HIDDEN VARIABLE THEORIES

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The usual interpretation of quantum theory is based on an assumption having very far-reaching implications, viz., that the physical state of an individual system is completely specified by a wave function that determines only the probabilities of actual results that can be obtained in a statistical ensemble of similar esperiments. This assumption has been the object of severe criticisms, notably on the part of Einstein, who always believed that even at the quantum level, there must exist precisely definable elements of dynamical variables determining (as in classical physics) the actual behavior of each individual system, and not merely its probable behavior. Since these elements or variables are not now incluced in the quantum theory and have not yet been detected experimentally, Einstein always regarded the present form of the quantum theory as incomplete, although he admitted its internal consistency. The following is from a letter from Einstein to Born, dated 4. December, 1926:

> Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the "old one". I, at any rate, am convinced that He is not playing at dice.

Several of the other "giants" of the twenties, including Schroedinger and deBroglie, have never been satisfied with quantum theory. In fact, deBroglie is still writing articles on this subject.

The problem centers mainly around the quantum theory of measurement. In its simplest form the quantum theory of measurement considers a world composed of just two entities, a system and an apparatus. Both are subject to quantum-mechanical laws, so you can form a combined state function that can be expanded in terms of an orthonormal set of basis functions

$$\psi_{s,A} = \psi_s \psi_A$$

where ψ_s is an eigenstate of the system and ψ_A is an eigenstate of the apparatus. The above product reflects the fact that the system and the apparatus are isolated, independent and distinguishable. For convenience assume s ranges over a discrete set of eigenvalues and A ranges over a continuous set. Now suppose that at some instant the world is represented by the function

$$\Psi_0 = \Psi_S \Psi_A \qquad (1)$$

where Ψ_S refers to the system and Ψ_A to the apparatus. In such a state the system and apparatus are uncorrelated. For the apparatus to learn something about the system the two must be coupled for a certain period of time so that their combined state will not retain the form of eqn. (1) as time passes. The final result of the coupling will be described by the operator U,

$$\Psi_1 = U\Psi_0 = U\Psi_S\Psi_A$$

with U defined so that when acting on the basis vectors,

$$U\psi_s\psi_A = \psi_s\psi_{A+gs}$$

where g is a coupling constant. Now if the initial state of the system were ψ_s and the initial state of the apparatus were ψ_A this coupling would be said to result in an "observation" by the apparatus, since the measurement is stored by virtue of the permanent shift from ψ_A to ψ_{A+gs} in the apparatus state functions. Now consider what happens to the state function Ψ_0. In general,

$$\begin{aligned}\Psi_0 &= \Psi_S\Psi_A \\ &= \left(\sum_s c_s\psi_s\right)\left(\int \psi_A \Phi_A \, dA\right)\end{aligned}$$

where
$$c_s = \sum_S \psi_s^* \Psi_s ,$$
$$\Phi(A) = \int \psi_A^* \Psi_A \, dA .$$

Now if the operator U is applied
$$\begin{aligned}\Psi_1 &= U \Psi_0 = U \Psi_s \Psi_A \\ &= (\textstyle\sum_s c_s \psi_s) U \int \psi_A \Phi(A) \, dA \\ &= (\textstyle\sum_s c_s \psi_s) \int \psi_{A+gs} \Phi(A) \, dA . \end{aligned} \qquad (2)$$

The final state function Ψ_1 does not represent the system observable as having any distinct value unless of course
$$\Psi_0 = \psi_s \psi_A , \qquad \text{an eigenstate.}$$
Rather it is a linear superposition of functions, each of which represents the system observable as having assumed one of its possible values and the apparatus as having observed that value.

Here is where the problem begins. The apparatus has apparently entered a kind of schizophrenic state in which it is unable to decide what value it has found for the system observable. Yet classically, practically, some value is chosen. At this point a second apparatus is introduced to help the first "make up its mind," but this is no better. It also becomes schizophrenic. And so would a third, fourth and so on. This is known as "von Neumann's catastrophe of infinite regression." The conventional quantum theory escapes the von Neumann catastrophe by postulating the so-called "reduction of the wave packet." Any time the apparatus enters a schizophrenic state it immediately collapses to a single element of the superposition. To which element it reduces one cannot say. One instead assigns a probability distribution to the possible outcomes, with weights given by
$$w_s = |c_s|^2 .$$
The collapse of the state vector does not follow Schroedinger's equation.

There are other possibilities. Everett, Wheeler, and Graham propose the simultaneous <u>reality</u> of the superposition (eqn. (2)). The universe is viewed as constantly splitting up into a stupendous number of branches, all resulting from the measurement-like interactions between its myriads of components. Wigner has proposed a solution in which the entry of the measurement signal into the consciousness of a human observer triggers the decision and breaks von Neumann's chain. Wigner actually sketches a possible mathematical description of the process, which might come about as a result of the grossly nonlinear departures from the normal Schroedinger equation that he believes must occur when conscious beings enter the picture. He also proposes that a search be made for unusual effects of consciousness acting on matter. Another possibility, and this is the primary focus of this seminar, is to introduce the so-called "hidden variables." This cures even the first apparatus of its schizophrenia, by describing an underlying deterministic substructure.

Another motivation for looking at hidden variable theories is the classic paper by Einstein, Podolsky, and Rosen (EPR), which "cries out for a hiddenvariable interpretation." EPR define Reality by the following criteria. "If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." EPR show that the state

$$\psi = e^{i\frac{p_0 x}{\hbar}}$$

has a definite momentum, p_0, but that the coordinate is completely unpredictable. "The usual conclusion from this in quantum mechanics is that when the momentum of a particle is known, its coordinate has no physical reality. More generally if the operators corresponding to two physical quantities, say A and B, do not commute, i.e. if $AB \neq BA$, then precise knowledge of one of them precludes such knowledge of the other. Any attempt to measure the latter will destroy the knowledge of the first. From this follows that either (1) the quantum mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness. If then the wavefunction provided such a complete description of reality it would contain these values; these would then be predictable. This not being the case, we are left with the alternatives stated." Now the alternatives as stated are somewhat difficult to understand. Without so many negatives, they read: (1) If the operators corresponding to two physical quantities do not commute, the two quantities cannot have simultaneous reality, as predicted by quantum mechanics. If the two physical quantities do have simultaneous reality then (2) the quantum mechanical description of reality given by the wavefunction is not complete. In quantum mechanics it is usually assumed that the wavefunction does contain a complete description of the physical reality of the system in the state to which it corresponds. EPR show that this assumption, together with the criteria of reality, and, although not explicitly considered an assumption by EPR, a certain locality, leads to a contradiction.

Now suppose that we have two systems, I and II, which are permitted to interact from t=0 to t=T, after which time we suppose that there is no longer any interaction between the two parts. That this is possible was considered obvious by EPR. It is however not clear and will be discussed further below. For the time being assume for t>T there is no interaction between the two parts. The wave function for the total system will be designated by Ψ. Let a_1, a_2, a_3, . . . be the eigenvalues of some physical quantity A pertaining to system I, and $u_1(x_1)$, $u_2(x_1)$, $u_3(x_1)$ . . . the corresponding eigenfunctions, where x_1 stands for the variables used to describe the first system. Then

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2)\, u_n(x_1) \tag{3}$$

where $\psi_n(x_2)$ are to be regarded as coefficients of the expansion of Ψ into a series of orthogonal functions $u_n(x_1)$. Now if A is measured and found to have eigenvalue a_k, it is concluded that after the measurement the first system is left in the state $u_k(x_1)$ and that the second state is given by $\psi_k(x_2)$. This is the process of the reduction of the wave packet, which was discussed earlier. Eqn. (3) is reduced to the single term

$$\Psi = \psi_k(x_2)\, u_k(x_1).$$

If now instead another physical quantity B is used for the same process with b_1, b_2, b_3, . . . and eigenfunctions $v_1(x_1)$, $v_2(x_1)$, $v_3(x_1)$, . . . ,

$$\Psi(x_1, x_2) = \sum_{s=1}^{\infty} \varphi_s(x_2)\, v_s(x_1). \tag{4}$$

If B is measured and thefirst state is found to be $v_s(x_1)$, then the second state is $\varphi_s(x_2)$. Thus as a consequence of two different measurements performed upon the first system, the second may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything done to the first system. Thus it is possible to assign two different wave functions to the same reality. EPR then show that it is possible for these wavefunctions to be eigenfunctions of two noncommuting operators, corresponding to physical quantities, say P and Q. Thus by measuring A or B we can predict with certainty, without disturbing the second system, either the value of P (i.e. p_k) or the value of Q (i.e. q_k). In the first case we must consider the quantity p as an element of physical reality and in the second case we must consider the quantity Q as an element of physical reality. Thus the two physical quantities P and Q can have simultaneous reality, but have non-commuting operators. This implies that quantum mechanics is incomplete.

Now it can be objected that their criterion of Reality is not sufficiently restrictive. Indeed, EPR would not arrive at their conclusion if it were required that two or more physical quantities be regarded as simultaneous elements of reality only when they can be simultaneously predicted or measured. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P and Q depend on the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

Thus the EPR paradox arises from the conjunction of these propositions: (1) the complete describability of the physical world in quantum mechanical terms, (2) their criteria of Reality, and (3) the ability to isolate systems or the non-occurence of action at a distance.

In discussing the status of hidden-variable theories one must carefully distinguish among the various kinds of theories. One can classify hidden-variable theories as "noncontextualistic" or "contextualistic." A noncontextualistic hidden variable theory makes a kind of intrinsic ascription to each observable of the system when the state is specified. In the case of a deterministic noncontextualistic hidden variable theory, the definite value $A(\lambda)$ is given to each observable A, and therefore in any state λ there is a simultaneous ascription of definite values to all observables, even though their simultaneous exhibition may be operationally impossible. The essential characteristic of noncontextualistic hidden variable theories can be conveyed by considering several pieces of apparatus, each of which is ideal for measuring A, i.e. if the system is in the eigenstate that has eigenvalue α, then the result of a measurement by the apparatus will yield the value α. Now according to a noncontextualistic hidden variable theory, the result which each of these pieces of apparatus yields for A is the same when the system is in any specified state λ, i.e. the same value $A(\lambda)$ results from each. The reason that this agreement is noteworthy is that there may be great differences of construction and of function among the different pieces of apparatus for the same observable. For example, one ideal apparatus for measuring the total angular momentum J^2 may simultaneously measure J_x, while another may simultaneously measure J_z. A noncontextualistic theory asserts that the results of measuring J^2 with the two pieces of apparatus are independent of the "context" supplied by the additional measurement. By contrast, a contextualistic hidden variable theory is one in which the result yielded by an ideal apparatus depends upon context, and hence upon the state λ' of the apparatus. The outcome of a measurement may be written as $A(\lambda, \lambda')$.

Now one decisive mathematical result is that all noncontextualistic deterministic hidden variable theories are inconsistent except in the case of extremely simple systems. This result was first established as a corollary of a theorem of Gleason (1957), and more simply by Bell (1966), in another way by Kochen and Specker (1967) with further simplifications by Belinfante (1971).

The famous theorem of von Neumann (1955) on the nonexistence of hidden variables is not decisive, because it relies upon an addition premise which is not physically acceptable.

Von Neumann observed that in quantum mechanics an observable whose operator is a linear combination of operators for other observables,

$$A = \beta B + \gamma C$$

has for expectation value the corresponding linear combination of expectation values:

$$\langle A\rangle = \beta\langle B\rangle + \gamma\langle C\rangle .$$

He considered more general schemes in which this particular feature was preserved. Now for hypothetical dispersion-free states there is no distinction between expectation values and eigenvalues—for each such state must yield with certainty a particular one of the possible results for any measurement. But eigenvalues are not additive. Consider for example the components of spin for a particle of spin $\frac{1}{2}$. The operator for the component along the direction halfway between the x and y axes is

$$(\sigma_x + \sigma_y)/\sqrt{2}$$

whose eigenvalues are certainly not the corresponding linear combinations

$$(\pm 1 + \pm 1)/\sqrt{2}$$

of the eigenvalues of σ_x and σ_y. Thus the requirement of additive expectation values excludes the possibility of dispersion-free states. Von Neumann concluded that a hidden variable interpretation is not possible for quantum mechanics.

It seems that von Neumann considered the additivity of eigenvalues more as an obvious axiom than as a possible postulate. But consider what it means in terms of the actual physical situation. Measurements of the three quantities

$$\sigma_x \,,\ \sigma_y \,,\ (\sigma_x+\sigma_y)/\sqrt{2}\,,$$

require three different orientations of the Stern-Gerlach magnet, and cannot be performed simultaneously. It is just this which makes intelligible the non-additivity of eigenvalues—the values observed in specific instances. It is by no means a question of simply measuring different components of a preexisting vector, but rather of observing different products of different physical procedures. That the statistical averages should then turn out to be additive is really a quite remarkable feature of quantum mechanical states, which could not be guessed a priori. It is by no means a "law of thought" and there is no reason to exclude the possibility of states for which it is false. The component states need only have such properties that ensembles of them have the statistical properties of observed states.

Now among deterministic contextualistic hidden variable theories one can classify some as "local" and others as "non-local." In order to define locality consider a composite physical system consisting of two spatially well-separated components 1 and 2. Suppose $\{A_a\}$ is a family of observables of 1 parametrized by a, and $\{B_b\}$ is a family of observables of 2 parametrized by b. In a contextualistic hidden variable theory the outcome of a measurement of A_a is determined by the state λ and the state of the apparatus, and similarly for B_b. A theory is local if the outcome of the measurement of A_a is independent of the value of the parameter b and of the state of the app ratus used to observe component 2, and similarly for the measurement of B_b.

Now consider the case of two measuring devices. The first has a setting $\hat{a}$ and measures A. The second has a setting $\hat{b}$ and measures B. Suppose the hypothetical complete description of the initial state is in terms of the hidden variables λ with probability distribution $\rho(\lambda)$ for the given quantum-mechanical state. A has values ± 1 and clearly depends on λ and $\hat{a}$. B also can only take on values ± 1, and depends on λ and $\hat{b}$. Our notion of locality requires that A not depend on $\hat{b}$ nor B on $\hat{a}$. We then ask if the mean value $P(\hat{a},\hat{b})$ of the product AB, i.e.

$$P(\hat{a},\hat{b}) = \int d\lambda\ \rho(\lambda)\ A(\hat{a},\lambda)B(\hat{b},\lambda),$$

is equal to the quantum mechanical predictions.

Actually the instruments themselves could contain hidden variables which could influence the results. If we average first over these instrument variables,

$$P(\hat{a},\hat{b}) = \int d\lambda\ \rho(\lambda)\ \bar{A}(\hat{a},\lambda)\ \bar{B}(\hat{b},\lambda).$$

Now since $A=\pm 1$ and $B=\pm 1$, $|\bar{A}|\leq 1$ and $|\bar{B}|\leq 1$. This is enough to derive an interesting restriction on P, called Bell's inequality:

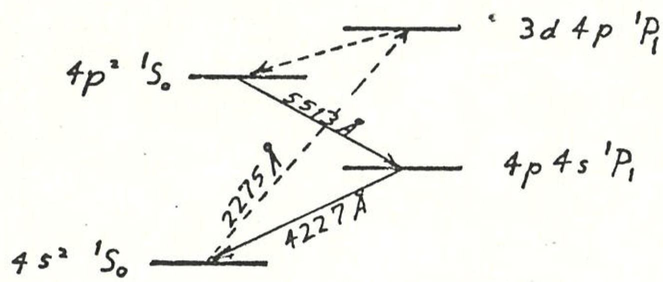
Let $\hat{a}'$ and $\hat{b}'$ be alternate settings of the instruments. Then

$$|P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}')| + |P(\hat{a}',\hat{b}') + P(\hat{a}',\hat{b})| \leq 2 \ .$$

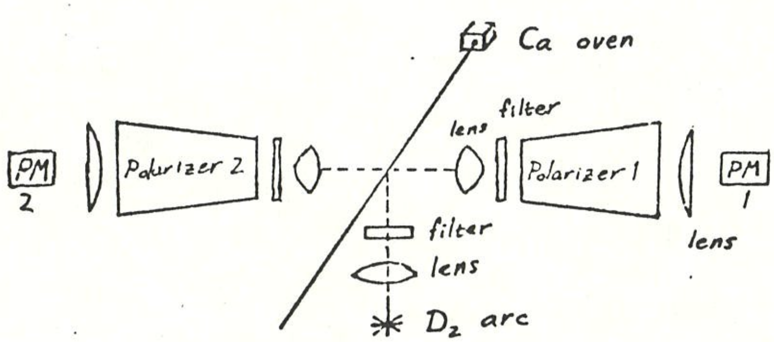
A proof of Bell's inequality is contained in the appendix.

Now there are situations in which the quantum mechanical predictions violate Bell's inequality. Hence no <u>local</u> hidden variable theory can agree with all the statistical predictions of quantum mechanics. This is in conflict with the expectations of EPR. Bell's result gives rise to a new question: are the statistical predictions of quantum mechanics always empirically correct, particularly in the domain of correlation phenomena, where they conflict with local hidden variable theories?

An experiment designed to answer this question has been done by Stuart J. Freedman and John F. Clauser. Basically, it involves examining the correlations of polarization of pairs of photons emitted in atomic cascades. A calcium atomic beam effused from a tantalum oven. The continuum output of a deuterium arc lamp was passed through a filter and focused on the beam. Resonance absorption of a 2275Å photon excited calcium atoms to the $3d4p\,^1P_1$ state. Of the atoms that did not decay directly to the ground state, about 7% decayed to the $4p^2\,^1S_0$ state, from which they cascaded through the $4s4p\,^1P_1$ intermediate state to the ground state with the emission of two photons at 5513Å and 4227Å.



The decaying atoms were viewed by two symmetrically-placed optical systems, each consisting of two lenses, a wavelength filter, a rotatable and removable polarizer, and a single photon detector.



The following quantities were measured: $R(\varphi)$, the coincidence rate for two-photon detection, as a function of the angle φ between the planes of linear polarization defined by the orientation of the inserted polarizers; R_1, the coincidence rate ~~with polarizer~~ with polarizer 2 removed; R_2, the coincidence rate ~~with polarizer~~ with polarizer 1 removed; and R_0, the coincidence rate with both removed. Local hidden variables constrain the coincidence rates by the following inequalities:

$$-1 \leq \Delta(\varphi) \leq 0,$$

where

$$\Delta(\varphi) = \frac{3R(\varphi)}{R_0} - \frac{R(3\varphi)}{R_0} - \frac{R_1 + R_2}{R_0}.$$

These are not satisfied by the quantum mechanical predictions. Maximum violations occur at $\varphi=22\frac{1}{2}^\circ$ $[\Delta(\varphi)>0]$ and $\varphi=67\frac{1}{2}^\circ$ $[\Delta(\varphi)<-1]$. At these angles the inequalities above can be combined into the simpler expression

$$\delta = \left|R(22\tfrac{1}{2}^\circ)/R_0 - R(67\tfrac{1}{2}^\circ)/R_0\right| - \tfrac{1}{4} \leq 0,$$

which does not involve R_1 or R_2.

The experiment yields $\delta=0.050\pm0.008$, in clear violation of the above inequality.

Now does this mean that local hidden variable theories are ruled out? One objection is that even if there is no action at a distance in nature there is still plenty of time for the hidden variables of one polarizer to communicate with those of the other, after the two are installed and oriented. Clauser has suggested a device for changing the orientation of the polarizer axes while the photons are in flight by means of Kerr cells, thereby thwarting any conspiracy which employs signals of light velocity or slower. There are still other objections and other experiments (for example Kasday, Ullmann, and Wu). In particular, there is one reference (<u>A Survey of Hidden Variable Theories</u>, Belinfante, 1973, footnote on p. 290) to an as yet uncompleted experiment by Holt at Harvard. Results obtained by Holt up until April 1973 apparently contradict quantum theory. So the question of locality is not yet settled, but a definite experimental answer to this question should be found soon.

The correlation experiments say nothing, however, about non-local theories. These include those of Bohm (1952), Wiener and Siegel (1953, 1955), and Bohm and Bub (1966). In order to be more complete, only Bohm's 1952 theory will be discussed.

As an introduction to the 1952 theory of Bohm, there is a fairly interesting quotation from the <u>Born-Einstein Letters</u>, in a letter from Einstein to Born, dated 12 May 1952:

> Have you noticed that Bohm believes (as deBroglie did, by the way, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me.

The basic idea is as follows. Consider a particle in a potential $V(\vec{r})$. If no measuring apparatus desturbs the particle in some hard-to-describe fashion, the particle is described quantum mechanically by the Schroedinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right] \psi .$$

If $V(\vec{r})$ is real it is easy to derive the continuity equation

$$\frac{\partial(\psi^*\psi)}{\partial t} = div \left\{ \frac{i\hbar}{2m} \left[\psi^* \nabla \psi - (\nabla \psi^*) \psi \right] \right\}$$

Now if

$$\rho = \psi^* \psi$$

is regarded as the probability density of the ensemble of particles described by ψ in x-space, the continuity equation may be written

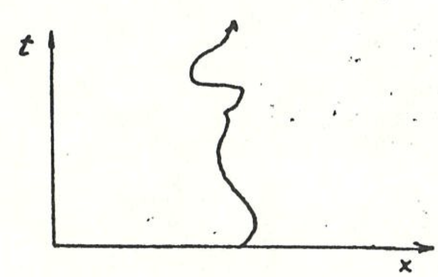
$$\frac{\partial \rho}{\partial t} = -div(\rho v)$$

where the local ensemble velocity v is defined by

$$v = \frac{\hbar}{2im} \frac{\psi^* \nabla \psi - (\nabla \psi^*)\psi}{\psi^* \psi}$$

$$= \frac{\hbar}{2im} \nabla \ln(\psi / \psi^*) .$$

In x,y,z,t-space we visualize the "sample particles" of which the ensemble is made up, as points distributed on the corresponding surface t=constant at positions $\vec{r} = \vec{\xi}$. We then connect these points from surface to surface by world-lines with a slope corresponding to the velocity v given above. The result will look like a picture of the world-lines of the particles of a fluid, though, of course, in reality, it may have been the existence of one single particle that made us write down the Schroedinger equation. We then might imagine that any single particle from the ensemble described by ψ would perform on its own a well-defined continuous motion by following one of the world-lines just described, if somehow its initial (or final) position were known. That this would be so is the basic claim made by Bohm's 1952 theory. Th perhaps make this a little clearer, consider the case of one particle moving in one dimension, x. The world-line will look something like this:



The surfaces of constant t are in this case horizontal lines. The position on one horizontal line is connected to the position on another infinitesimally close horizontal line by the velocity vector which gives the slope of the world-line.

Now returning to the general case, we shall define for our particle a quantity called the momentum π, such that

$$\pi = m\frac{d\xi}{dt} = mv$$

$$= \frac{\hbar}{2i}\nabla \ln(\psi/\psi^*).$$

There is thus a quantum-mechanical force given by

$$f(\vec{r},t) = \frac{\partial\pi}{\partial t}.$$

Calculations are simplified if we put

$$\psi = R\exp(iS/\hbar),$$

$$R = |\psi| = \sqrt{\rho},$$

$$S = \frac{\hbar}{2i}\ln(\psi/\psi^*).$$

Then the equation for π above becomes

$$\pi = \nabla S.$$

It can be shown that

$$f = \frac{\partial\pi}{\partial t} = m\frac{\partial^2\vec{\xi}}{\partial t^2} = -\nabla(V+U),$$

where the right-hand number is to be taken at $\vec{r} = \vec{\xi}$ and where

$$U = \frac{-\hbar^2}{2m}\frac{\nabla^2 R}{R}$$

$$= \frac{-\hbar^2}{4m}\left[\frac{1}{\rho}\nabla^2\rho - \frac{1}{2}\left(\frac{\nabla\rho}{\rho}\right)^2\right].$$

This U is called by Bohm the quantum mechanical potential.

This potential is responsible for the difference between the motion of a particle according to this Bohm interpretation and the motion of a particle according to classical mechanics.

It is possible in some simple cases to integrate the equation of motion in a known ψ field. But in most cases $-\nabla \mathcal{U}$ produces a motion that is wildly complicated. In order to integrate the equation of motion it is also necessary to know the initial value of $\vec{\xi}$. But there is no known way of doing this without disturbing ψ. Therefore the $\vec{\xi}$ are for us "hidden" as are the motions of individual molecules in a gas. Like the latter in kinetic theory, therefore, for given ψ the $\vec{\xi}(\psi)$ can only be used statistically, by averaging over all possible motions. To summarize, there are four essential assumptions:

(1) The ordinary Schroedinger equation holds for $\psi(\vec{r}, t)$.

(2) the particles cannot have arbitrary velocities. Their initial velocities are determined by

$$\pi =' (\nabla S)_{\vec{r}=\vec{\xi}}$$

(3) The velocities change according to

$$f = \frac{\partial \pi}{\partial t} = m \frac{\partial^2 \vec{\xi}}{\partial t^2} = -\nabla(V + \mathcal{U}),$$

maintaining $\pi = \nabla S$ at all times.

(4) The probability density of $\vec{\xi}$, which we may call $P(\vec{\xi})$ is equal to $\rho(\vec{r}) \equiv |\psi(\vec{r})|^2$ at $\vec{r} = \vec{\xi}$.

The theory determined by these four assumptions will then yield all the results of ordinary wave mechanics.

Now the EPR paradox and the double slit experiment need to be explained in the context of Bohm's theory. First, the double slit experiment will be explained. Consider a particle incident in the z direction with an initial momentum, p_0, on a system consisting of two slits. After the particle passes through the slit system its position is measured and recorded, for example on a photographic plate. Now in the usual interpretation of quantum theory the particle is described by a wave function. The initial part is

$$\psi_0' \sim \exp(i p_0 z/\hbar)$$

but when it passes through the slit system it is modified by interference and diffraction effects. The probability that the particle will be detected between x and $x+dx$ is $|\psi(x)|^2 dx$. In the usual interpretation of quantum theory the origin of the interference pattern is difficult to understand. For there may be certain points where the wave function is zero when both slits are open, but not zero when only one slit is open. How can the opening of a second slit prevent the particle from reaching certain points that it could reach if this slit were closed?

In Bohm's theory this experiment is described causally and continuously. The initial momentum of the particle is obtained from the initial wave function $\psi_0 \sim \exp(i p_0 z/\hbar)$ as

$$p = \frac{\partial}{\partial z} S = p_0 .$$

The initial position of the particle is however not known, so that although the particle goes through a definite slit we cannot predice which one this will be. The particle is at all times acted upon by the "quantum mechanical" potential

$$U = \frac{-\hbar^2}{2m} \frac{\nabla^2 R}{R} .$$

While the particle is incident, this potential vanishes because R is then a constant; but after it passes through the slit system, the particle encounters a quantum mechanical potential that changes rapidly with position. The subsequent motion of the particle may therefore be quite complicated. Nevertheless, the probability that the particle will enter a given region, dx, is as in the usual interpretation $|\psi(x)|^2 dx$. We therefore deduce that the particle can never reach a point where the wavefunction vanishes. The reason is that the "quantum mechanical" potential, U, becomes infinite when R becomes zero. If the approach to infinity happens through positive values of R, there will then be an infinite repelling force on the particle. If the approach is negative, the particle will go through the point with infinite speed and thus spends no time there. Clearly the closing of a slit will alter the ψ field and thus the particle's motion.

Now consider the paradox of EPR. The system is described in terms of a precisely definable trajectory in a six-dimensional space. If the first particle's position is measured we introduce uncontrollable fluctuations in the wave function for the whole ensemble, which through the "QM" forces bring about corresponding uncontrollable fluctuations in the momentum of each particle. The "quantum mechanical" forces may be said to transmit uncontrollable disturbances instantaneously from one particle to another through the medium of the ψ field.

What does this transmission of forces at infinite rate mean? There is no problem in a non-relativistic theory. In a relativistic theory, it is quite complicated. Since Bohm's theory gives precisely the same predictions as regular quantum mechanics and QM is consistent with relativity, Bohm's theory is consistent with relativity. The reason why no contradictions with relativity arise in his interpretation despite the instantaneous transmission of momentum between particles is that no signal can be carried in this way. For such a transmission of momentum could constitute a signal only if there were some practical means of determining precisely what the second particle would have done if the first particle had not been observed, and this information cannot be obtained as long as the present form of quantum theory is valid. To obtain such information we require conditions (such as might occur in connection with disturbances of the order of 10^{-13} cm) under which the usual form of the quantum theory breaks down, so that the positions and momenta of the particles can be determined precisely and simultaneously.

The usual interpretation of quantum theory implies that we must renounce the possibility of describing an individual system in terms of a single precisely defined conceptual model. Bohm has, however, provided an alternative interpretation which does not imply such a renunciation, but which instead leads us to regard a quantum-mechanical system as a synthesis of a precisely definable particle and a precisely definable ψ-field which exerts a force on this particle. No experimental choice between the two interpretations can be made in a domain where the present mathematical formulation of the quantum theory is a good approximation; but such a choice is conceivable in domains such as those associated with dimensions of the order of 10^{-13} cm, where the extrapolation of the present theory seems to break down and where Bohm's theory can lead to completely different kinds of predictions. Bohm's theory provides a consistent alternative to the usual assumption that no objective and precisely defined description of reality is possible at the quantum level of accuracy.

Excellent!

A well prepared paper on a stimulating topic

R.H. 4-24-75.

Final grade 1-

APPENDIX: PROOF OF BELL'S INEQUALITY

We have (p. 11)

$$P(\hat{a},\hat{b}) = \int d\lambda\, \rho(\lambda)\, \bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda)$$

and

$$|\bar{A}| \le 1 \;,\quad |\bar{B}| \le 1 . \tag{1}$$

Let $\hat{a}'$ and $\hat{b}'$ be alternate settings of the instruments.

$$P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}') = \int d\lambda\, \rho(\lambda)\left[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda) - \bar{A}(\hat{a},\lambda)\bar{B}(\hat{b}',\lambda)\right]$$

$$= \int d\lambda\, \rho(\lambda)\left[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b}',\lambda)\right)\right]$$

$$- \int d\lambda\, \rho(\lambda)\left[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b}',\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(b,\lambda)\right)\right] .$$

Using (1),

$$P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}') = \left|\int d\lambda\, \rho(\lambda)\left\{\left[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b}',\lambda)\right)\right] - \left[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b}',\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b},\lambda)\right)\right]\right\}\right|$$

$$\le \left|\int d\lambda\, \rho(\lambda)[\ldots]\right| + \left|\int d\lambda\, \rho(\lambda)[\ldots]\right|$$

$$\le \int d\lambda\, \rho(\lambda)\left|[\ldots]\right| + \int d\lambda\, \rho(\lambda)\left|[\ldots]\right|$$

using (1) again,

$$\le \int d\lambda\, \rho(\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b}',\lambda)\right) + \int d\lambda\, \rho(\lambda)\left(1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b},\lambda)\right)$$

or

$$P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}') \le 2 \pm \left(P(\hat{a}',\hat{b}') + P(\hat{a}',\hat{b})\right)$$

or more symmetrically

$$\left|P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}')\right| + \left|P(\hat{a}',\hat{b}') + P(\hat{a}',\hat{b})\right| \le 2 .$$

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