

Episodic volcanism on Enceladus: Application of the Ojakangas–Stevenson model

Jennifer Meyer*, Jack Wisdom

Massachusetts Institute of Technology, 54–410 MIT, 77 Massachusetts Avenue, Cambridge, MA 02139, USA

ARTICLE INFO

Article history:

Received 22 August 2007

Revised 19 June 2008

Available online 12 July 2008

Keywords:

Enceladus

Saturn, satellites

Satellites, dynamics

Resonances, orbital

Tides, solid body

ABSTRACT

The main equations in the paper “Episodic volcanism of tidally heated satellites with application to Io” by Ojakangas and Stevenson [Icarus 66, 341–358] are presented; numerical integration of these equations confirms the results of Ojakangas and Stevenson [Icarus 66, 341–358] for Io. Application to Enceladus is considered. It is shown that Enceladus does not oscillate about the tidal equilibrium in this model by both new nonlinear stability analysis and numerical integration of the model equations.

© 2008 Elsevier Inc. All rights reserved.

1. Introduction

We have shown that equilibrium tidal heating in Enceladus cannot account for the nonsolar heat emanating from Enceladus (Meyer and Wisdom, 2007): equilibrium tidal heating can account for only $1.1(18000/Q_S)$ GW of the observed 5.8 ± 1.9 GW (Spencer et al., 2006). Provided the origin of the observed heating is tidal heating, it is possible that Enceladus is oscillating about equilibrium. A model for oscillation about equilibrium has been presented for Io by Ojakangas and Stevenson (1986). Fischer and Spohn (1990) presented similar oscillation models for Io, emphasizing different rheologies. Ojakangas and Stevenson (1986) mentioned the possible application of their model to Enceladus.

The Ojakangas and Stevenson model would only apply to Enceladus if heat transport is mainly by convection. Squyres et al. (1983) discuss whether convection occurs in Enceladus. They consider a convecting region overlain by a nonconvecting, conductive crustal ice layer. They find that for crustal thicknesses larger than 30 km, heat transport is dominated by convection. Here we assume convection occurs and that the Ojakangas and Stevenson model is applicable to Enceladus.

We first review the Ojakangas and Stevenson (1986) model. We carry out a new linear stability analysis for their full model. We show that, in fact, Enceladus does not oscillate about the tidal equilibrium within the Ojakangas and Stevenson (1986) model.

2. Ojakangas and Stevenson evolution equations

Consider the thermal evolution of a satellite, with index 0, in resonance with another satellite, with index 1, exterior to it. Let m_i be the mass of the satellite i , and n_i be the mean motion. The semimajor axis of the inner satellite is a and its orbital eccentricity is e .

The physical parameters of the inner satellite are the heat capacity C_p , the temperature T , the radius R , the density ρ , the thermal diffusivity K , the surface gravitational acceleration g , the thermal expansion coefficient α_T , the kinematic viscosity $\nu(T)$, the critical Rayleigh number Ra_c , and the Love number $k(T)$ and the tidal quality factor $Q(T)$. The values adopted for these physical parameters are listed in Table 1.

The basic equation for the thermal state states that the rate of change of the thermal energy in the satellite is a balance between tidal heating and loss from thermal convection:

$$m_0 C_p \frac{dT}{dt} = \frac{21}{2} \frac{GM_p^2 R^5 n_0}{a^6} \left(\frac{k(T)e^2}{Q(T)} \right) - 4\pi R^2 \rho C_p K T \left(\frac{g \alpha_T T}{\nu(T) K Ra_c} \right)^{1/3}. \quad (1)$$

The temperature dependence of the factor Q/k is unknown, but approximated by a power law near the solidus and a constant at low temperature. A form that interpolates these characteristics is

$$\frac{Q(T)}{k(T)} = f(T/T_m) = \left[\left(\frac{Q}{k} \right)_0^{-1} + A \left(\frac{T}{T_m} \right)^n \right]^{-1}, \quad (2)$$

* Corresponding author.

E-mail addresses: meyerj@mit.edu (J. Meyer), wisdom@mit.edu (J. Wisdom).

Table 1
The adopted physical parameters for Io and Enceladus are presented

	Io	Enceladus
M_p [10^{24} kg]	1898.8	568.5
m_0 [10^{20} kg]	893.3	1.08
m_1 [10^{20} kg]	479.7	10.95
R_p [km]	71,492	60,330
Q_p	10^5	1.8×10^4
a [km]	421,769	238,400
R [km]	1821.3	252.3
k_{2p}	0.38	0.341
Ra_c	800	800
α_T [K^{-1}]	3.0×10^{-5}	5.1×10^{-5}
K [$m^2 s^{-1}$]	1.0×10^{-6}	1.35×10^{-7}
C_p [$J kg^{-1} K^{-1}$]	800	2100
T_m [K]	1400	273
ρ [$kg m^{-3}$]	3500	1602
g [$m s^{-2}$]	1.8	0.11
k_2	0.027	0.0018
ν_{TM} [$m^2 s^{-1}$]	$10^{12}-10^{13}$	10^{10}
β	13	30.7
γ	0.32	0.49
n	20–30	?
m	8–12	13
L	20–30	35–37

with

$$A = \left(\frac{Q}{k}\right)_{\min}^{-1} - \left(\frac{Q}{k}\right)_0^{-1}, \quad (3)$$

where T_m is the melting temperature, $(Q/k)_0$ is the value of Q/k at low temperature, and $(Q/k)_{\min}$ is the minimum value of Q/k that is reached near the melting temperature. The viscosity is taken as a power law

$$\nu(T) = \nu_{TM}(T/T_m)^{-L}, \quad (4)$$

where, in silicates, $20 < L < 30$, and $\nu_{TM} = 10^{12}-10^{13} m^2 s^{-1}$. In water ice, $35 < L < 37$, and $\nu_{TM} = 10^{10} m^2 s^{-1}$ (Durham et al., 1997).

The equilibrium value of the temperature is

$$T_0 = \left[\frac{7}{2\beta} \frac{k_p}{Q_p} \frac{Gm_0 R R_p^5 n_0}{C_p a^6} \left(\frac{\nu_{TM} T_m^L R a_c}{g \alpha_T K^2} \right)^{1/3} \right]^{1/m}, \quad (5)$$

where $m = (L + 4)/3$, and $\beta = 13$ for Io in the Io–Europa–Ganymede resonance and $\beta = 30.69$ for Enceladus in the Enceladus–Dione resonance (Meyer and Wisdom, 2007). The equilibrium value of the square of the eccentricity is

$$e_0^2 = \frac{m_0^2 R_p^5 k_p}{\beta M_p^2 R^5 Q_p} f(T_0/T_m). \quad (6)$$

The convective cooling timescale is

$$\tau_{th} = \frac{R}{3} \left(\frac{\nu_{TM} T_m^L R a_c}{g \alpha_T K^2} \right)^{1/3} T_0^{1-m}. \quad (7)$$

The characteristic timescale for the equilibration of eccentricity near equilibrium is

$$\tau_e = \frac{m_1 \alpha |C(\alpha)|}{2M_p \gamma c_0 e_0}, \quad (8)$$

where α is the semimajor axis ratio a_0/a_1 , $C(\alpha)$ is about -1.19 for a 2:1 mean motion resonance,

$$c_0 = \frac{9}{2} \left(\frac{R_p}{R} \right)^5 \frac{m_0}{M_p} n_0 \frac{k_p}{Q_p}, \quad (9)$$

and γ is about 0.32 for the Io–Europa–Ganymede resonance. For a two-body $j:j-1$ resonance,

$$\gamma = j - 1 - j \frac{m_1}{m_0} \left(\frac{a_0}{a_1} \right)^8. \quad (10)$$

For the Enceladus–Dione 2:1 resonance, $\gamma = 0.49$.

Let

$$p = \frac{\tau_{th}}{\tau_e} = \frac{6\beta}{7} \frac{C_p a \gamma}{Gm_1 \alpha |C(\alpha)|} e_0 T_0. \quad (11)$$

For Io this constant is about 3.

Define the nondimensional temperature $T_N = T/T_0$, the scaled eccentricity $e_N = e/e_0$, and the nondimensional time $t_N = t/\tau_{th}$. With these definitions the nondimensionalized evolution equations are

$$\frac{dT_N}{dt_N} = \left[\frac{f(T_0/T_m)}{f(T_N T_0/T_m)} \right] e_N^2 - T_N^m, \quad (12)$$

$$\frac{de_N}{dt_N} = e_N^2 \frac{p}{2} \left[1 - \frac{f(T_0/T_m)}{f(T_N T_0/T_m)} e_N^2 \right]. \quad (13)$$

The equilibrium heat flow is

$$q_0 = \frac{m_0 C_p T_0}{4\pi R^2 \tau_{th}} \quad (14)$$

$$= \frac{7}{2\beta} \frac{k_p}{Q_p} \frac{Gm_0 R R_p^5 n_0 \rho}{a^6}, \quad (15)$$

which is $0.53(100,000/Q_J) W m^{-2}$ for Io, and $1.48(18,000/Q_S) mW m^{-2}$ for Enceladus. The latter corresponds to a total power of $1.1(18,000/Q_S) GW$ emanating from Enceladus.

In integrating the evolution equations, several assumptions have to be made. The temperature T does not rise above T_m . During an interval in which $T = T_m$, the tidal heating rate is greater than the convective cooling rate, and the excess energy is assumed to be released through volcanism. The interval of $T = T_m$ is terminated when the rate of tidal heating falls (due to the declining eccentricity) below the rate of convective cooling. The heat flow during an interval in which $T < T_m$ is given by the convective cooling term; the heat flow during an interval in which $T = T_m$ is given by the tidal heating term.

In Fig. 1, we show a typical evolution of Io's eccentricity and heat flow in the Ojakangas–Stevenson model. We confirm the oscillating behavior found by Ojakangas and Stevenson (1986). In Fig. 2, we show a typical evolution of Enceladus's eccentricity and heat flow in the same model. Both the eccentricity and heat flow rapidly damp to equilibrium. No oscillations were found for Enceladus.

3. Stability analysis

Ojakangas and Stevenson carry out a linear stability analysis for their simplified model in which $Q(T)/k(T)$ is a power law. They then introduce a more realistic form for $Q(T)/k(T)$ that approaches a constant for small T [see Eq. (2)]. However, they do not carry out the stability analysis for this case. Here we describe how the results of the linear stability analysis are modified for their more realistic model.

For the simplified model they found that the equilibrium state, $e_N = T_N = 1$, was unstable if $n > m + p$. Further, they found that there were linear oscillations (either growing or decaying) provided

$$m^2 + n^2 + p^2 - 2mp - 2mn - 2np < 0. \quad (16)$$

When the tidal equilibrium is linearly unstable, the nonlinear system oscillates.

We have carried out a linear stability analysis for the more realistic model in which $Q(T)/k(T)$ is given by Eq. (2). We find that the results of their analysis for the simplified model continue to hold in the more realistic model if n is replaced by

$$n' = nA(T_0/T_m)^n f(T_0/T_m). \quad (17)$$

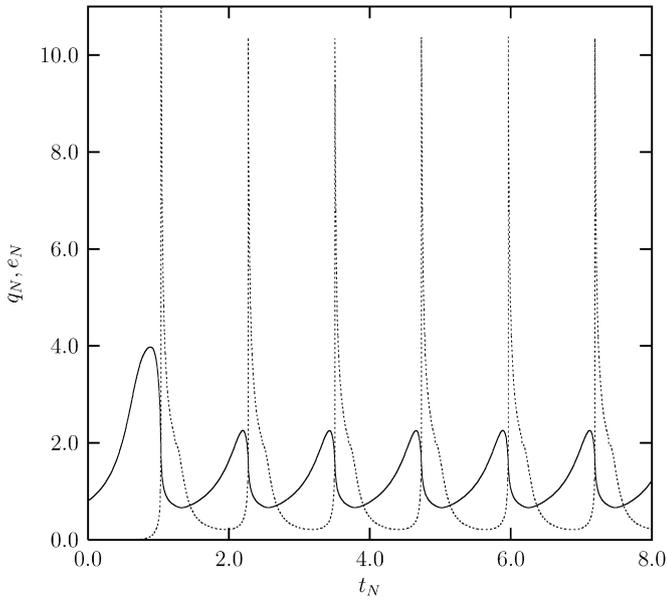


Fig. 1. The scaled eccentricity e_N (solid) and the nondimensional heat flow q_N (dotted) for Io are plotted versus the nondimensional time. The timescale τ_{th} is about 135 Myr, the scale for the heatflow is 0.53 W m^{-2} , and the scale for the eccentricity is 0.0052. Here $m = 12$, $n = 25$, $(Q/k)_0 = 200/0.027$, $(Q/k)_{\min} = 3/0.027$, and $Q_J = 10^3$.

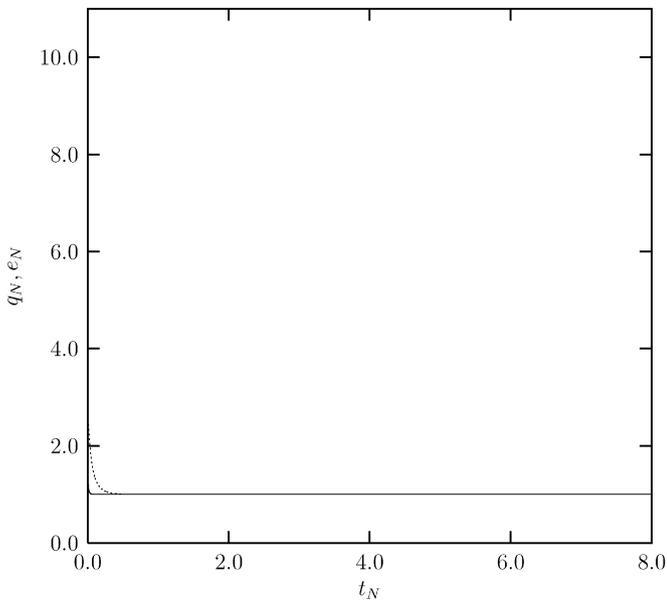


Fig. 2. The scaled eccentricity e_N (solid) and the nondimensional heat flow q_N (dotted) for Enceladus are plotted versus the nondimensional time.

Thus the equilibrium is linearly unstable if $n' > m + p$, and there are linear oscillations (growing or decaying) if

$$m^2 + (n')^2 + p^2 - 2mp - 2mn' - 2n'p < 0. \quad (18)$$

For Io, $n' \approx n$, for n in the range of interest $20 < n < 30$, and $T_0/T_m \approx 0.94$. The system is linearly unstable and develops non-linear oscillation at moderate $n \approx n'$.

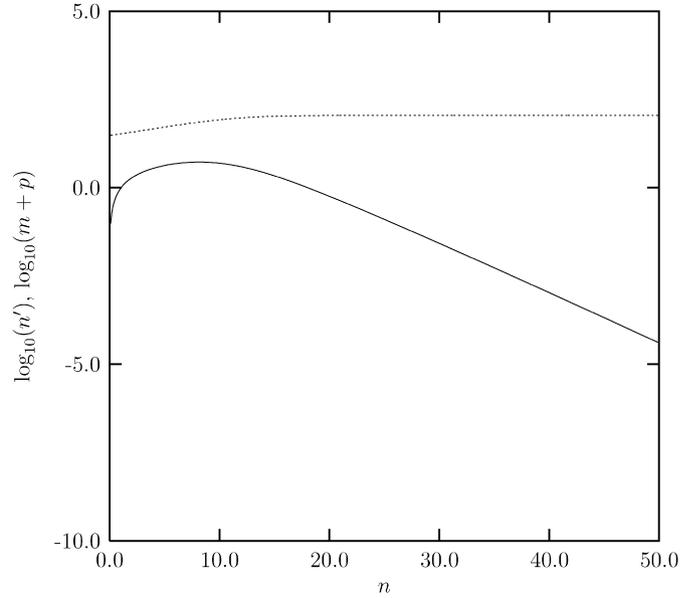


Fig. 3. The parameter n' (solid) is plotted as a function of n , for Enceladus. This is to be compared with $m + p$ (dotted) plotted as a function of n . For $n' < m + p$ the system damps to the equilibrium state. The system is stable for all n . Here $(Q/k)_0$ is $100/0.0018$; $(Q/k)_0$ is $100/0.0018$; $T_0/T_m = 0.70$; and $m = 13$.

For Enceladus, T_0/T_m is smaller ($T_0/T_m \approx 0.70$) so the dropoff in n' at large n is more rapid. In fact, the peak of n' is about 5, for n about 8. At this n , $p \approx 51.2$. Fig. 3 shows a graph of n' and $m + p$ as a function of n . Enceladus is not in the unstable region for any n ; instability requires $n' > m + p$. This criterion cannot be fulfilled for Enceladus for two reasons: the large value of p requires a large value for n' for instability, and the maximum value of n' as a function of n is small. Thus for any n the state of Enceladus damps down to the equilibrium state. This conclusion is insensitive to the values we have adopted for the physical parameters.

4. Conclusion

We have shown that Enceladus does not oscillate about the tidal equilibrium within the [Ojakangas and Stevenson \(1986\)](#) model. If Enceladus is oscillating about equilibrium, then another model must be developed to describe those oscillations. One possibility is the idea expressed by [Yoder \(1981\)](#) that there might be oscillations about equilibrium if the Q of Enceladus was stress dependent, but this idea has not been developed.

References

- Durham, W.B., Kirby, S.H., Stern, L.A., 1997. Creep of water ices at planetary conditions: A compilation. *J. Geophys. Res.* 102, 16293–16302.
- Fischer, H.-J., Spohn, T., 1990. Thermal-orbital histories of viscoelastic models of Io. *Icarus* 83, 39–65.
- Meyer, J.A., Wisdom, J., 2007. Tidal heating in Enceladus. *Icarus* 188, 535–539.
- Ojakangas, G.W., Stevenson, D.J., 1986. Episodic volcanism of tidally heated satellites with application to Io. *Icarus* 66, 341–358.
- Spencer, J.R., Pearl, J.C., Segura, M., Flasar, F.M., Mamoutkine, A., Romani, P., Buratti, B.J., Hendrix, A.R., Spilker, L.J., Lopes, R.M.C., 2006. Cassini encounters Enceladus: Background and the discovery of a south polar hot spot. *Science* 311, 1401–1405.
- Squyres, S.W., Reynolds, R.T., Cassen, P.M., Peale, S.J., 1983. The evolution of Enceladus. *Icarus* 53, 319–331.
- Yoder, C.F., 1981. Tidal friction and Enceladus' anomalous surface. *Eos* 62, 939.