

# Experimental Validation of the Kinematic Theory of Unsteady Separation

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**We present a study of a new kinematic theory of unsteady separation. The approach allows for the determination of both the location and orientation of separation in a two-dimensional unsteady fluid flow based solely upon surface shear-stress and pressure measurements. The experimental results obtained for a rotor-oscillator flow are compared with complementary numerical simulations. It is clearly identified that in both periodic and certain aperiodic unsteady fluid flows, separation does not occur at a point of instantaneous zero skin-friction, but rather at a point where the time-averaged skin-friction is zero.**

## Nomenclature

|              |   |                                                        |
|--------------|---|--------------------------------------------------------|
| $A$          | = | peak-to-peak amplitude of oscillation                  |
| $a$          | = | cylinder radius                                        |
| $c$          | = | cylinder distance from boundary                        |
| $\gamma$     | = | location of the separation point on the $y=0$ boundary |
| $\nu$        | = | kinematic viscosity                                    |
| $p$          | = | pressure                                               |
| $\mathbf{p}$ | = | the separation point $(\gamma, 0)$                     |
| $\rho$       | = | fluid density                                          |
| $r, s, t, T$ | = | generic time variables                                 |
| $t_0$        | = | the present time                                       |
| $\tau_w$     | = | wall shear-stress                                      |
| $\mathbf{v}$ | = | velocity $(u, v)$                                      |
| $u, v$       | = | horizontal and vertical velocity components            |
| $\mathbf{x}$ | = | coordinates $(x, y)$                                   |
| $x, y$       | = | horizontal and vertical coordinates                    |
| $Re$         | = | Reynolds number                                        |
| $Sr$         | = | Strouhal number                                        |

## I. Introduction

UNSTEADY flow separation plays a pivotal role in the stability, maneuverability and efficiency of aerospace technology. For example, dynamic stall at high Reynolds numbers, initiated by boundary-layer separation near the leading edge of an airfoil, limits the performance of modern air vehicles. The ability to monitor and control unsteady separation is therefore crucial for the next generation of helicopters, aircraft and projectiles.

The goal of this research is to implement a new kinematic theory of unsteady separation that identifies both the location and geometry of two-dimensional unsteady separation based on distributed surface shear-stress and pressure measurements. The study utilizes both experiments and numerical simulations of a rotor-oscillator flow to provide the first set of results towards achieving this goal.

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## II. Background

In his classic paper, Prandtl<sup>1</sup> derived a separation criterion for two-dimensional steady incompressible velocity fields  $\mathbf{v}(\mathbf{x})=(u(x,y),v(x,y))$  that satisfy the no-slip boundary condition  $u(x,0)=v(x,0)=0$  on the  $y=0$  boundary. Prandtl's criterion states that separation takes place at a boundary point  $\mathbf{p}=(\gamma,0)$  whenever

$$u_y(\gamma,0) = 0, \quad u_{xy}(\gamma,0) < 0. \quad (1)$$

These conditions are necessary for separation on all length scales, including both boundary-layer separation and small-scale separation within a boundary layer. The zero-skin-friction principle<sup>1</sup> has become the most broadly used indicator of separation, even though numerical work in the 1970's by Sears and Tellionis<sup>2,3</sup> and others showed that the principle fails for unsteady flows. Furthermore, it is now widely understood that even in steady three-dimensional flows, separation is rarely associated with the vanishing of wall shear-stress<sup>4</sup>.

Concluding that vanishing skin friction "does not denote separation in any meaningful sense in unsteady flow", Sears and Tellionis<sup>3</sup> proposed a separation criterion known as the Moore-Rott-Sears (MRS) principle. This principle locates separation at off-boundary points where the wall-tangential shear component vanishes, and the local streamwise velocity equals the velocity of the moving separation structure. This postulate, however, requires the a priori knowledge of the separation speed, making the MRS principle practically inapplicable. Indeed, the principle relies on information that is not foreknown for rapidly maneuvering air vehicles<sup>5,6</sup>.

As an alternative, Van Dommelen<sup>5</sup> and Van Dommelen and Shen<sup>7</sup> proposed that separation takes place at singularities in the solution of the boundary layer equations. Analytic results show, however, that separation in the boundary layer equations has no direct connection with velocity singularities<sup>8</sup>. In addition, physical velocity fields display no singularities, making Van Dommelen's principle inapplicable to Navier-Stokes flows. Nevertheless, Van Dommelen's Lagrangian view -- that unsteady separation is to be understood as the formation of a material spike -- was a major advance. The so-called Van-Dommelen-Shen process<sup>9</sup> is consistent with the narrow eruptive spires readily observable in careful separation experiments at both high and low Reynolds numbers<sup>10</sup>. On pitching airfoils, the erupting shear layer is unstable, and the subsequent roll-up leads to the formation of the dynamic stall vortex.

Studying time-periodic incompressible flows, Shariff, Pulliam and Ottino<sup>11</sup> realized that fixed separation (i.e., separation at a constant location) in unsteady flows can indeed be viewed as material ejection from the boundary along a time-dependent unstable manifold, which is a material line that shrinks to the separation point in backward time. A schematic of fixed separation along an unstable manifold is shown in figure 1. Due to the no-slip boundary condition, the unstable manifold of figure 1 is non-hyperbolic: it cannot be located from a linear analysis of the flow at the wall. For this reason, locating such manifolds in compressible flows with general time-dependence has remained a formidable technical challenge. As a further difficulty, moving separation (i.e., separation at time-varying locations) cannot be explained by unstable manifolds, because material lines emanating from a no-slip boundary remain anchored at the same boundary point for all times. In this paper we will concern ourselves with fixed separation; moving separation will be described in later publications.

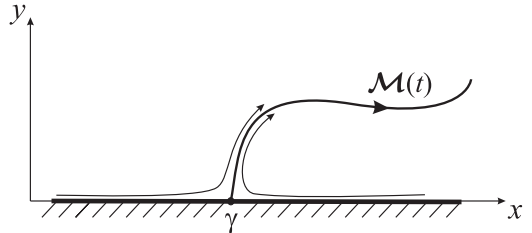


Figure 1: Fluid separation along an unstable manifold, originating from a fixed point  $(\gamma,0)$  on the boundary.

In the absence of a theoretically-sound and practical separation criterion for unsteady two- and three-dimensional flows, experimentalists have been forced to approach separation empirically. Examples include using oil films or directionally sensitive hot-films to locate converging skin-friction lines<sup>12</sup>, and determining local minima in the skin friction magnitudes accompanied by a 180° phase relationship between adjacent shear-stress sensors<sup>13</sup>. Despite such approaches, the experimental detection and control of unsteady separation has remained ambiguous<sup>14</sup>.

## III. An exact two-dimensional theory

Recently, Haller and co-workers<sup>15,16</sup> have derived an analytic criterion for the location of wall-based unstable manifolds in a two-dimensional velocity field  $\mathbf{v}(\mathbf{x},t)=(u(x,y,t),v(x,y,t))$ . These wall-based unstable manifolds are the eruptive material spires reported in both low and high Reynolds number separation<sup>9</sup>. According to the new criterion, any fixed separation point  $\mathbf{p}=(\gamma,0)$  satisfies the following conditions:

$$\begin{aligned}
& \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^t v_y(\gamma, 0, s) ds} u_y(\gamma, 0, t) dt = 0, \\
& \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^t v_y(\gamma, 0, s) ds} \left[ u_{xy}(\gamma, 0, t) + u_y(\gamma, 0, t) \int_{t_0}^t v_{xy}(\gamma, 0, s) ds \right] dt < 0, \\
& \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^t v_y(\gamma, 0, s) ds} v_{yy}(\gamma, 0, t) dt > 0.
\end{aligned} \tag{2}$$

The conditions in (2) are valid for any mass-conserving fluid flow, compressible or incompressible, subsonic, supersonic or transonic. They require a knowledge of the time history of kinematic quantities along the wall, such as velocity gradients  $u_y(\gamma, 0, t)$ , up to the present time  $t_0$ . An example in which such a fixed separation point has been identified is separating flow past a cylinder<sup>11</sup>.

The first condition in (2) is the most important one: it is a necessary condition for the existence of a material separation or reattachment profile at  $x=\gamma$ , requiring that the unstable manifold remains bounded away from the  $y=0$  boundary. The second inequality in (2) -- a non-degeneracy condition -- guarantees a unique separation profile by ensuring that all material lines, other than the unstable manifold, align with the wall in backward time. The third condition in (2) is to distinguish  $\mathbf{p}$  from a reattachment point, ensuring material ejection from  $\mathbf{p}$  into the mean flow. This condition is intimately related to the strength of separation, and can be used to distinguish between small scale separation and boundary-layer separation in a physical situation. Together, the three conditions in (2) yield a sufficient criterion for fixed separation in unsteady two-dimensional flows field that admit a finite asymptotic average in time.

As an example of the application of criterion (2), consider incompressible flows, for which  $u_{xy}(x, 0) = -v_{yy}(x, 0)$ , and the no-slip boundary condition implies  $u_x(x, 0, t) = v_y(x, 0, t) = 0$ . Thus, the separation criterion (2) simplifies to

$$\begin{aligned}
& \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0-T}^{t_0} u_y(\gamma, 0, t) dt = 0 \\
& \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0-T}^{t_0} u_{xy}(\gamma, 0, t) dt < 0.
\end{aligned} \tag{3}$$

For steady incompressible flows, (3) further simplifies to the original Prandtl criterion<sup>1</sup>.

All of the conditions in (2) can be experimentally verified by measuring the skin friction  $\tau_w(x, t)$ , the fluid density at the wall  $\rho(x, 0, t)$  and the viscosity  $\nu$ , since it can readily be shown that

$$u_y(x, 0, t) = \tau_w(x, 0, t) / (\nu \rho(x, 0, t)), \quad e^{-\int_{t_0}^t v_y(x, 0, s) ds} = \rho(x, 0, t) / \rho(x, 0, t_0). \tag{4}$$

For incompressible flow, (3) and (4) show that a knowledge of the time history of wall shear-stress is all that is required to identify the location of fixed unsteady separation. Indeed, one can also consider condition (3) to apply more generally, as the effects of compressibility do not become important until the free stream Mach number reaches approximately 4 (and the formation of shocklets is not seen in boundary layer turbulence until the free stream Mach number of the flow reaches approximately 7)<sup>17</sup>.

In addition to monitoring the location of a separation point in a two-dimensional flow, Haller<sup>15</sup> also derived analytic formulae for the time-dependent unstable manifold (separation profile) in compressible flow along which particles are ejected from the wall. The time-dependent separation angle can be calculated from distributed skin-friction and pressure measurements along the wall, and for the particular case of incompressible flow the relation simplifies to

$$\tan \alpha(t_0) = - \lim_{T \rightarrow \infty} \frac{3 \int_{t_0-T}^{t_0} \tau_{w,x}(\gamma, s) ds}{\int_{t_0-T}^{t_0} \left[ p_x(\gamma, 0, s) + 3 \tau_{w,x}(\gamma, s) \int_{t_0}^s \frac{\tau_w(\gamma, r)}{\nu \rho} dr \right] ds}, \quad (5)$$

where  $\alpha$  is the angle the separation profile makes with the boundary and  $p(x, 0, t)$  is the surface pressure.

#### IV. The rotor-oscillator flow

The goal of this study is to implement criteria (2) in a physical setting for the first time. For practical reasons we choose a low Reynolds number flow ( $Re < O(1)$ ), for which the time- and length scales of unsteady separation are readily amenable to both experimental investigation and simulation. The emphasis of these experiments is on proof-of-concept rather than immediate practical application. In making this choice, however, we emphasise an important point. The theoretical basis for the approach of Haller<sup>15</sup> is kinematical, and therefore independent of Reynolds number. Thus, these experiments are a logical first step towards implementation of the method in more practically relevant geometries, such as flow past a backwards facing step or an airfoil.

The flow we investigate is the so-called rotor-oscillator flow - a two-dimensional flow generated by a local source of angular momentum. This may be realized experimentally by considering a cross section of the flow generated by a rotating cylinder adjacent to a rigid boundary (the flow is considered uniform along the length of the cylinder and therefore nominally two-dimensional). The rotor-oscillator flow has been investigated in detail by Hackborn and co-workers<sup>18</sup>. To the best of our knowledge, this arrangement is the simplest geometry in which one can establish and influence a low Reynolds number separating flow. A sketch of the separation generated by a rotating cylinder near a flat boundary is presented in figure 2. Flow separates from the boundary at point to the right of the cylinder, driven by the anti-clockwise rotation. Unsteadiness can be introduced into the system by moving the position of the cylinder from side-to-side. At low Reynolds number, when the flow is to a good approximation a Stokes flow, time-dependence only enters the problem through the boundary conditions and the velocity field associated with the rotating cylinder simply translates along with the source<sup>15</sup>.

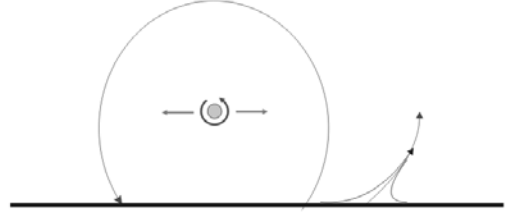


Figure 2: Sketch of separation driven by the translation of a rotating cylinder, adjacent to a boundary.

The experiment comprised a 40cm x 10cm x 10cm acrylic tank filled with viscous fluid (glycerol) that sat inside an aluminum support stand. A cylinder 10cm long and 6 mm in diameter was positioned vertically in the tank, parallel to one of the sidewalls. Rotation of the cylinder was driven by a micro-stepper motor, which in turn was mounted on a linear traverse. The rotation generated a nominally two-dimensional flow, uniform along the length of the cylinder. In a horizontal cross-section of the experiment the rotating cylinder generated a recirculation bubble that separated and reattached to the vertical sidewall. In some of the experiments to be described, as the cylinder rotated it was translated back and forth with desired time dependence (e.g. periodic or random) using a National Instruments motion control system.

Visualization of flow within the tank was achieved through the mechanical injection of neutrally buoyant dye adjacent to the boundary. The dye was either injected as a material line prior to commencement of the experiment using a syringe, or as streaklines through small holes in the boundary surface. When separation occurred the dyed fluid was drawn away from the wall and into the bulk. The images of separation were captured using a CCD camera looking up through the tank, via a 45 degree mirror placed beneath the experiment, and the location of separation was identified by eye from the images taken. An image showing an example of the separating material spike is presented in figure 3.

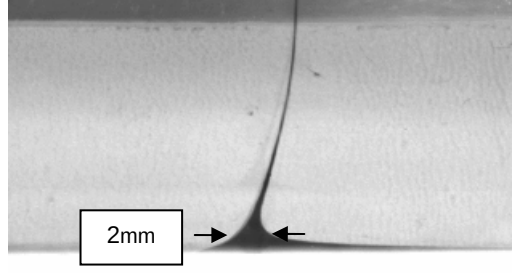


Figure 3: Dye visualization of material spike separating from the wall.

Shear-stress sensors were used to investigate the steady flow in the experiment. The sensors were hot wires flush-mounted running in constant temperature mode. A sensor comprised a 1mm piece of 15 micron diameter platinum wire, mounted between electrode connections on a circuit board substrate. The sensors, which have been successfully utilized in active control of turbulent boundary layers<sup>19</sup>, were fabricated and calibrated 'in-house'. The calibration was achieved by positioning the sensors in the experimental tank, under known flow conditions driven by the rotating cylinder. The directionality of shear-stress for the two-dimensional experiments, which is required by criterion (3), is readily inferred from knowledge of the magnitude of the shear-stress field and the position of the cylinder. Consequently, there was no need to use angled hotwire pairs to achieve directionally sensitive shear-stress sensors in this experiment.

Finally, we simulated the flow numerically using two different approaches. Firstly, a direct numerical simulation of the 2D flow was performed using FLUENT. This numerical solution could also be very closely reproduced the analytical solution for a rotor-oscillator flow obtained by Hackborn<sup>18</sup>, and for ease of processing we therefore used time-dependent translations of the analytical profile to simulate the periodic and aperiodic flow.

## V. Experimental Results

### A. Fixed separation in steady flow

The first set of experimental results concern steady flow in the experiment. These results were obtained (i) to reproduce the classic result that in a steady flow separation occurs at a point of zero skin-friction and negative skin-friction gradient; and (ii) to confirm the realization of the theoretical flow within our experimental arrangement.

Initially the position of the fixed separation point on the boundary was studied as a function of both the cylinder distance from the wall  $c$  and the rotational speed of the cylinder  $\omega$ . The location of the experimental separation point was determined using the thermal shear-stress sensors to detect the location of minimum (zero) skin-friction. This procedure could typically be achieved with an accuracy of  $\pm 1$  mm. For comparison, we also simulated the two-dimensional flow field using FLUENT and identified the location of separation using the zero skin-friction point in the numerical data. The results of one set of experiments for  $c=2.5$  cm are plotted in figure 4, in which the vertical axis is ratio of the numerical to experimental separation location, and the horizontal axis is the Reynolds number  $Re = \omega a^2 / \nu$ , where  $a$  is the cylinder radius (3 mm) and  $\nu$  is the kinematic viscosity ( $1.19 \times 10^{-3} \text{ m}^2/\text{s}$ ). We note that the distance from the cylinder to the wall,  $c$ , is not included in the Reynolds number, which is a consequence of the scaling by Hackborn<sup>18</sup>. The agreement between experiment and numerics in figure 4 is good, with the location of the experimentally determined separation point differing by no more than 2% (which corresponded to the experimental resolution of  $\pm 1$  mm).

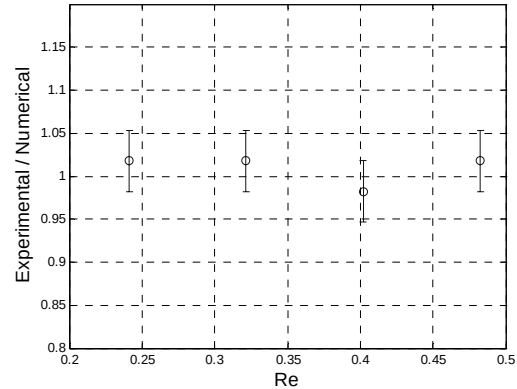


Figure 4: Comparison of experimental and numerical location of zero skin-friction point for  $c=2.5$  cm.

To investigate in more detail the nature of the flow field in the experiment we also made measurements of the steady shear-stress distribution along the wall using the thermal shear-stress sensors. The sensors were calibrated in situ, as described in section IV, and the results for a steady flow with  $\omega=60$  rad/s and  $d=2.5$  cm are shown in figure 5. The typical shear stress levels in the experiment were on the order of 6 Pa, which is obtained by multiplying the vertical scale in figure 5 by the dynamic viscosity of pure glycerol (1.5 Pas). There is again good agreement between experiment and the numerical simulation obtained using FLUENT (on the order of 2%), emphasizing that the experimental apparatus did indeed reproduce the expected shear-stress field on the sidewall boundary.

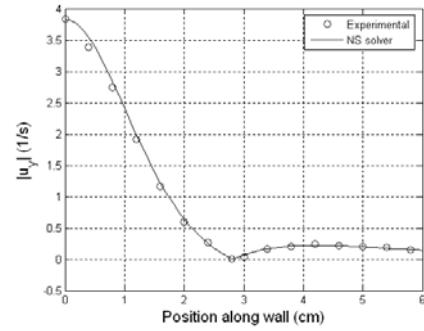


Figure 5: Experimental (circles) and numerical (line) shear-stress profile, measured at a depth of 4.5cm in the tank. The values of the shear-stress are obtained by multiplying the vertical axis by the dynamic viscosity (1.5 Pas).

## B. Fixed separation in periodic flow

The focus of the second set of experiments was to identify the location of a fixed separation point in a time periodic flow. According to criteria (3), the location of separation should be at the boundary position at which the time average shear stress is zero. We performed a series of experiments for sinusoidal side-to-side oscillations of the cylinder, with  $c=2.5$ cm and  $\omega=60$  rad/s. The amplitude of oscillation  $A$  was set at 1, 2 or 3 cm and the forcing frequency of the side-to-side oscillations  $\alpha$  was varied in the range 0.2 rad/s and 2.5 rad/s. The corresponded to a Strouhal number for the experiment ( $Sr=c^2 \alpha / 2\pi^2 \omega$ ) in the range 0 to 2.

After a short transient time period following the start of the experiment, a fixed separation point was clearly identified in these experiments using either a dye material line or streaklines injected at the boundary - the observed location of the separating material spike being independent of the dye method used. The results are plotted as a function of the governing parameters in figure 6, in which the experimentally determined location of the fixed separation point has been non-dimensionalized using results of the Stokes flow numerical simulations. The close agreement between experiments and theory supports the assumption of a Stokes flow in the experiment.

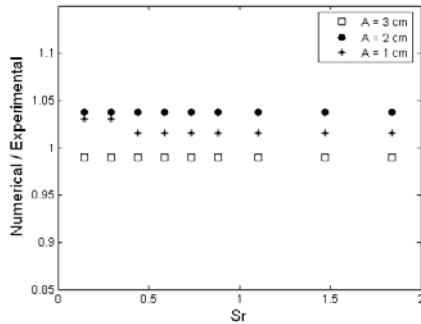


Figure 6: The location of the fixed separation point in periodic flow, plotted as a function of the Strouhal number for  $A=1$ cm , 2 cm, 3 cm.

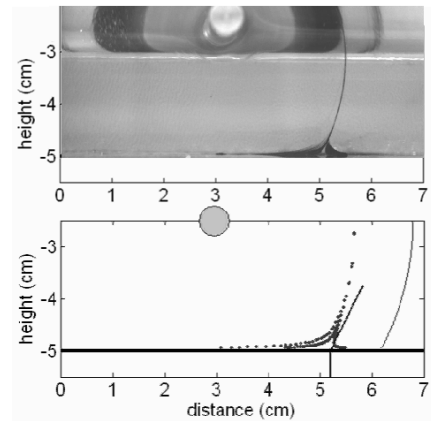


Figure 7: Experimental visualization of separating material line (Upper image). Corresponding numerical simulation (Lower image).

We attempted to use the shear-stress sensors to identify the point of zero time-average skin-friction directly from surface measurements in the experiment. However, this was not possible as the characteristic response time of the

sensors became impractically long at very low shear stress levels, preventing their being able to follow the shear-stress field. Instead, we lend support to criteria (3) and (5) through numerical simulation, using the quasi-steady Hackborn solution<sup>18</sup>. In this numerical study, boundary shear-stress and pressure data was processed to track both the location and angle of separation, and the results compared to the corresponding experimental fluid flow. One set of results from this study is presented in figure 7, in which we directly compare the experimental visualizations and numerical simulation for  $A=2$  cm and  $Sr=1.5$ . In the numerical simulation, material separation has been visualized using streaklines originating from three different locations on the boundary. Also shown on the right hand side of the numerical visualization is an instantaneous streamline that attaches to the point of zero skin-friction.

Both the fixed location and time-dependent orientation of the separating material spike were in close agreement for the experiments and numerics, throughout the course of an oscillation. This evidenced by the good agreement between the two images in figure 7, which were taken at the time when the cylinder was at the right-most extreme of its oscillation. This particular instant in time was chosen because the results also highlight an important point – that the zero skin-friction point at this instant lay 1cm to the right of the point of material separation and cannot therefore be considered a good indicator of separation in an unsteady flow. In contrast, the solid straight line originating from the boundary near 5.2 cm is the separation profile determined using (3) and (5), which captures both the location and orientation of separation.

### C. Fixed separation in aperiodic flow

The final set of experiments we report here concern the location of a fixed separation point in an aperiodic flow. Aperiodicity was introduced by virtue of randomly moving the cylinder from side-to-side, whilst rotating at a constant rate  $\omega=60$  rad/s. Specifically, the cylinder was able to move randomly from side to side within a step size that was set by the performance characteristics of the linear traverse (typically of the order of 3 cm). The nature of the random motion was chosen so that there was a point on the boundary at which the mean shear-stress approached zero in the long time limit. In this case, equation (3) predicts that this will be a fixed point of separation

We show one set of experimental and numerical results for this configuration in figure 8. In this case we can clearly see a separating material spike around 3.3 cm, and this spike remained fixed at that location. The angle of separation changed in time, driven by the aperiodic motion of the cylinder. Also included in the numerical simulation is a solid line originating near 3.3cm that was determined using (3) and (5), and an instantaneous streamline 0.5 cm to the right of it that attaches to the point of zero skin-friction. These results again support the notion that (3) and (5) are capable of detecting separation in an unsteady - in this case random - flow field, while the zero skin-friction principle fails

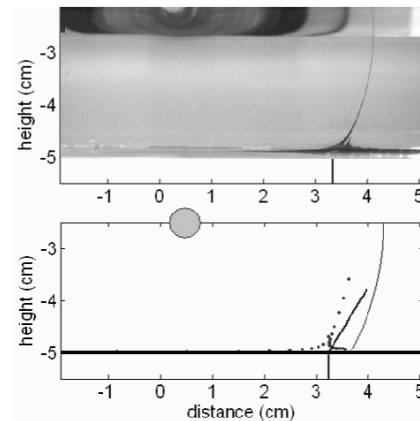


Figure 8: Experimental visualization of separating material line in a random flow field (Upper image). Corresponding numerical simulation (Lower image).

## VI. Conclusions

We have presented some of the first results from our experimental investigations of fixed separation in an unsteady rotor-oscillator flow. The main results concern the close correspondence between the experimental visualizations and numerical simulations, supporting the ability of the new criterion to detect unsteady separation and emphasizing the failure of the zero skin-friction principle to do so. The experiment is intended as a proof-of-concept for unsteady flows before scaling to higher Reynolds number, and thus more practically relevant, flows

In closing, we re-iterate that in this study we have been concerned with fixed separation in unsteady flows. Perhaps even more important is the notion of moving separation in unsteady flows. In this case material does not separate from a fixed location on the boundary but rather from a point that is free to translate along the boundary. The notion of a finite time unstable manifold has been developed by Haller<sup>15</sup> to address such a situation. We are currently implementing this method in the rotor-oscillator flow experiments, with a view to their further validation.

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