Faraday's Law Summary

Related Reading: Course Notes (Liao et al.):Chapter 10 ; Sections 11.1 – 11.4

So far in this class magnetic fields and electric fields have been fairly well isolated. Electric fields are generated by static charges, magnetic fields by moving charges (currents). In each of these cases the fields have been static – we have had constant charges or currents making constant electric or magnetic fields. Today we make two major changes to what we have seen before: we consider the interaction of these two types of fields, and we consider what happens when they are not static. We will discuss the final Maxwell's equation, Faraday's law, which explains that electric fields can be generated not only by charges but also by magnetic fields that vary in time.

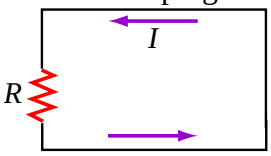
Faraday's Law

It is not entirely surprising that electricity and magnetism are connected. We have seen, after all, that if an electric field is used to accelerate charges (make a current) that a magnetic field can result. Faraday's law, however, is something completely new. We can now forget about charges completely. What Faraday discovered is that a changing magnetic flux generates an EMF (electromotive force). Mathematically:

$\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, where $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$ is the magnetic flux, and $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$ is the EMF

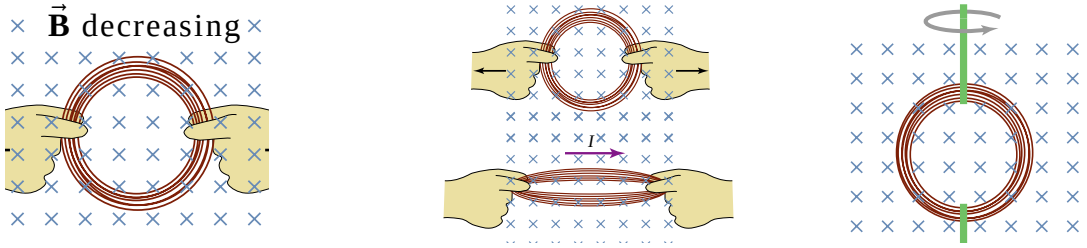
In the formula above, $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving. The above formula is deceptively simple, so I will discuss several important points to consider when thinking about Faraday's law.

WARNING: First, a warning. Many students confuse Faraday's Law with Ampere's Law. Both involve integrating around a loop and comparing that to an integral across the area bounded by that loop. Aside from this mathematical similarity, however, the two laws are completely different. In Ampere's law the field that is "curling around the loop" is the magnetic field, created by a "current flux" $\left(I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} \right)$ that is penetrating the looping B field. In Faraday's law the electric field is curling, created by a *changing* magnetic flux. In fact, there need not be any currents at all in the problem, although as we will see below typically the EMF is measured by its ability to drive a current around a physical loop – a circuit. Keeping these differences in mind, let's continue to some details of Faraday's law.



EMF: How does the EMF become apparent? Typically, when doing Faraday's law problems there will be a physical loop, a closed circuit, such as the one pictured at left. The EMF is then observed as an electromotive force that drives a current in the circuit: $\mathcal{E} = IR$. In this case, the path walked around in calculating the EMF is the circuit, and hence the associated area across which the magnetic flux is calculated is the rectangular area bordered by the circuit. Although this is the most typical initial use of Faraday's law, it is not the only one – we will see that it can be applied in "empty space" space as well, to determine the creation of electric fields.

Changing Magnetic Flux: How do we get the magnetic flux Φ_B to change? Looking at the integral $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA\cos(\theta)$, hints at three distinct methods: changing the strength of the field, the area of the loop, or the angle of the loop. These methods are shown below.



In each of the cases pictured above, the magnetic flux into the page is decreasing with time (because the (1) B field, (2) loop area or (3) projected area are decreasing with time). This decreasing flux creates an EMF. In which direction? We can use Lenz's Law to find out.

Lenz's Law

Lenz's Law is a non-mathematical statement of Faraday's Law. It says that systems will always act to *oppose* changes in magnetic flux. For example, in each of the above cases the flux into the page is decreasing with time. The loop doesn't want a decreased flux, so it will generate a clockwise EMF, which will drive a clockwise current, creating a B field into the page (inside the loop) to make up for the lost flux. This, by the way, is the meaning of the minus sign in Faraday's law. I recommend that you use Lenz's Law to determine the direction of the EMF and then use Faraday's Law to calculate the amplitude. By the way, just as with Faraday's Law, you don't need a physical circuit to use Lenz's Law. Just pretend that there is a wire in which current could flow and ask what direction it would need to flow in order to *oppose* the changing flux. In general, *opposing* a change in flux means *opposing* what is happening to change the flux (e.g. forces or torques *oppose* the change).

Applications

A number of technologies rely on induction to work – generators, microphones, metal detectors, and electric guitars to name a few. Another common application is eddy current braking. A magnetic field penetrating a metal spinning disk (like a wheel) will induce eddy currents in the disk, currents which circle inside the disk and exert a torque on the disk, trying to stop it from rotating. This kind of braking system is commonly used in trains. Its major benefit (aside from eliminating costly service to maintain brake pads) is that the braking torque is proportional to angular velocity of the wheel, meaning that the ride smoothly comes to a halt.

Experiment 5: Faraday's Law of Induction

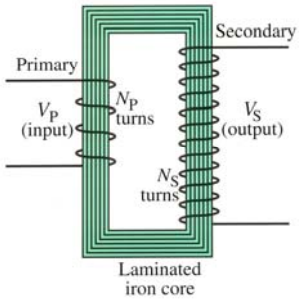
Preparation: Read pre-lab and answer pre-lab questions

In this lab you will have a chance to measure and even feel Faraday's law in action. The lab basically consists of moving a loop of wire over a magnetic dipole. You will (we hope) develop an intuition for how currents flow through the wire loop as it moves in the magnetic field of the dipole, and for the direction of the resultant force on the loop.

Mutual Inductance

Since magnetic fields are typically generated by currents, Faraday's law implies that changing currents also generate EMFs. This is the idea of mutual inductance: given any two circuits, a changing current in one will induce an EMF in the other, or, mathematically, $\mathcal{E}_2 = -M\, dI_1/dt$, where M is the mutual inductance of the two circuits. How does this work? The current in loop 1 produces a magnetic field (and hence flux) through loop 2. If that current changes in time, the flux through 2 changes in time, creating an EMF in loop 2. The mutual inductance, M, depends on geometry, both on how well the current in the first loop can create a magnetic field and on how much magnetic flux through the second loop that magnetic field will create.

Transformers



A major application of mutual inductance is the transformer, which allows the easy modification of the voltage of AC (alternating current) signals. At left is the schematic of a step up transformer. An input voltage V_P on the primary coil creates an oscillating magnetic field, which is "steered" through the iron core (recall that ferromagnets like iron act like wires for magnetic fields) and through the secondary coils, which induces an EMF in them. In the ideal case, the amount of flux generated and received is proportional to the number of turns in each coil. Hence the ratio of the output to input voltage is the same as the ratio of the number of turns in the secondary to the number of turns in the primary. As pictured we have more turns in the secondary, hence this is a "step up transformer," with a larger output voltage than input.

The ease of creating transformers is a strong argument for using AC rather than DC power. Why? Before sending power across transmission lines, voltage is stepped way up (to 240,000 V), leading to smaller currents and losses in the lines. The voltage is then stepped down to 240 V before going into your home.

Self Inductance

Self inductance is very similar to mutual inductance, obeying a similar equation: $\mathcal{E} = -L\dfrac{dI}{dt}$, and the same concept: when a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the "back EMF."

To calculate the self inductance (or inductance, for short) of an object, imagine that a current I flows through it, and determine how much magnetic field and hence flux Φ_B that makes through the object. The self inductance is then $L = \Phi_B / I$.

An inductor is a circuit element whose main characteristic is its inductance, L. It is drawn as a coil in circuit diagrams. The strong resemblance to a solenoid is intentional – solenoids make very good inductors both because of their ability to make a strong field inside

themselves, and also because the field they produce is fairly well contained, and hence doesn't produce much flux (and induce EMFs) in other, nearby circuits.

The role of an inductor is to oppose changing currents. At steady state, in a DC circuit, an inductor is off – it induces no EMF as long as the current through it is constant. As soon as you try to change the current through an inductor though, it will fight back. In this sense an inductor is the opposite of a capacitor. If a capacitor is placed in a steady state current it will eventually fill up and "open" the circuit, whereas an inductor looks like a short in this case. On the other hand, when starting from its uncharged state, a capacitor looks like a short when you first try to move current through it, while an inductor looks like an open circuit, as it prevents the change (from no current to some current).

Energy in B Fields

Where do inductors get the energy to source current when they need to? In capacitors we found that energy was stored in the electric field between their plates. In inductors, energy is similarly stored, only now its in the magnetic field. Just as with capacitors, where the electric field was created by a charge on the capacitor, we now have a magnetic field created when there is a current through the inductor. Thus, just as with the capacitor, we can discuss both the energy in the inductor, $U = \frac{1}{2} L I^2$, and the more generic energy density $u_B = \frac{B^2}{2\mu_0}$, stored in the magnetic field. Again, although we introduce the magnetic field energy density when talking about energy in inductors, it is a generic concept – whenever a magnetic field is created it takes energy to do so, and that energy is stored in the field itself.

Important Equations

Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

Magnetic Flux: $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

EMF: $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$ where $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving.

Mutual Inductance: $\mathcal{E}_2 = -M \frac{dI_1}{dt}$

Self Inductance, *L*: $L = \frac{\Phi_B}{I}$

EMF Induced by Inductor: $\mathcal{E} = -L \frac{dI}{dt}$

Energy stored in Inductor: $U = \frac{1}{2} L I^2$

Energy Density in B Field: $u_B = \frac{B^2}{2\mu_0}$