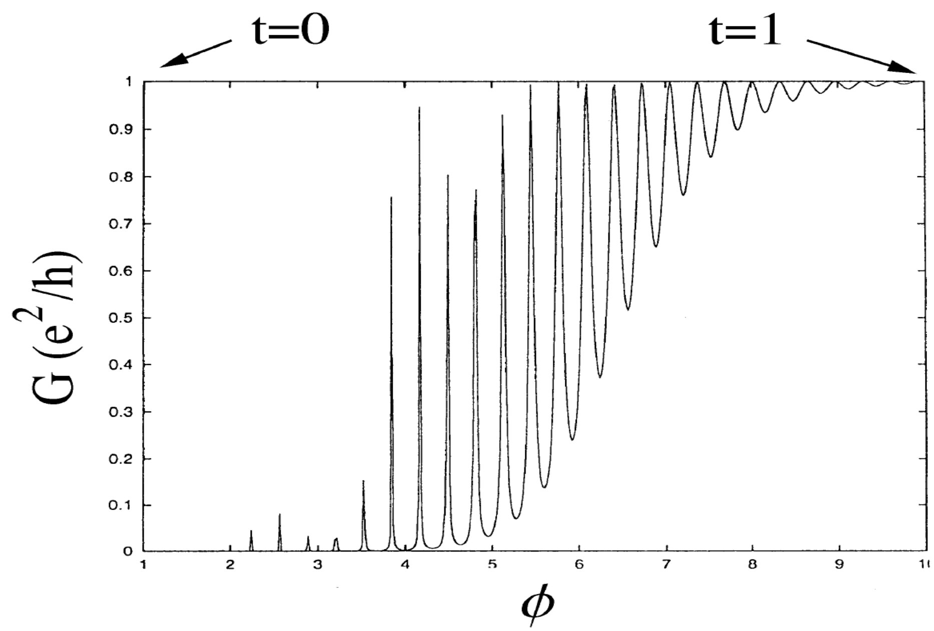
Possible presence of composite Fermions

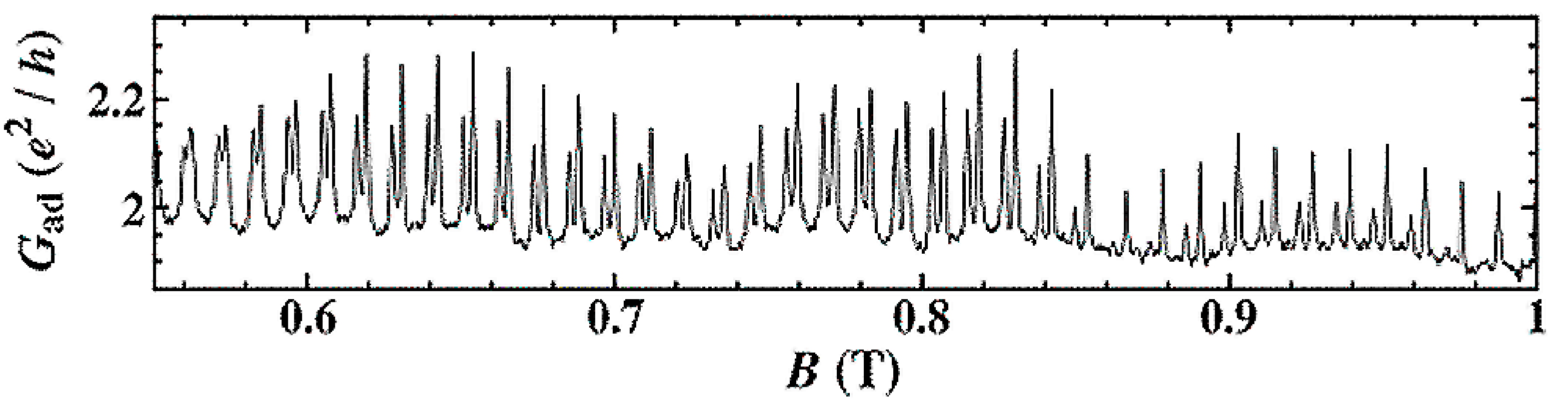


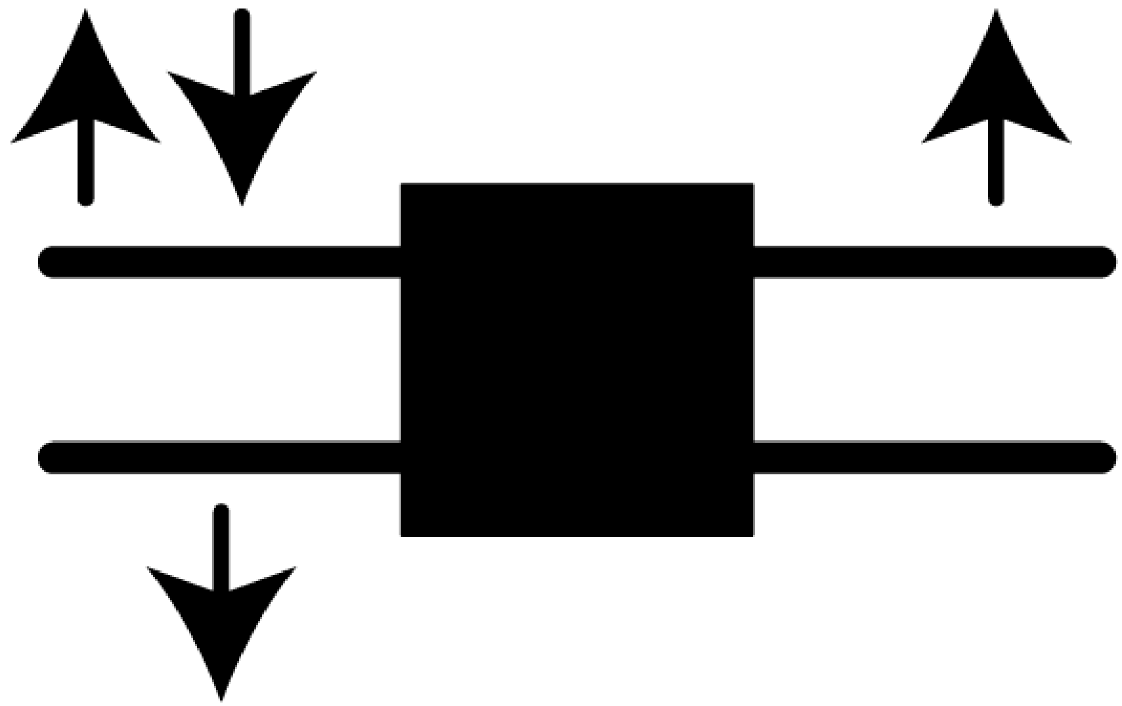


Experimental Result: D. Mace *et al* Phys. Rev. B





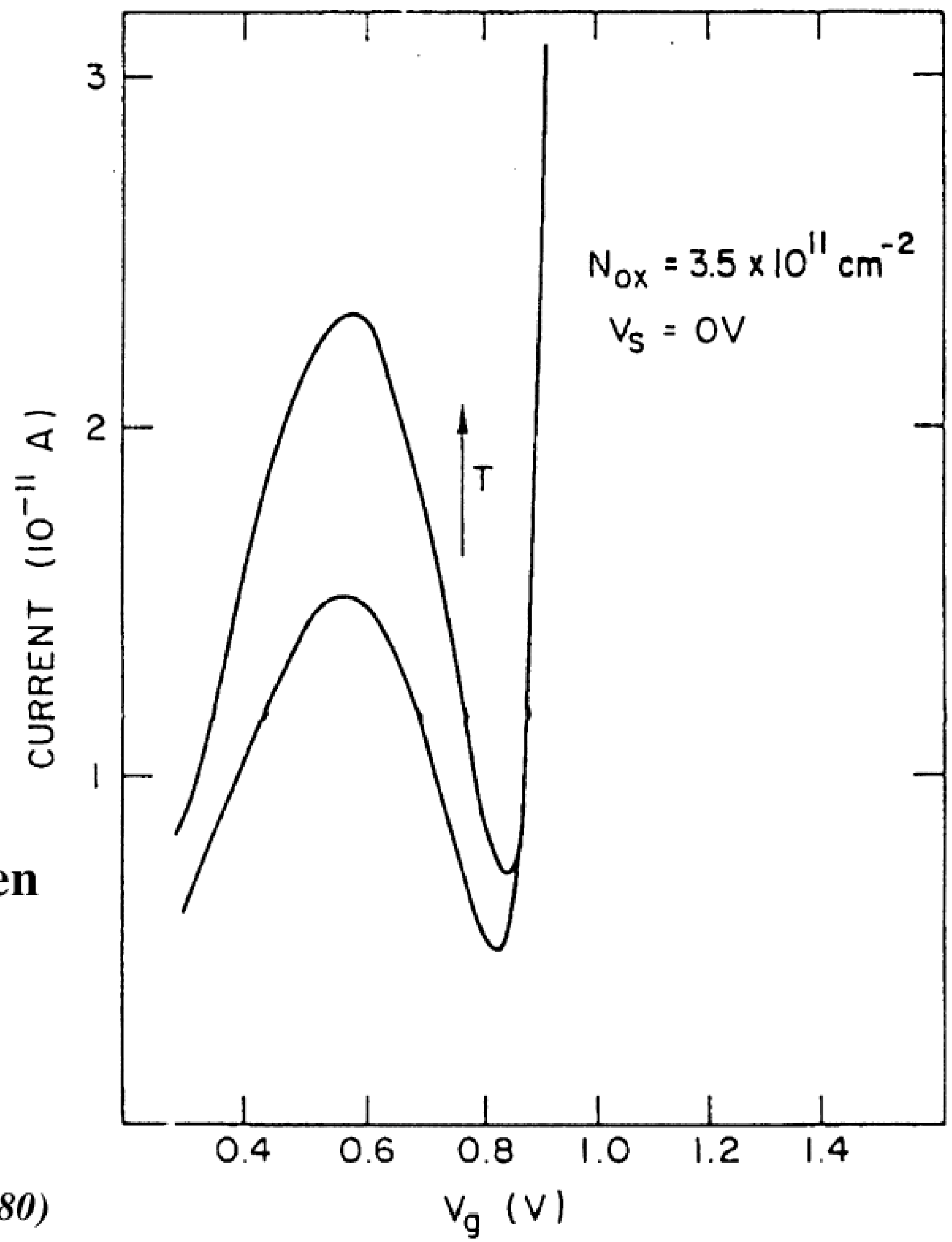




Quantum processor 1.

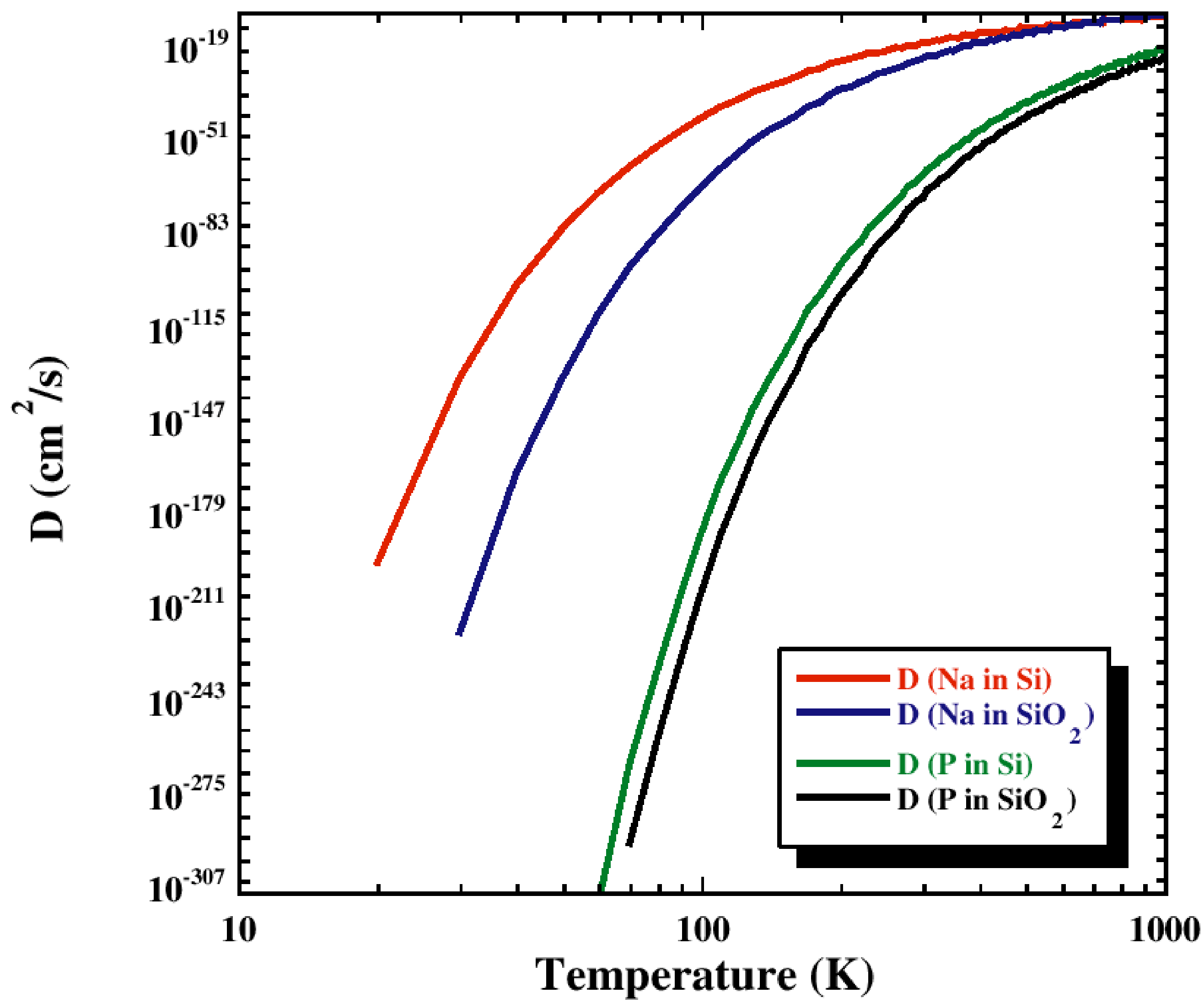
Sodium in MOSFETS

- **Na deposited before Al gate metal**
- **Gate bias to drift Na to Si/SiO_2 interface**
- **Na laterally spaced between 10 nm and 32 nm**

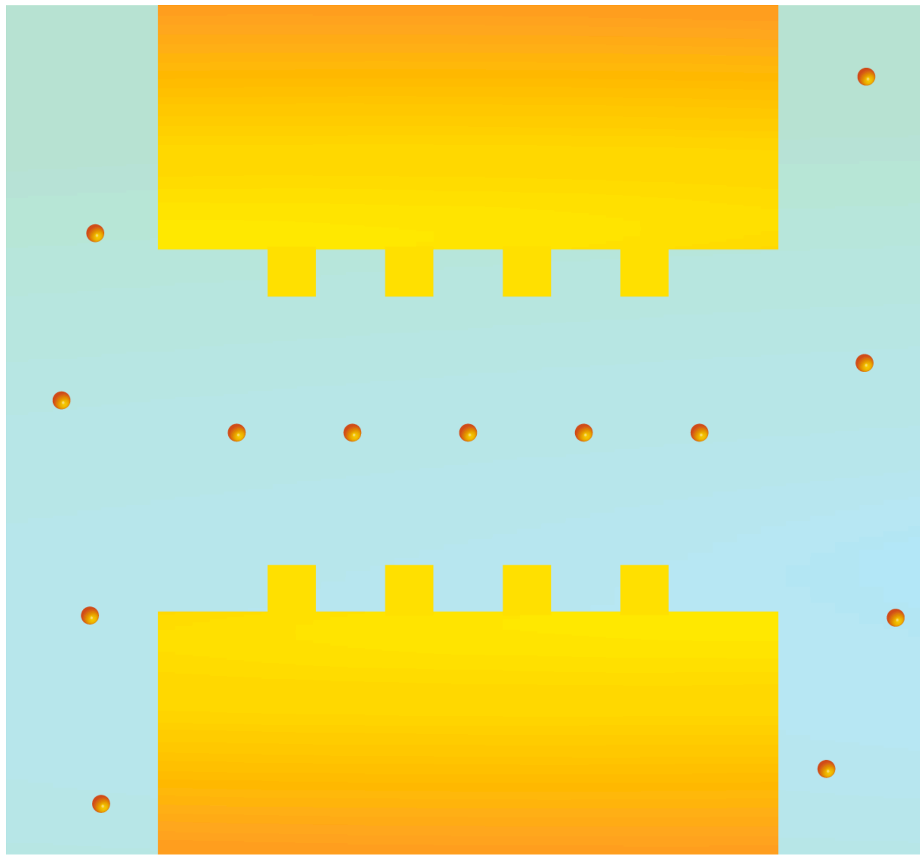


Fowler + Hartstein, Phil. Mag. B 42, 949 (1980)

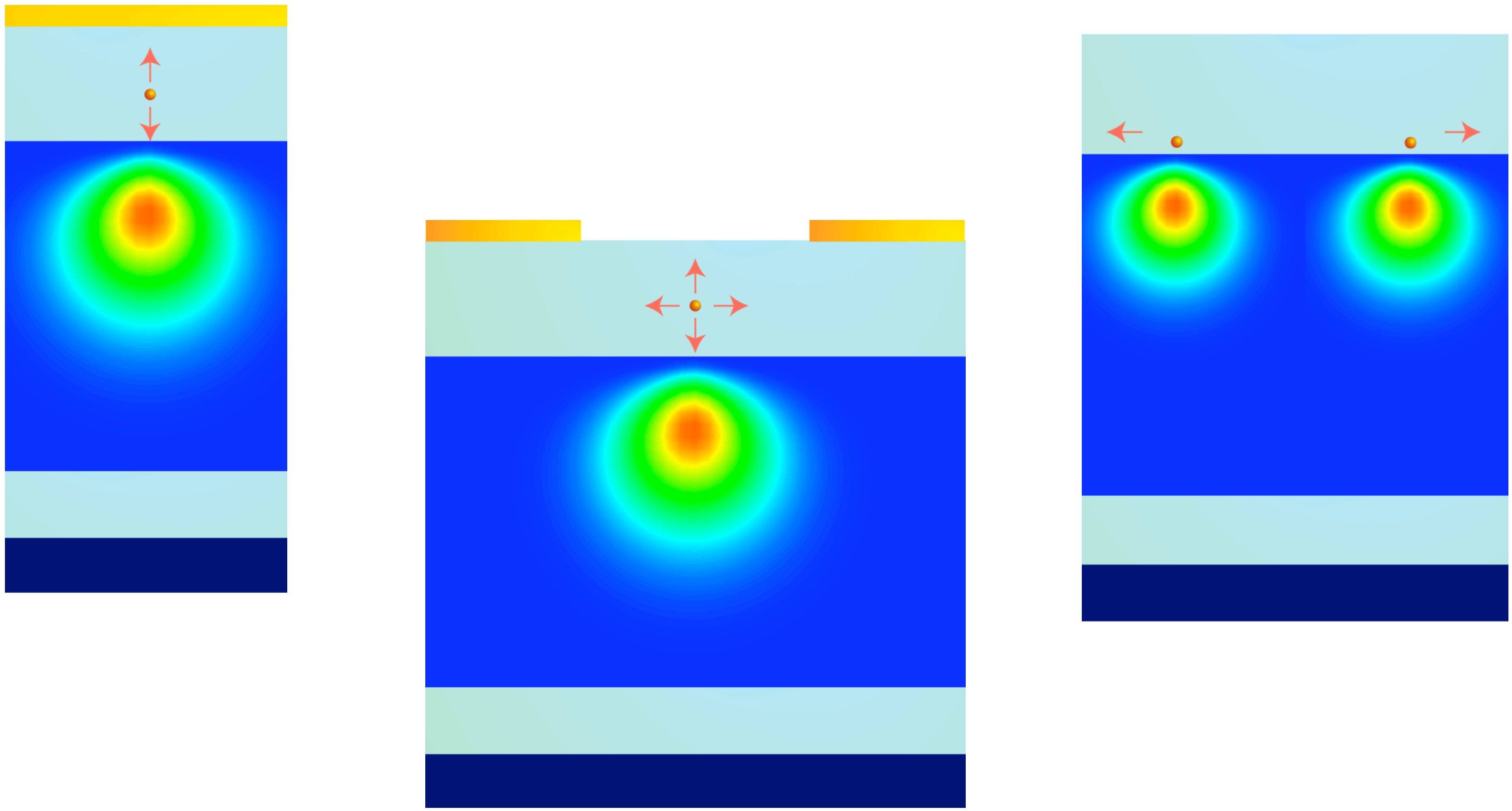
Temperature Dependence Ion Diffusivity



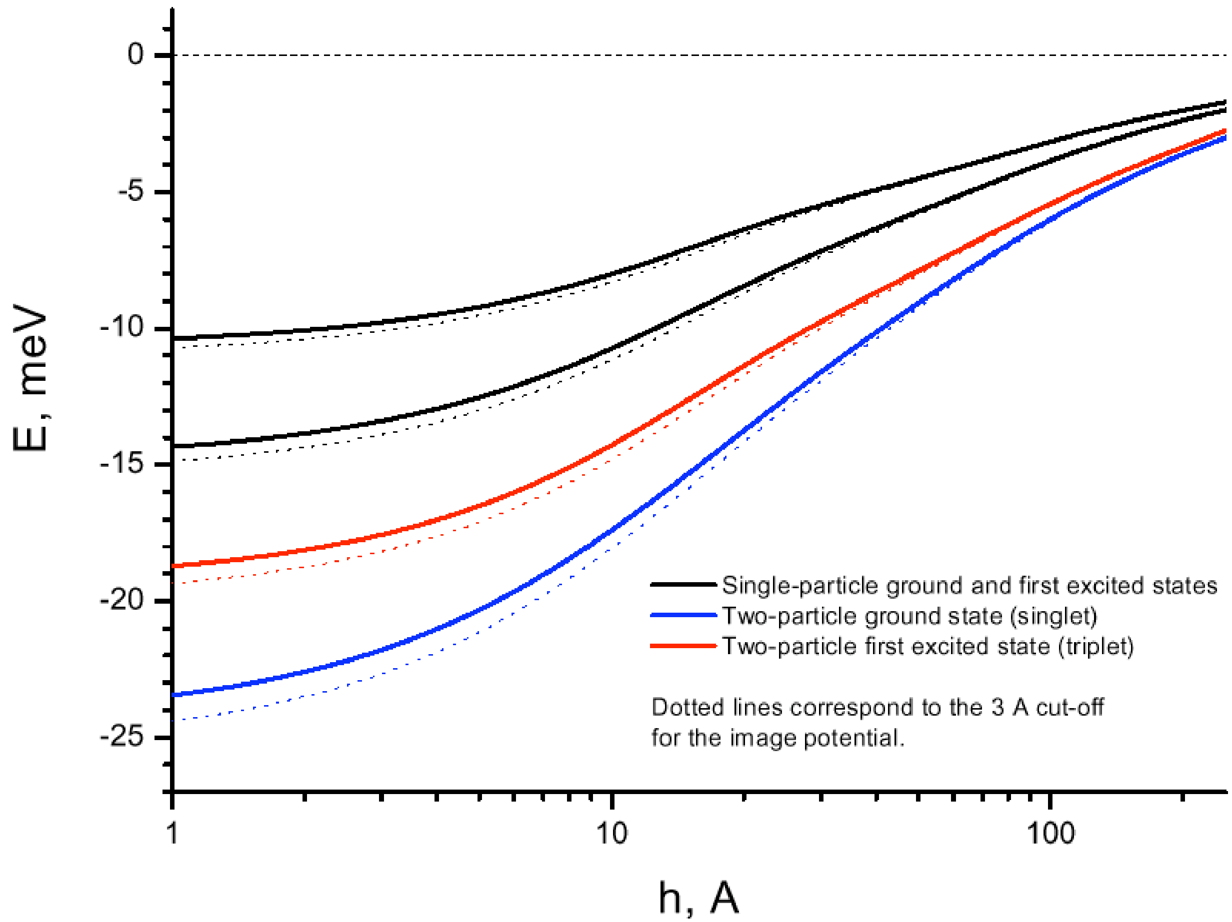
Creation of a surface Ion trap.

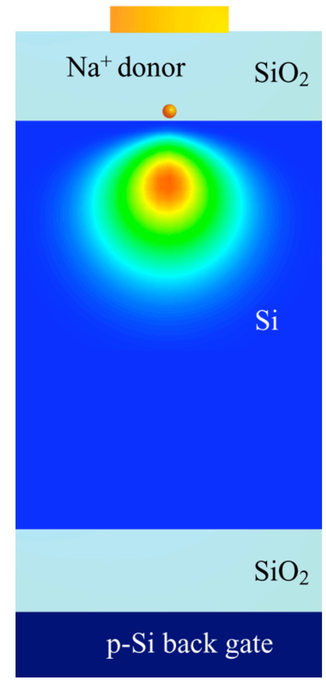


Creation of a surface Ion trap.

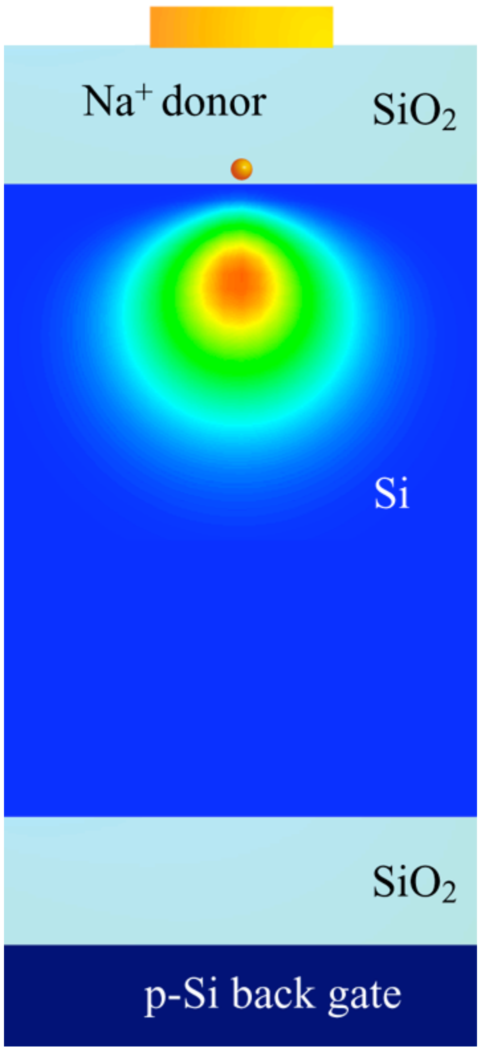


Single and two-particle states for Na^+ at a Si/SiO_2 boundary.





Electrons bound to Na^+ as qubits.



(1.) Scalable physical system with
well characterised qubits.

(5.) Long relevant decoherence times,
much longer than gate operation time.

Possible decoherence mechanisms

Spin-orbit interaction:

$$H_{SO} = \frac{\hbar}{(2M_0c)^2}\nabla V(r).(\hat{\sigma} \times \hat{\mathbf{p}})$$

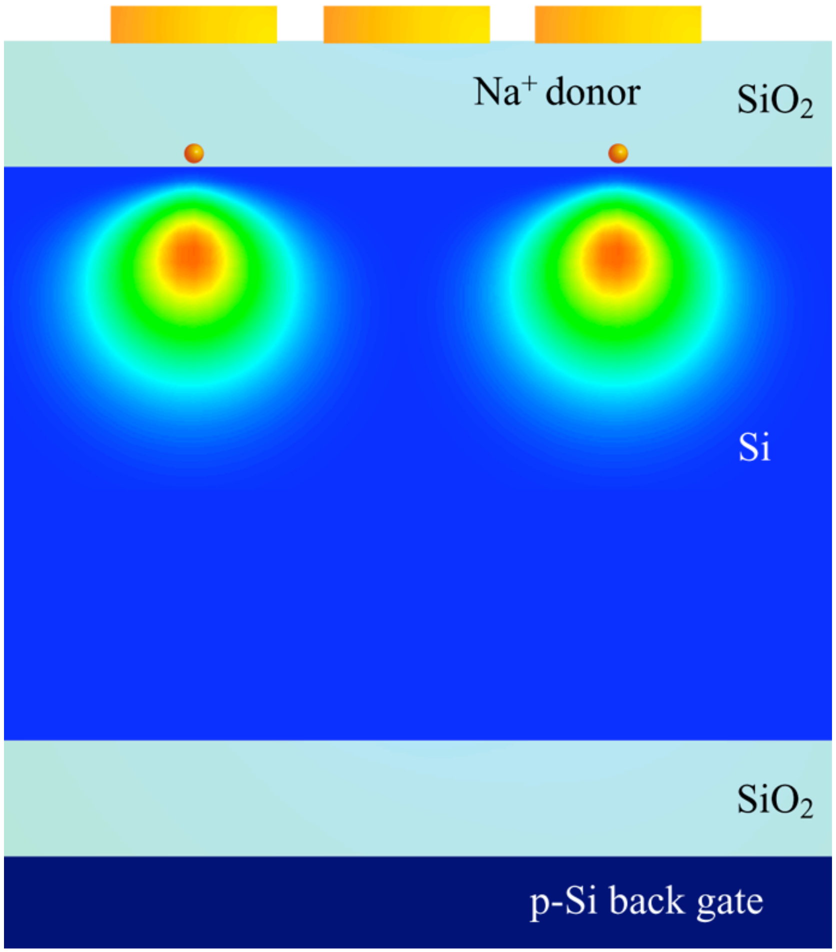
V(r) : - crystal potential, confining/disorder potentials, acoustic phonons, Coulomb interaction.

Exchange interaction:

$$H_{EX} = J\hat{\mathbf{S}}_1.\hat{\mathbf{S}}_2$$

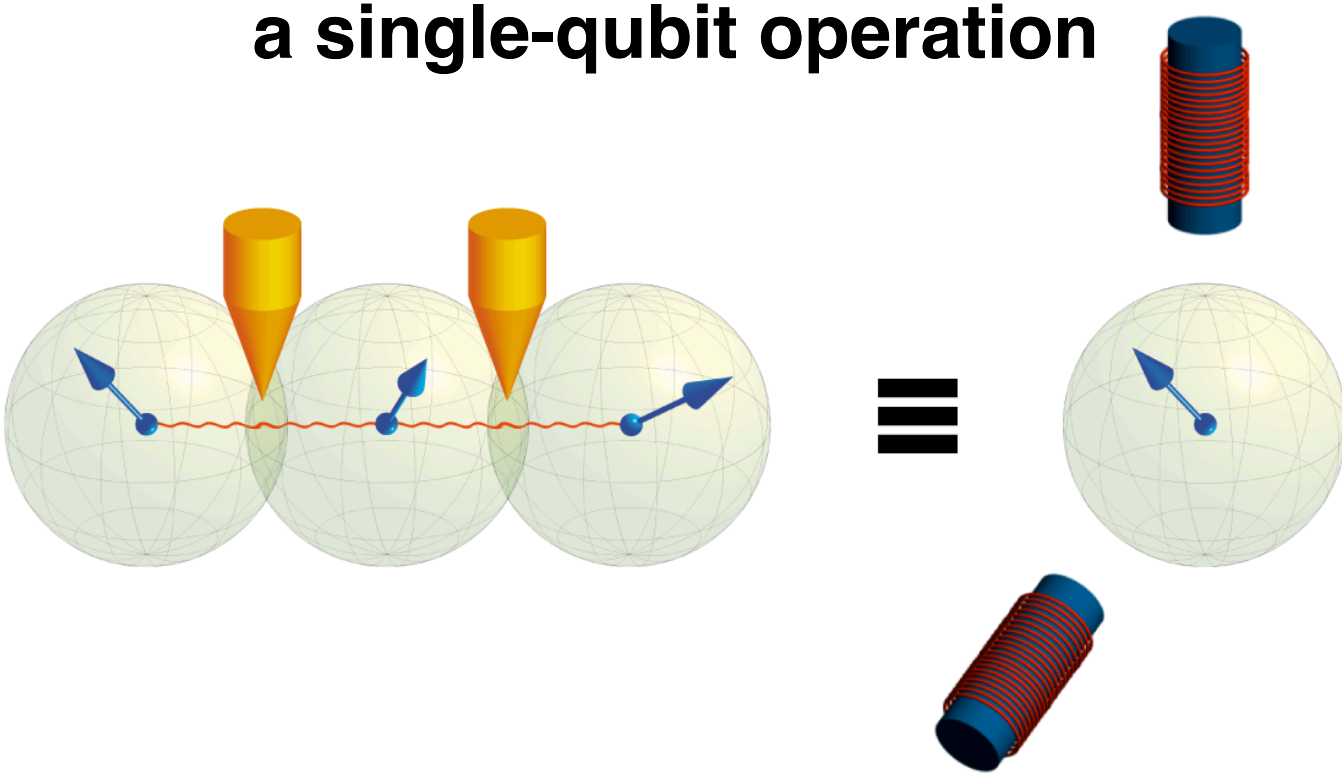
S_2 : - impurity or crystal fault spin, spin of other electrons , nuclear spin.

Entanglement of bound electrons



(4.) A “universal” set of gates

Using the exchange interaction to perform a single-qubit operation



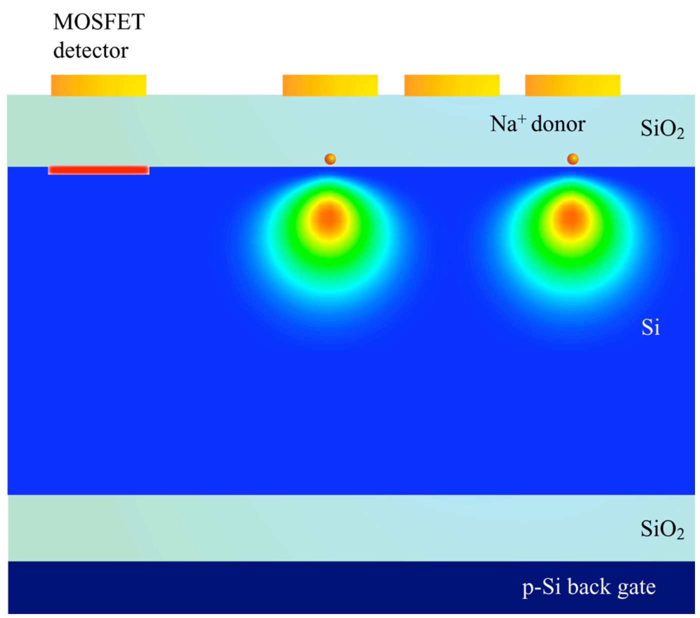
$$|\Uparrow\rangle = |S\rangle|\uparrow\rangle$$
$$|\Downarrow\rangle = \frac{1}{\sqrt{2}}\left(|T_+\rangle|\downarrow\rangle + |T_0\rangle|\uparrow\rangle\right)$$

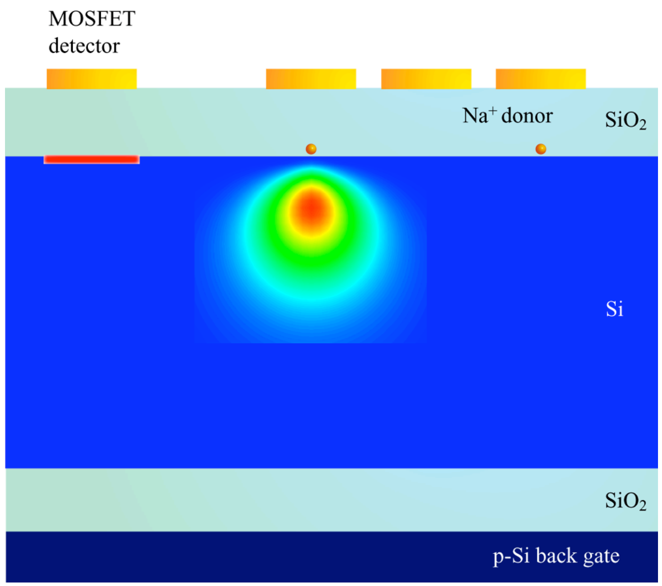
$$|T_+\rangle = |\uparrow\uparrow\rangle$$
$$|T_0\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$$
$$|T_-\rangle = |\downarrow\downarrow\rangle$$

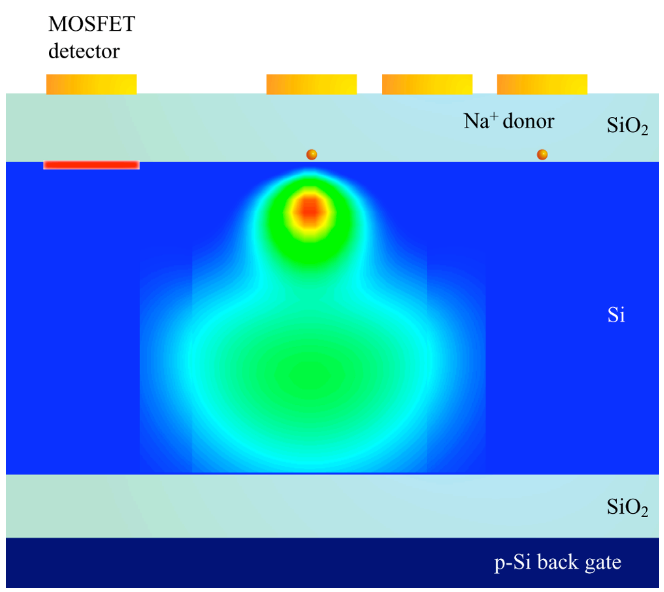
$$|S\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$$

D. DiVincenzo, Nature

Spin-dependent measurement.





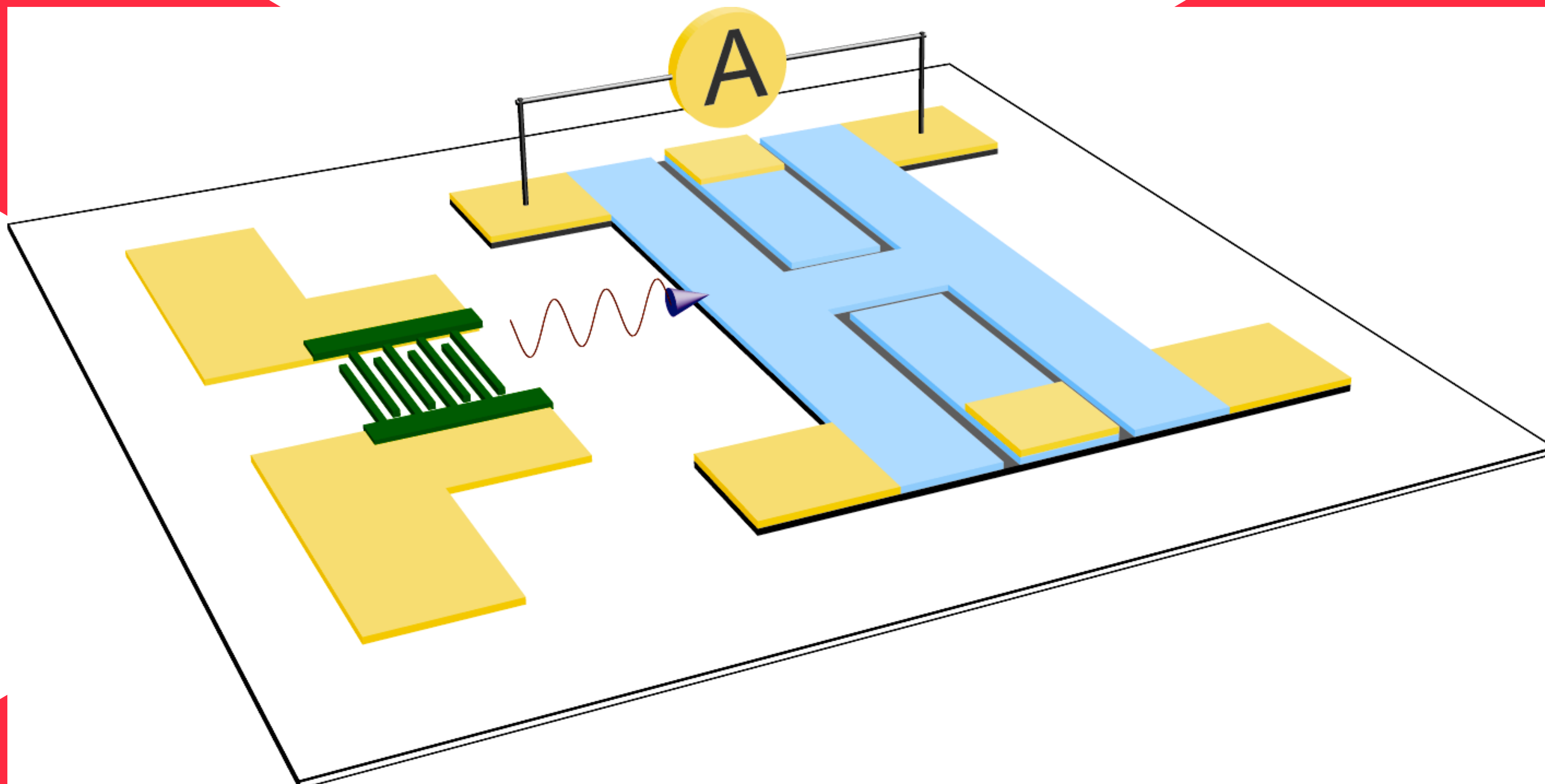


Singlet and triplet states:

- Both bound.
- Different charge distributions.

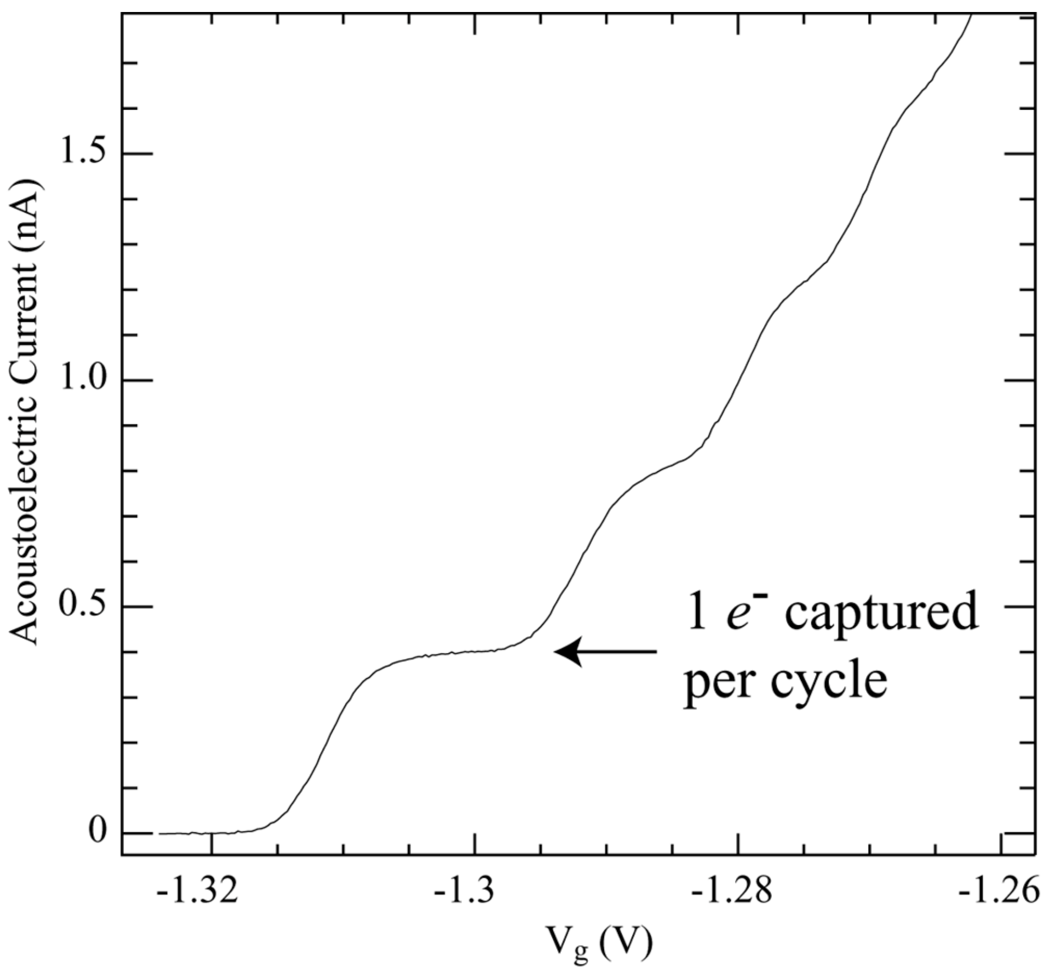
Quantum processor 2.

SAW quantised current device in GaAs



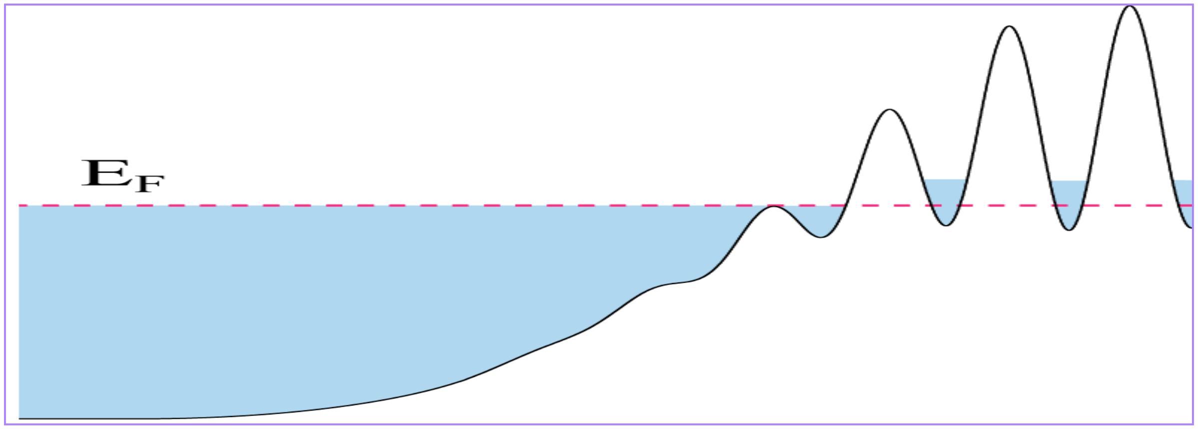
J. M. Shilton *et al*
V. I. Talyanskii *et al*

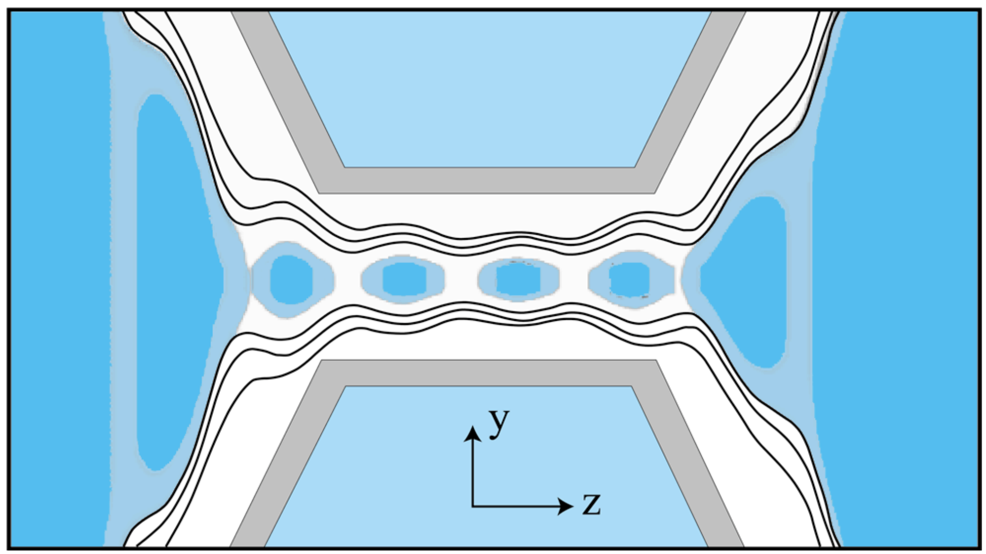
Quantised acoustoelectric current



J. M. Shilton Ph.D Thesis University of Cambridge

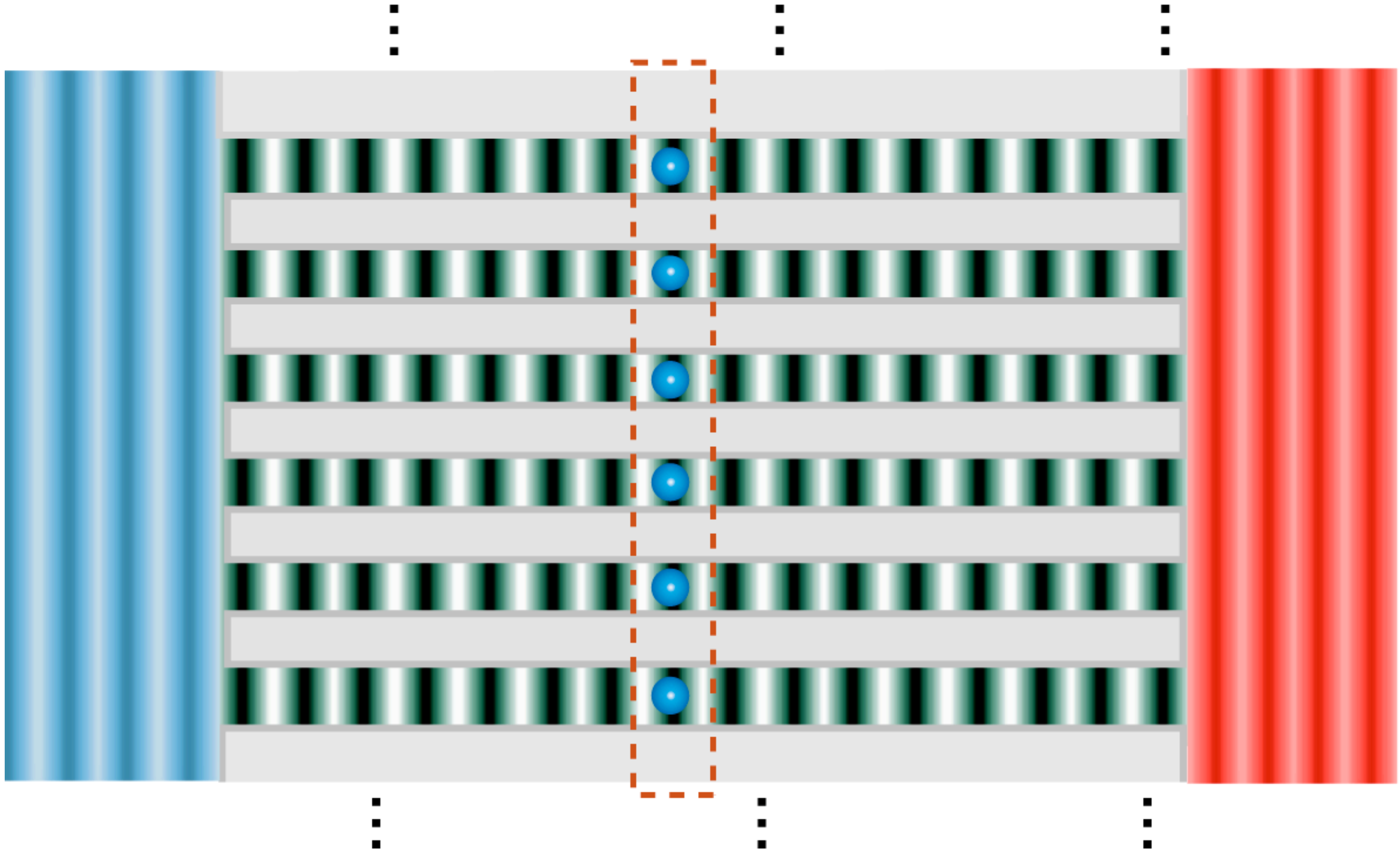
Electron capture





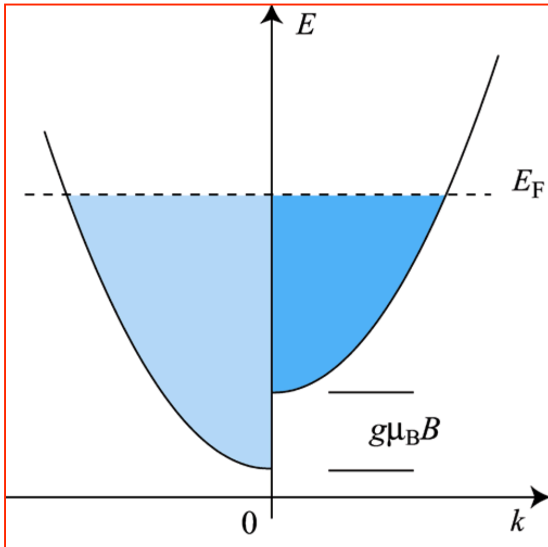
Robinson and Barnes PRB

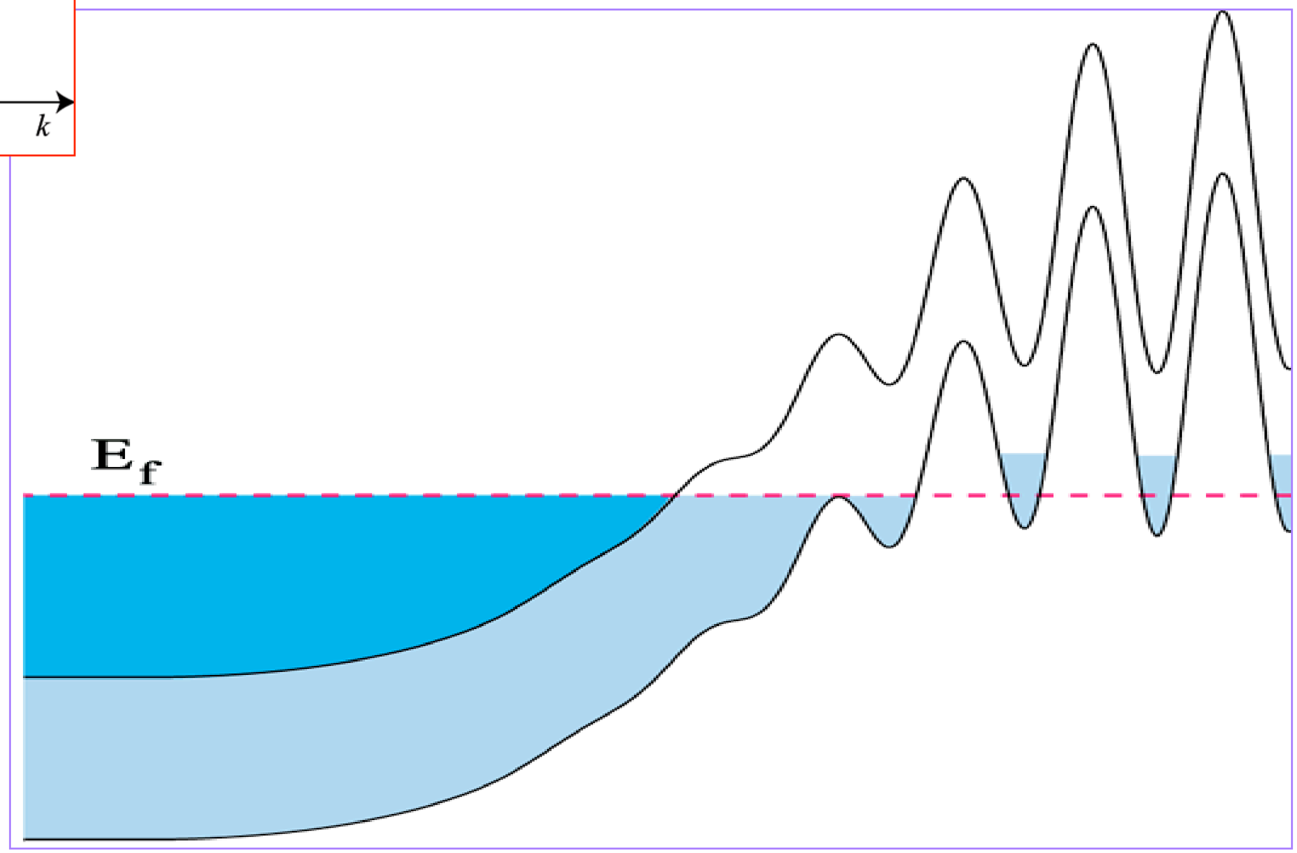
N - qubits



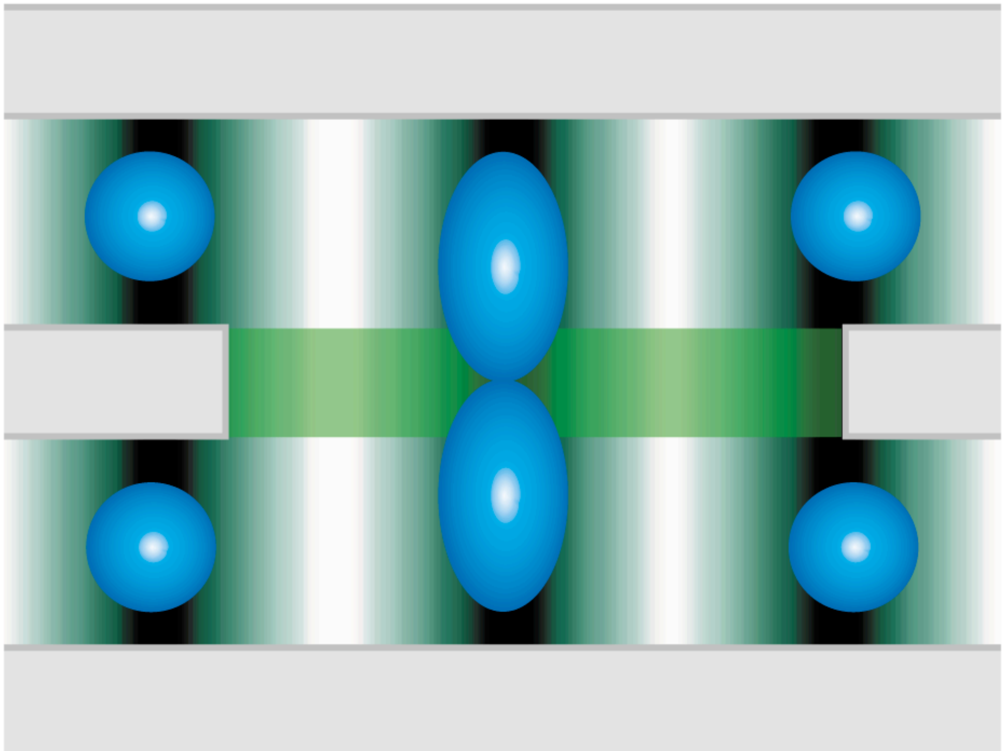
B. Kardynal *et al* PRB
J. Ebbecke *et al* cond matt/0006487
A. North

The initial state





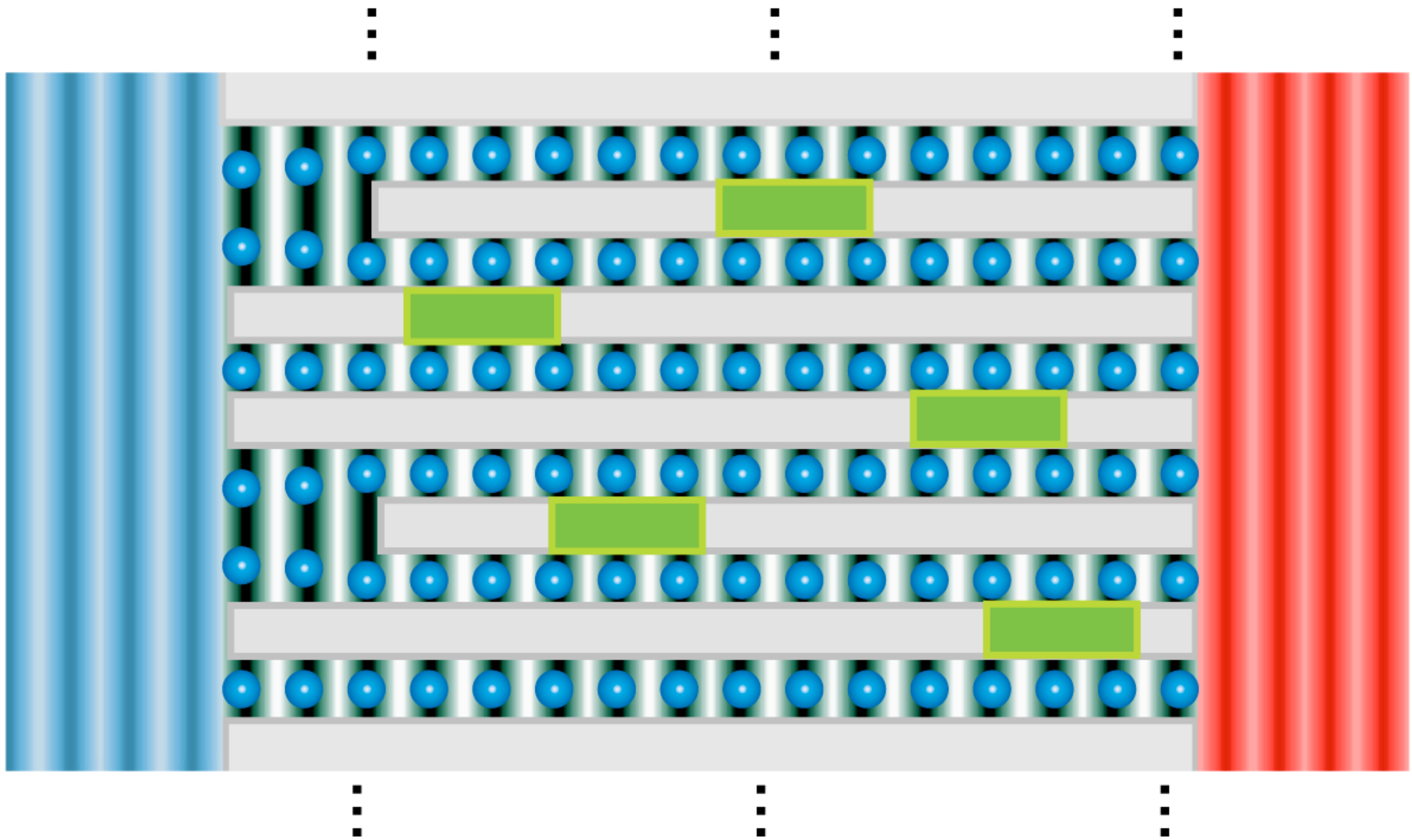
A universal set of gates



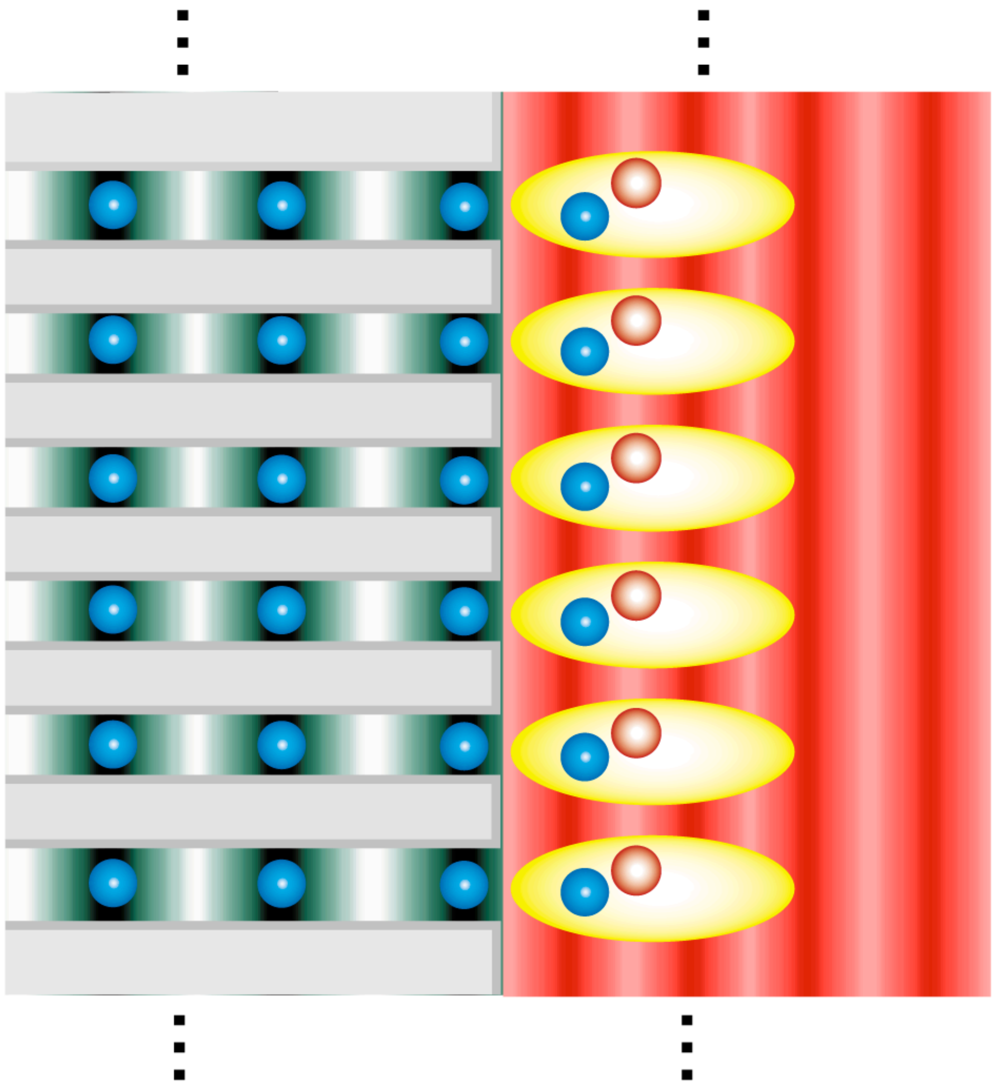
DiVincenzo *et al* Nature
Barrett and Barnes PRB

$$H = \sum_{\sigma=\uparrow,\downarrow}\left(\frac{1}{2}(\sigma S - V)c^\dagger_{1,\sigma}c_{1,\sigma} + \frac{1}{2}(\sigma S + V)c^\dagger_{2,\sigma}c_{2,\sigma} + \nu\left(c^\dagger_{1,\sigma}c_{2,\sigma} + c^\dagger_{2,\sigma}c_{1,\sigma}\right)\right) + U\left(n_{1,\uparrow}n_{1,\downarrow} + n_{2,\uparrow}n_{2,\downarrow}\right)$$

A quantum processor



Optical readout



Electron-Hole recombination in GaAs

$$e^{-\frac{1}{2}} + h^{+\frac{3}{2}} \to \gamma^{+1}$$

$$e^{+\frac{1}{2}} + h^{-\frac{3}{2}} \to \gamma^{-1}$$

C. L. Foden *et al*

Decoherence mechanisms in semiconductors

$$H_{SO} = \frac{\hbar}{(2M_0c)^2}\nabla V(r).(\hat{\sigma} \times \hat{\mathbf{p}})$$

Spin-orbit interaction:

V(r) : - crystal potential, confining/disorder potentials, acoustic phonons, Coulomb interaction.

$$H_{EX} = J\hat{\mathbf{S}}_1.\hat{\mathbf{S}}_2$$

Exchange interaction:

S_2 : - impurity or crystal fault spin, spin of other electrons , nuclear spin.