Statistical Inference



Two simple examples

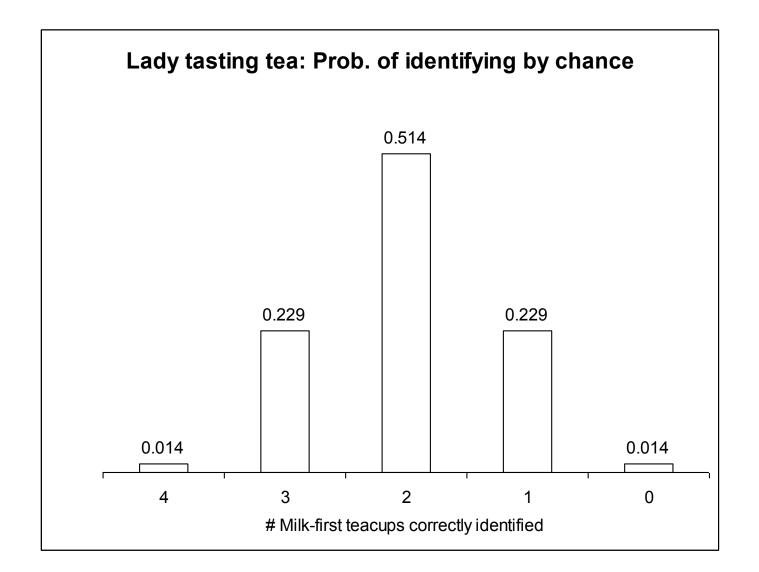
- Lady tasting tea
- Human energy fields



Fisher's exact test

- A simple approach to inference
- Only applicable when outcome probabilities known
- Lady tasting tea example
 - ☐ Claims she can tell whether the milk was poured first
 - □ In a test, 4/8 teacups had milk poured first
 - □ The lady correctly detects all four
- What is the probability she did this by chance?
 - □ 70 ways of choosing four cups out of eight
 - □ How many ways can she do so correctly?





Second simple example Healing touch: human energy field detection

"A Close Look at Therapeutic Touch"

Linda Rosa; Emily Rosa; Larry Sarner; Stephen Barrett. 1998.

JAMA

(279:1005-1010)

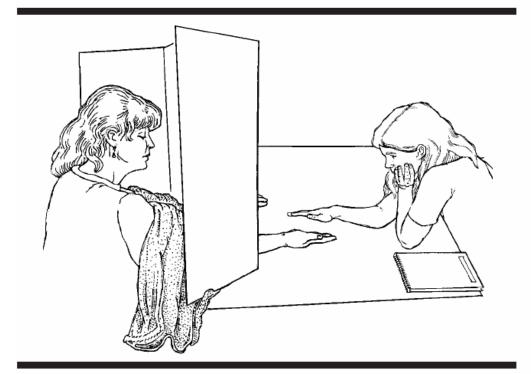


Figure 1.—Experimenter hovers hand over one of subject's hands. Draped towel prevents peeking. Drawing by Pat Linse, Skeptics Society.

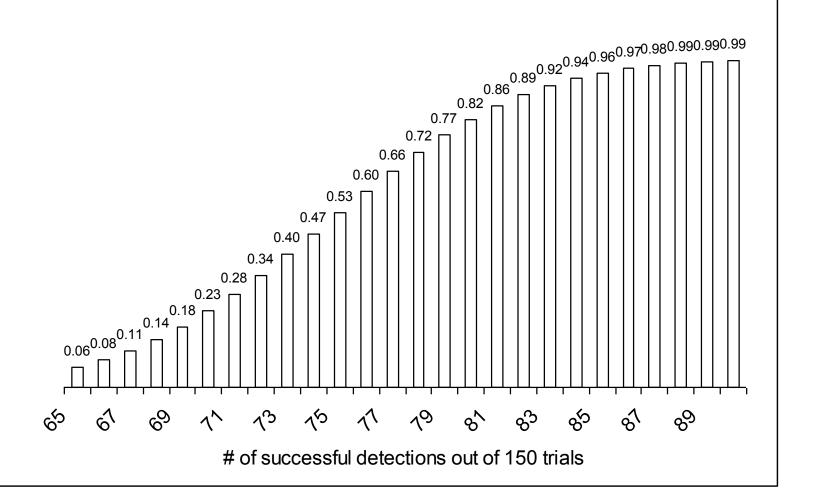
M

Human energy field: Prob. of success by chance

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$
0.21
0.21
0.12
0.04
0.04
0.04
0.04
0.04
0.05
of successful detections



Human energy field detection: Confidence in ability



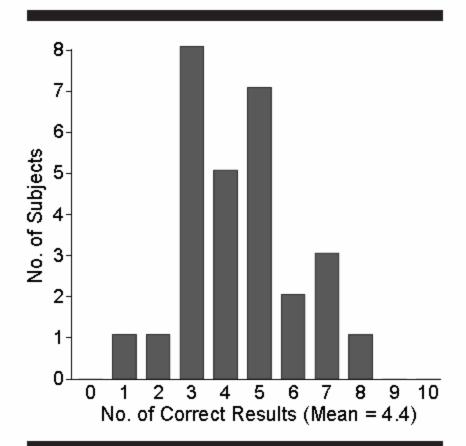


Figure 2.—Distribution of test results.

Table 2.—Statistical Analysis

	-	
Statistical Function	Initial Test (n = 15)	Follow-up Test (n = 13)
Mean (95% confidence interval)	4.67 (3.67-5.67)	4.08 (3.17-4.99)
SD	1.74	1.44
α (1-tailed test)	.05	.05
t statistic	-0.7174	-2.222
Upper critical limit of Student <i>t</i> distribution	1.761	1.782
Alternative hypothesis, µ = 6.67	0.9559	0.9801
Alternative hypothesis, μ = 7.50	0.999644	0.999953



Null hypothesis

- In both cases, we calculated the probability of making the correct choice by chance and compared it to the observed results.
- Thus, our <u>null hypothesis</u> was that the lady and the therapists lacked any of their claimed ability.
- What's the null hypothesis that Stata uses by default for calculating p values?
- Always consider whether other null hypotheses might be more substantively meaningful.
 - □ E.g., testing whether the benefits from government programs outweigh the costs.



Two types of inference

- Testing underlying traits
 - □ E.g., can lady detect milk-poured first?
 - □ E.g., does democracy improve human lives?
- Testing inferences about a population from a sample
 - What percentage of the population approves of President Bush?
 - What's average household income in the United States?



Assessing uncertainty

- Today we will cover
 - ☐ Standard error
 - Confidence intervals
 - □ Central limit theorem



- In 2006, Manny Ramírez hit .321
- How certain are we that, in 2006, he was a .321 hitter?
- To answer this question, we need to know how precisely we have estimated his batting average
- The standard error gives us this information, which in general is (where s is the sample standard deviation)

std. err. =
$$\frac{s}{\sqrt{n}}$$



Baseball example

 The standard error (s.e.) for proportions (percentages/100) is

$$\sqrt{\frac{p(1-p)}{n}}$$

- ightharpoonup N = 400, p = .321, s.e. = .023
- Which means, on average, the .321 estimate will be off by .023

M

Baseball example: postseason

- 20 at-bats
 - \square N = 20, p = .400, s.e. = .109
 - Which means, on average, the .400 estimate will be off by .109

- 10 at-bats
 - \square N = 10, p = .400, s.e. = .159
 - Which means, on average, the .400 estimate will be off by .159

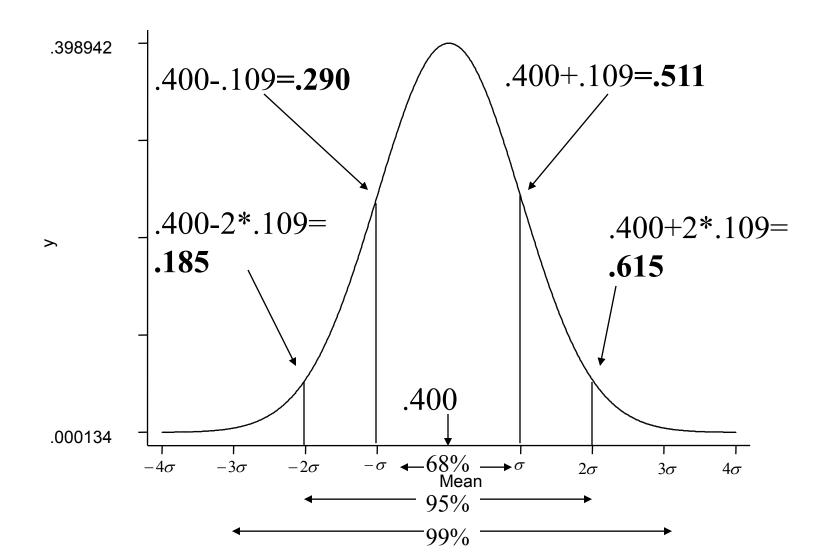


Using Standard Errors, we can construct "confidence intervals"

Confidence interval (ci): an interval between two numbers, where there is a certain specified level of confidence that a population parameter lies

ci = sample parameter <u>+</u> multiple * sample standard error

Confidence interval

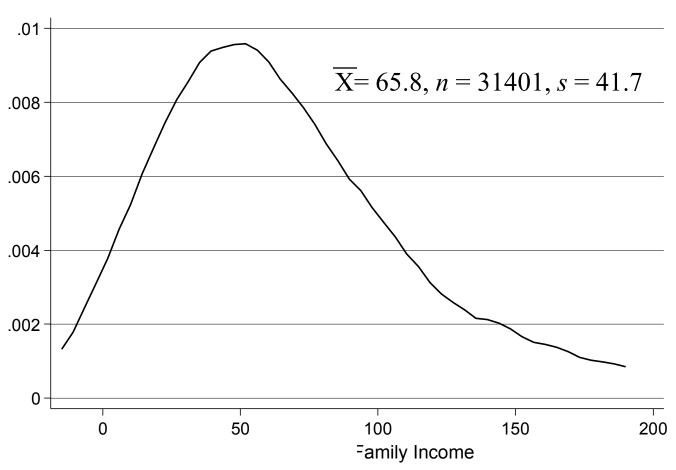




- Much of the time, we fail to realize the uncertainty in statistical estimates
 - □ Postseason statistics
 - □ Competitions



Certainty about mean of a population based on a sample: Family income in 2006



Source: 2006 CCES

Calculating the Standard Error

std. err. =
$$\frac{s}{\sqrt{n}}$$

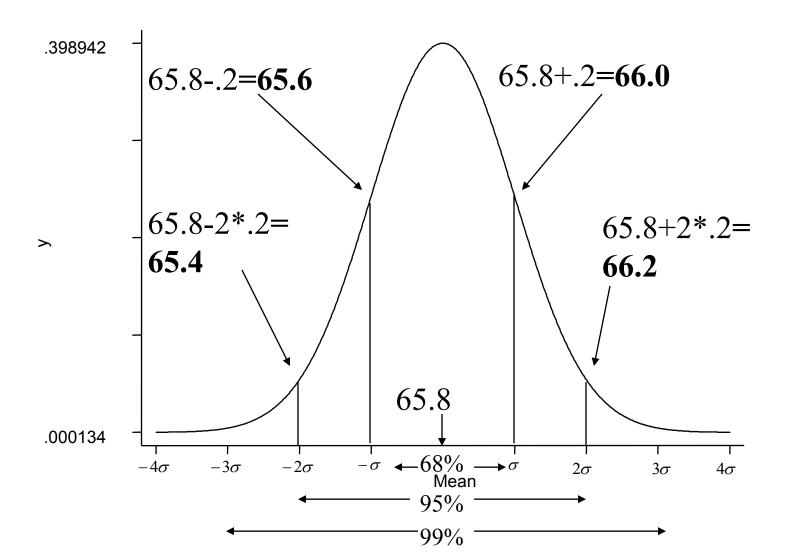
For the income example,

std. err. =
$$41.6/177.2 = .23$$

$$\overline{X}$$
= 65.8, n = 31401, s = 41.7

N = 31401; avg. = 65.8; s = 41.6; s.e. = s/\sqrt{n} = .2

The Picture





Where does the bell-shaped curve come from?

Central limit theorem

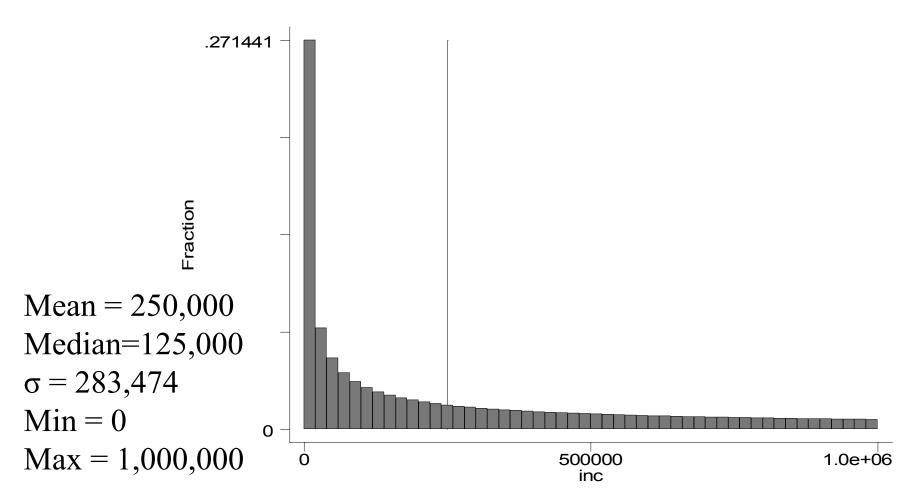


Central Limit Theorem

As the sample size n increases, the distribution of the mean \overline{X} of a random sample taken from **practically any population** approaches a *normal* distribution, with mean μ and standard deviation σ/\sqrt{n}

м

Illustration of Central Limit Theorem: Exponential Distribution

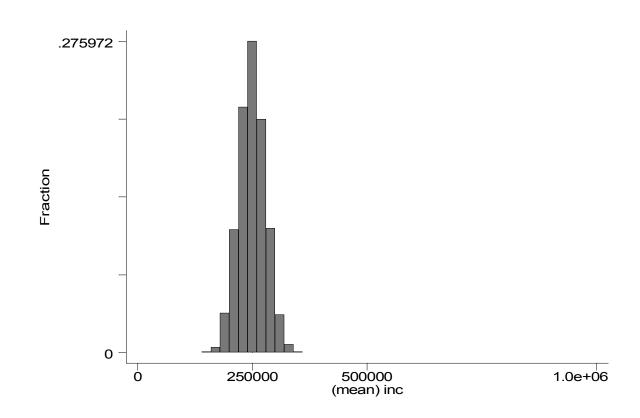




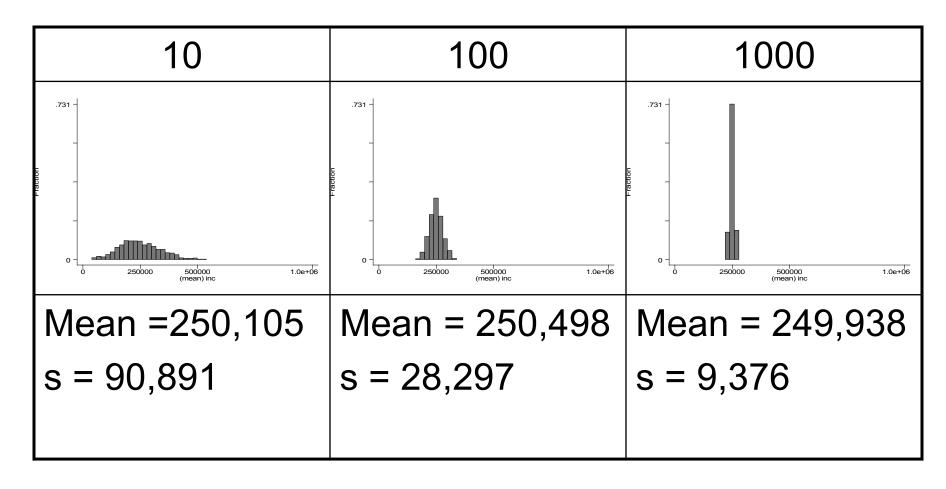
Consider 10,000 samples of n = 100

$$N = 10,000$$

Mean = 249,993
 $s = 28,559$



Consider 1,000 samples of various sizes





Play with some simulations

- http://www.ruf.rice.edu/~lane/stat_sim/sam pling_dist/index.html
- http://www.kuleuven.ac.be/ucs/java/index. htm

Most important standard errors Mean s

Mean **Proportion** Diff. of 2 means Diff. of 2 $\frac{p_1(1-p_1)}{p_1(1-p_2)} + \frac{p_2(1-p_2)}{p_2(1-p_2)}$ proportions Diff of 2 means (paired data) Regression (slope) coeff.

M

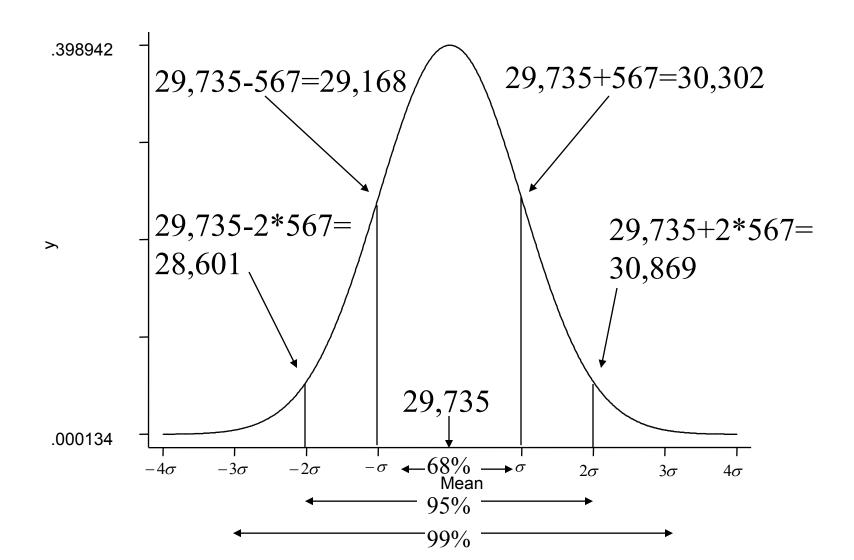
Another example

- Let's say we draw a sample of tuitions from 15 private universities. Can we estimate what the average of all private university tuitions is?
- N = 15
- Average = 29,735

• s = 2,196
• s.e. =
$$\frac{s}{\sqrt{n}} = \frac{2,196}{\sqrt{15}} = 567$$

N = 15; avg. = 29,735; s = 2,196; s.e. = s/\sqrt{n} = 567

The Picture



10

Confidence Intervals for Tuition Example

- 68% confidence interval = 29,735+567 = [29,168 to 30,302]
- 95% confidence interval = 29,735<u>+</u>2*567 = [28,601 to 30,869]
- 99% confidence interval = 29,735+3*567 = [28,034 to 31,436]



What if someone (ahead of time) had said, "I think the average tuition of major research universities is \$25k"?

- Note that \$25,000 is well out of the 99% confidence interval, [28,034 to 31,436]
- Q: How far away is the \$25k estimate from the sample mean?
 - □ A: Do it in *z*-scores: (29,735-25,000)/567 = 8.35



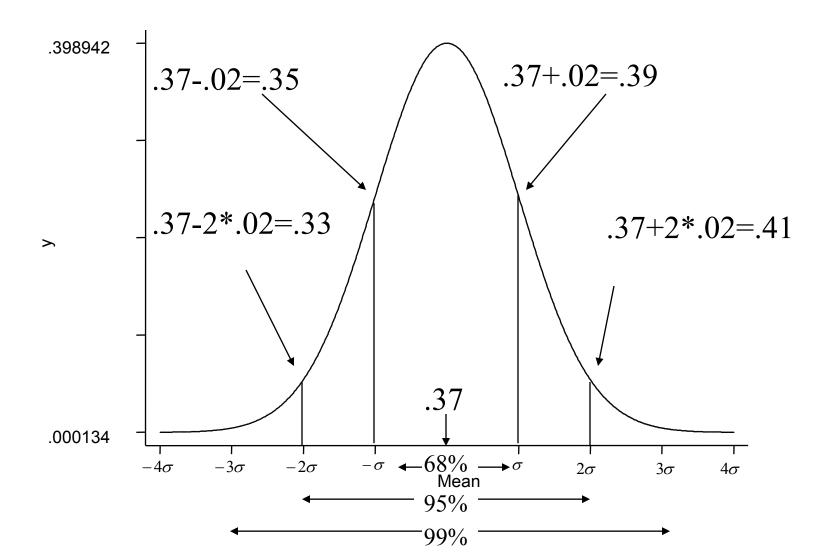
Constructing confidence intervals of proportions

- Let us say we drew a sample of 1,000 adults and asked them if they approved of the way George Bush was handling his job as president. (March 13-16, 2006 Gallup Poll) Can we estimate the % of all American adults who approve?
- N = 1000

p = .37
s.e.
$$= \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.37(1-.37)}{1000}} = 0.02$$

N = 1,000; p. = .37; s.e. =
$$\sqrt{p(1-p)/n}$$
 = .02

The Picture



r,e

Confidence Intervals for Bush approval example

- 68% confidence interval = .37±.02 = [.35 to .39]
- 95% confidence interval = .37<u>+</u>2*.02 = [.33 to .41]
- 99% confidence interval = .37±3*.02 =[.31 to .43]



What if someone (ahead of time) had said, "I think Americans are equally divided in how they think about Bush."

- Note that 50% is well out of the 99% confidence interval, [31% to 43%]
- Q: How far away is the 50% estimate from the sample proportion?
 - □ A: Do it in z-scores: (.37-.5)/.02 = -6.5 [-8.7 if we divide by 0.15]

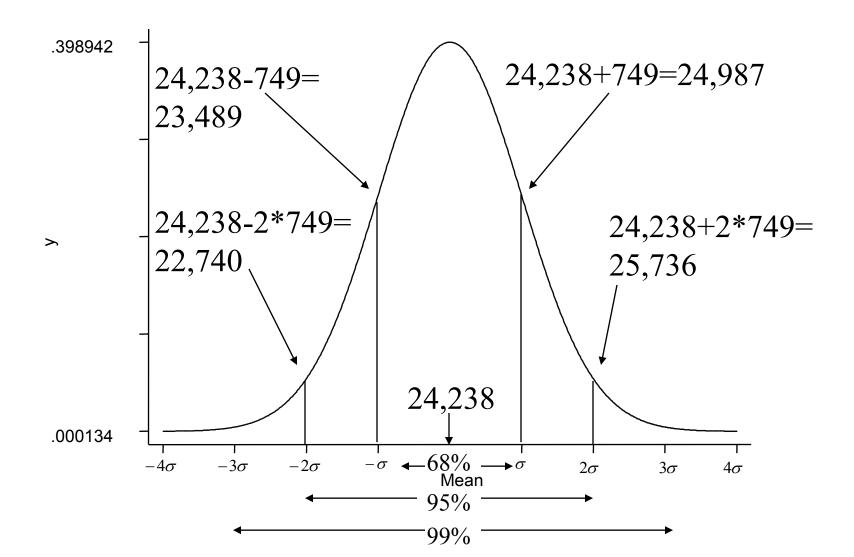
0

Constructing confidence intervals of differences of means

- Let's say we draw a sample of tuitions from 15 private and public universities. Can we estimate what the difference in average tuitions is between the two types of universities?
- N = 15 in both cases
- Average = 29,735 (private); 5,498 (public); diff = 24,238
- s = 2,196 (private); 1,894 (public)

s.e. =
$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{4,822,416}{15} + \frac{3,587,236}{15}} = 749$$

The Picture



re.

Confidence Intervals for difference of tuition means example

- 68% confidence interval = 24,238<u>+</u>749 = [23,489 to 24,987]
- 95% confidence interval = 24,238+2*749 = [22,740 to 25,736]
- 99% confidence interval =24,238<u>+</u>3*749 =
- **[21,991 to 26,485]**



What if someone (ahead of time) had said, "Private universities are no more expensive than public universities"

- Note that \$0 is well out of the 99% confidence interval, [\$21,991 to \$26,485]
- Q: How far away is the \$0 estimate from the sample proportion?
 - \square A: Do it in z-scores: (24,238-0)/749 = 32.4

100

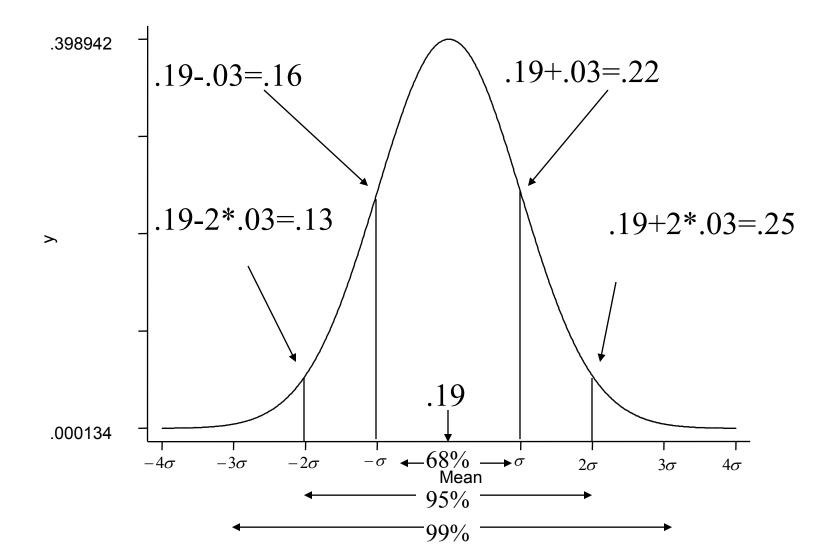
Constructing confidence intervals of difference of proportions

- Let us say we drew a sample of 1,000 adults and asked them if they approved of the way George Bush was handling his job as president. (March 13-16, 2006 Gallup Poll). We focus on the 600 who are either independents or Democrats. Can we estimate whether independents and Democrats view Bush differently?
- N = 300 ind; 300 Dem.
- p = .29 (ind.); .10 (Dem.); diff = .19

• s.e. =
$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = \sqrt{\frac{.29(1-.29)}{300} + \frac{.10(1-.10)}{300}} = .03$$

diff. p. = .19; s.e. = .03

The Picture



r,e

Confidence Intervals for Bush Ind/Dem approval example

- 68% confidence interval = .19±.03 = [.16 to .22]
- 95% confidence interval = .19<u>+</u>2*.03 =
- [.13 to .25]
- 99% confidence interval = .19<u>+</u>3*.03 =
- [.10 to .28]



What if someone (ahead of time) had said, "I think Democrats and Independents are equally unsupportive of Bush"?

- Note that 0% is well out of the 99% confidence interval, [10% to 28%]
- Q: How far away is the 0% estimate from the sample proportion?
 - \square A: Do it in z-scores: (.19-0)/.03 = 6.33



What if someone (ahead of time) had said, "Private university tuitions did not grow from 2003 to 2004"

- Stata command ttest
- Note that \$0 is well out of the 95% confidence interval, [\$1,141 to \$2,122]
- Q: How far away is the \$0 estimate from the sample proportion?
 - \square A: Do it in z-scores: (1,632-0)/229 = 7.13

M

The Stata output

- . gen difftuition=tuition2004-tuition2003
- . ttest diff=0 in 1/15

One-sample t test

```
Variable | Obs Mean Std. Err. Std. Dev. [95% Conf. Interval]

difftu~n | 15 1631.6 228.6886 885.707 1141.112 2122.088

mean = mean(difftuition) t = 7.1346

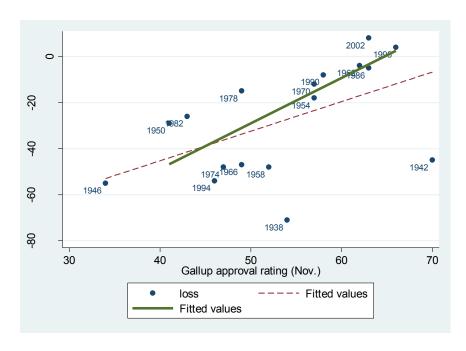
Ho: mean = 0 degrees of freedom = 14

Ha: mean < 0 Ha: mean != 0 Ha: mean > 0

Pr(T < t) = 1.0000 Pr(|T| > |t|) = 0.0000 Pr(T > t) = 0.0000
```

Constructing confidence intervals of regression coefficients

Let's look at the relationship between the midterm seat loss by the President's party at midterm and the President's Gallup poll rating



Slope = 1.97
N = 14
s.e.r. = 13.8

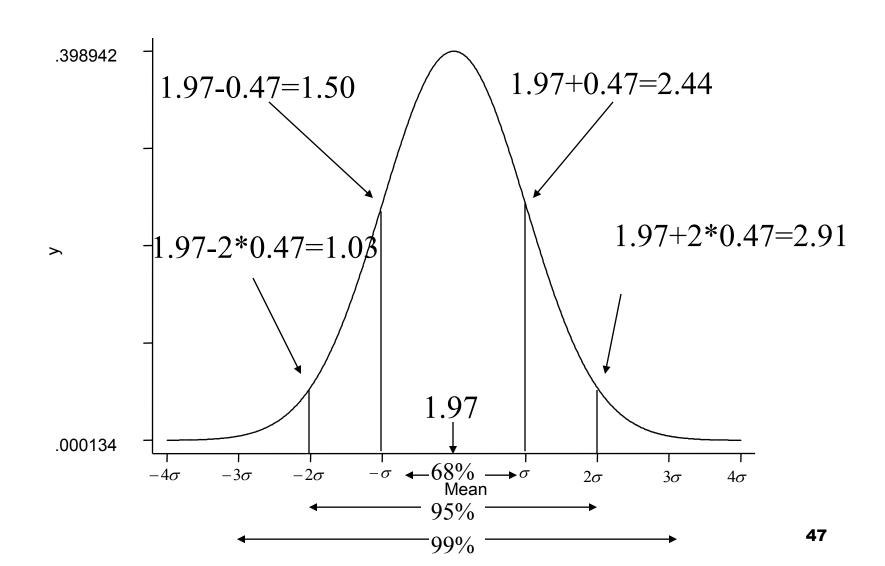
$$s_x = 8.14$$

s.e._{slope} =
$$\frac{s.e.r.}{\sqrt{n-1}} \times \frac{1}{s_x} = \frac{13.8}{\sqrt{13}} \times \frac{1}{8.14} = 0.47$$



N = 14; slope=1.97; s.e. = 0.45

The Picture



10

Confidence Intervals for regression example

- 68% confidence interval = 1.97<u>+</u> 0.47= [1.50 to 2.44]
- 95% confidence interval = 1.97+ 2*0.47 = [1.03 to 2.91]
- 99% confidence interval = 1.97+3*0.47 = [0.62 to 3.32]



What if someone (ahead of time) had said, "There is no relationship between the president's popularity and how his party's House members do at midterm"?

- Note that 0 is well out of the 99% confidence interval, [0.62 to 3.32]
- Q: How far away is the 0 estimate from the sample proportion?
 - \square A: Do it in z-scores: (1.97-0)/0.47 = 4.19

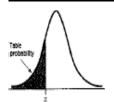


The Stata output

. reg loss gallup if year>1948

Source		df	MS			14
Model Residual	3332.58872 2280.83985	1 3332 12 190.	2.58872 .069988 		F(1, 12) = Prob > F = R-squared = Adj R-squared = Root MSE =	0.0013 0.5937
loss	Coef.	Std. Err.		P> t	=	terval]
gallup _cons	1.96812	.4700211 25.54753	4.19 -4.99	0.001 0.000	.9440315 2	.992208 1.76486





Reading a z table

The color of the											
33	I	.60	.01	.02	.03	.04	.65	.06	.01	.09	.09
32 0.007	-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.1 0010 0000 0000 0000 0000 0000 0000 0	33	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.0 0073 0073 0073 1073 1073 1072 0072 0071 0071 0071 1071 0070 0070	32	.0007	.0007	.0006	.0005	.0005	.0006	.0005	.0005	.0005	.0005
-2.0 0.009 0.018 0.018 0.017 0.016 0.016 0.015 0.015 0.014 0.014 -2.8 0.0029 0.0025 0.004 0.002 0.0023 0.0020 0.0020 0.0021 0.0021 0.0021 2.7 0.0035 0.004 0.0033 0.0032 0.0031 0.0020 0.0029 0.0028 0.0027 0.0026 -2.5 0.0047 0.045 0.044 0.043 0.044 0.040 0.039 0.0028 0.0031 0.0036 -2.5 0.0047 0.045 0.050 0.057 0.055 0.054 0.0022 0.051 0.0040 0.048 -2.4 0.002 0.000 0.059 0.057 0.055 0.054 0.002 0.051 0.049 0.048 -2.3 0.0107 0.0104 0.002 0.009 0.009 0.004 0.004 0.001 0.008 0.065 0.064 -2.3 0.0107 0.0104 0.002 0.009 0.009 0.004 0.004 0.001 0.008 0.065 0.064 -2.1 0.0139 0.0124 0.0120 0.029 0.0125 0.022 0.019 0.016 0.014 0.014 -2.1 0.0139 0.0174 0.0170 0.0166 0.0162 0.0188 0.0154 0.0150 0.0140 0.0140 -2.1 0.0139 0.0124 0.0104 0.0102 0.0189 0.0154 0.0160 0.0140 0.0143 -2.0 0.0228 0.0222 0.017 0.0212 0.0017 0.0020 0.0147 0.002 0.0188 0.0153 -1.0 0.0237 0.0281 0.0744 0.0268 0.022 0.0167 0.0140 0.0143 -1.1 0.0237 0.0281 0.0427 0.0418 0.0020 0.0326 0.0250 0.044 0.039 0.0334 -1.1 0.0354 0.0351 0.0427 0.0418 0.000 0.040 0.034 0.037 0.001 0.024 -1.1 0.0368 0.0537 0.0360 0.0361 0.0364 0.0364 0.0364 0.0362 0.0364 -1.1 0.0368 0.0537 0.0360 0.0361 0.0048 0.0018 0.0066 0.0544 0.036 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0364 0.0365 0.0364 0.03	-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-2.8	-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
2.7 0.035 0.034 0.033 0.032 0.081 0.030 0.029 0.028 0.027 0.026 -2.6 0.047 0.045 .044 .0043 .0041 .0040 .0039 .0038 .0037 .0038 -2.5 .0062 .0060 .0050 .0057 .0053 .0054 .0052 .0060 .0049 .0049 .0049 .0049 .0049 .0049 .0044 .0041 .0068 .0064	-2.9	.0019	.0018	.0018	.0017	.0015	.0016	.0015	.0015	.0014	.0014
-2.6 0047 0048 0044 0043 0041 0040 0039 0038 0037 0038 -2.5 0060 0060 0064 0044 0048 0064 0044 0048 0065 0064 0064 0044 0048 0065 0064 0064 0064 0064 0064 0064 0064	-2.8	.0025	.0025	.0024	.0023	.0023	0022	.0021	.0021	.0020	.0019
-2.5	2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0025
2.4 0.082 0.080 0.078 0.075 0.073 0.071 0.089 0.088 0.064 -2.3 0.070 0.014 0.002 0.009 0.0095 0.004 0.009 0.067 0.064 2.2 0.130 0.135 0.129 0.0125 0.0122 0.0119 0.016 0.0142 0.019 0.019 0.012 0.0119 0.016 0.0142 0.022 0.019 0.0143 0.012 0.019 0.0143 0.0183 0.020 0.0177 0.002 0.0177 0.002 0.0177 0.002 0.0177 0.002 0.0184 0.023 0.023 0.023 0.023 0.023 0.023 0.023 0.023 0.023 0.023 0.0222 0.0344 0.034 <t< td=""><td>-2.6</td><td>.0047</td><td>.0045</td><td>.0044</td><td>.0043</td><td>.0041</td><td>.0040</td><td>.0039</td><td>.0038</td><td>.0037</td><td>.0035</td></t<>	-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0035
-2.3	-2.5	.0062	.0000	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.2 .0133 .0136 .0132 .0109 .0125 .0122 .0110 .0116 .0113 .0110 -2.1 .0170 .0174 .0170 .0166 .0162 .0188 .0154 .0550 .0144 .0143 -2.0 .0228 .0222 .0217 .0212 .0207 .0002 .0167 .0160 .0168 .0163 .0167 .0160 .0168 .0163 .0229 .0224 .0239 .0233 .0233 .0233 .0233 .0233 .0233 .0233 .0233 .0234 .0294 .0239 .0234 .0234 .0239 .0233 .0236 .0247 .0241 .0248 .0260 .0240 .0234 .0234 .0235 .0241 .0237 .0247 .0418 .0400 .0401 .0302 .0284 .0257 .0567 .0455 .0445 .0445 .0445 .0445 .0445 .0445 .0445 .0445 .0445 .0445 .0445 .0445	2.4	.0082	.0800.	.0078	.0075	.0073	.0071	.0009			
-2.1 0.179 0.174 0.170 0.166 0.162 0.188 0.154 0.160 0.143 0.143 1.20 0.022 0.217 0.212 0.027 0.020 0.147 0.148 0.143 1.10 0.028 0.222 0.217 0.212 0.027 0.020 0.147 0.140 0.148 0.163 1.10 0.027 0.028 0.022 0.217 0.026 0.022 0.047 0.020 0.023 0.023 0.023 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.024 0.029 0.023 0.022 0.024 0.029 0.023 0.023 0.023 0.022 0.024 0.029 0.023 0.023 0.022 0.024 0.024 0.024 0.022 0.024 0.024 0.024 0.022 0.024 0.024 0.024 0.024 0.022 0.024 0.02	-2.3	.0107	.0104	.0102	.0099	.0095	.0094	.0091	.0089	.0067	.0084
-2.0	22	.0139	.0135	.0132	.0129	.0125	.0122	.0119	.0116	.0113	
-1.0 0.0287 0.081 0.074 0.0288 0.0202 0.0250 0.0250 0.044 0.0239 0.0233 -1.8 0.0389 0.0511 0.044 0.0386 0.0339 0.0222 0.0314 0.007 0.001 0.0244 -1.7 0.448 0.438 0.427 0.488 0.400 0.0222 0.0314 0.007 0.001 0.0244 -1.7 0.448 0.438 0.427 0.488 0.400 0.001 0.002 1.0384 0.075 0.087 -1.6 0.0388 0.637 0.0500 0.0516 0.0515 0.405 0.485 0.485 0.455 0.465 -1.5 0.068 0.657 0.653 0.053 0.0518 0.000 0.0544 0.682 0.671 0.059 -1.4 0.008 0.0703 0.078 0.0764 0.049 0.0735 0.0721 0.0708 0.064 0.081 -1.3 0.008 0.0091 0.004 0.018 0.000 0.0855 0.089 0.0833 0.023 -1.2 0.1511 0.1131 0.1112 0.003 0.007 0.0855 0.089 0.033 0.023 -1.2 0.1511 0.1131 0.1112 0.003 0.007 0.0855 0.089 0.033 0.023 -1.1 0.0557 0.0357 0.0357 0.035 0.007 0.0855 0.009 0.005 0.005 -1.1 0.0557 0.0357 0.0357 0.0357 0.002 0.005 -1.0 0.0557 0.005	-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0145	
-1.8	-2.0	.0228	.0222	.0217	.0212	.0297	.0202	.0197	.0192	.0188	.0183
-1.7 0.446 0.436 0.427 0.418 0.400 0.401 0.302 0.984 0.375 0.367 -1.6 0.548 0.637 0.506 0.516 0.505 0.446 0.485 0.445 0.445 0.445 0.465 0.465 -1.5 0.668 0.665 0.643 0.630 0.618 0.608 0.564 0.564 0.567 0.567 -1.4 0.568 0.073 0.778 0.774 0.749 0.735 0.721 0.708 0.644 0.881 -1.2 0.571 0.590 0.778 0.774 0.749 0.735 0.821 0.721 0.708 0.644 0.881 -1.2 0.571 0.571 0.570 0.823 0.823 -1.2 0.571 0.571 0.571 0.571 0.850 0.823 0	-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0344	.0239	
-1.6								.0314			
-1.5	-1.7	.0445	.0435	.0427	.0418						
-1.4 0808 0703 0778 0764 0740 0735 0721 0708 0844 0861 13 0968 0851 0924 0918 0901 0885 0869 0853 0833 0823 1.2 1.1 1731 17112 1903 1905 1905 1908 1900 1905 0823 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1900 1905 1905	-1.6	.0548		.0526	.0516	.0505	.0495	.0485	.0475		
13 0068 0051 0034 0018 0001 0885 0869 0853 0838 0823 -1.2 .1511 .1131 .1112 .1003 .1075 .1986 .1028 .1200 .1003 .0695 -1.1 .1351 .1344 .1202 .1271 .1251 .1220 .1210 .1010 .1100 .1100 .1100 .1101 .1170 .1101 .1101 .1170 .1171 .1251 .1220 .1210 .1100 .1170 .1170 .1101 .1101 .1170 .1171 .1051 .1422 .1440 .1440 .1442 .1440 .1440 .1441 .1370 .1011 .1051 .1412 .1440 .1443 .1441 .1370 .1011 .1065 .1600 .1035 .1611 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 .1001 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>											
-1.2 .1151 .1151 .1112 .1003 .1075 .1056 .1038 .1020 .1003 .0665 .1.1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .	-1.4	.0308	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.1 .1357 .1395 .1394 .1292 .1271 .1251 .1220 .1290 .1190 .1170 .1170 .1587 .1582 .1590 .1515 .1492 .1490 .5446 .1423 .1491 .1370 .0.9 .5441 .1584 .1782 .1782 .1793 .1711 .1685 .1900 .1605 .1001 .1005 .1001 .0.0 .1005 .1001 .1005 .1005 .1001 .1005 .10	13	.0968	.0951	.0034	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.0 .1587 .1582 .1530 .1515 .1482 .1490 .5446 .1423 .4401 .1370 .0.0 .1681 .1694 .1708 .1702 .1736 .1711 .1685 .1600 .1635 .1601 .0.3 .1701 .1708 .2719 .2000 .2001 .2033 .2005 .1771 .1649 .1702 .5844 .1867 .0.7 .3400 .2389 .2387 .2206 .2206 .2236 .2206 .2206 .2206 .2006 .2007 .2018 .1001 .1003 .100								.1038	.1020		
-0.0 1841 1814 1788 1762 1738 1711 1865 1860 1635 1861 -0.8 2119 2000 2001 2033 2005 1877 1849 1422 1884 1867 0.7 2400 2389 2387 2287 2286 2266 2266 2266 2277 248 0.6 2743 2700 2616 2843 2811 2518 2546 2314 2481 -0.5 3.065 3.060 3.015 2981 2049 2012 2677 2843 2810 2776 -0.4 3.446 3409 3372 3356 3300 3204 2228 3492 3159 3121 -0.3 3821 3783 3785 3707 3600 3832 3884 3957 3500 3483 -0.2 2677 2843 2850 3450 -0.3 3821 3783 3785 3707 3600 3632 3684 3957 3500 3483 -0.1 4602 4562 4562 4483 4443 4404 4564 4365 4365 4387	-1.1	.1357	.1335	.1314	1292	.1271	.1251	.1230	.1210	.1190	.1170
-0.8 2110 2090 2081 2033 2005 .1977 .1649 .1922 .1864 .1867 0.7 3420 2389 2388 2327 2296 2206 .2236 2206 .2177 .2148 0.6 2743 2709 2676 2043 .2611 .2518 .2546 .2914 .2483 .2481 -0.5 .3085 .3850 .3015 .2981 .2048 .2012 .2277 .2843 .2610 .2776 -0.4 .3440 .3400 .3372 .3336 .3300 .3264 .3228 .3162 .3156 .3121 -0.3 .3821 .3783 .3745 .3707 .3669 .3632 .3584 .3557 .3530 .3483 -0.2 .407 .4463 .4403 .4443 .4444 .4364 .4325 .4285 .4347 -0.1 .4062 .4522 .4483 .4443 .4444 .4364 .4325 <td></td>											
0.7 3450 2380 2381 2327 2298 2266 2276 2266 2277 2148 0.6 2743 2700 2876 2843 2611 2578 2546 2514 2483 2451 -0.5 3065 3890 3015 2841 2048 2912 2877 2843 2610 2776 -0.4 3440 3400 3372 3336 3300 3304 2228 3102 3156 3121 -0.3 3821 3483 3407 3689 3632 3584 3557 3520 3483 -0.2 4007 4768 4709 4482 4413 4404 4364 4325 4285 4347 -0.1 4002 4552 4483 4443 4404 4364 4325 4285 4347							.1711	.1685		.1635	
0.6 2743 2700 2676 2643 2611 2578 2546 2514 3483 2451 -0.5 3065 3090 3015 2881 2646 2912 2877 2843 2610 2776 -0.4 3446 3400 3372 3386 3300 3684 3288 3162 3155 3721 -0.3 3821 3783 3785 3707 3680 3682 3884 3557 3500 3483 -0.2 4507 4768 4700 4000 4852 4913 3074 3056 3897 3859 -0.1 4602 4502 4822 4483 4443 4404 4364 4325 4286 4347										.1894	.1867
-0.5 3.065 3.050 3.015 2981 2.049 2.912 2.877 2.843 2.810 2.776 -0.4 3.446 3.409 3.372 3.356 3.300 3.084 3.228 3.492 3.159 3.121 -0.3 3.821 3.783 3.785 3.707 3.690 3.682 3.684 3.957 3.500 3.483 -0.2 A.007 A.410 A.002 A.012 A.013 3.074 3.025 3.897 3.859 -0.1 A.002 A.502 A.483 A.443 A.404 A.564 A.205 A.285 A.247											
- 0.4 3.449 3.400 3372 3336 3300 3264 3228 3102 3159 3121 - 0.3 3821 3783 3745 3707 3690 3832 384 3857 3830 3483 - 0.2 4207 4168 4100 4000 4182 4013 3074 3036 3897 3850 - 0.1 4002 4582 4483 4443 4404 4364 4325 4285 4247											
-0.3 3821 3783 3745 3707 3860 3632 3614 3557 3520 3463 -0.2 4207 4168 4100 4000 4152 4013 3074 3036 3897 3850 -0.1 4602 4562 4522 4483 4443 4404 4364 4325 4286 4247											
-0.2 A207 A168 A109 A000 A152 A013 3074 3036 3897 3859 -0.1 A002 A562 A522 A483 A443 A404 A364 A325 A286 A247											
-0.1 AND ASSO ASSO ASSO ASSO ASSO ASSO											
-0.0 .5000 AWO AWO A880 A840 A801 A701 A721 A881 A841											
	-0.0	.5000	.4950	.4920	.4880	.4840	.4801	A761	A721	.4681	.4841

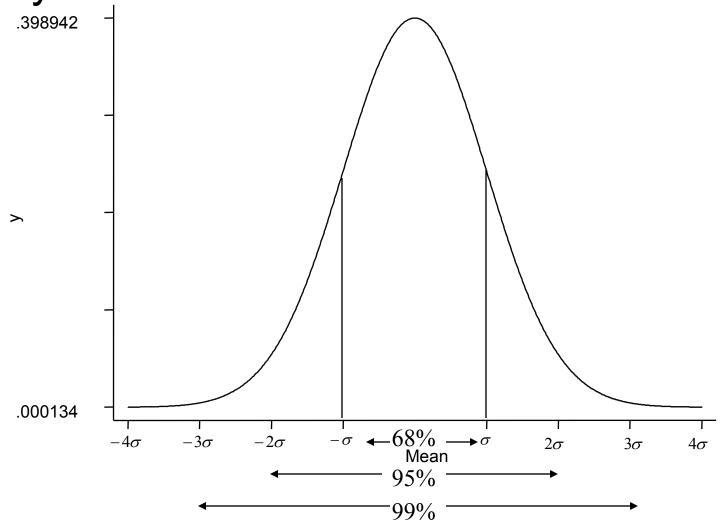
In the Extreme(forz<0)

	-3.09	-3.72	-4.26	-4.75	-5.20	-5.61	-6.00	
Indutiby	.011	,0001	.00001	.000001	.0000001	.00000001	.00000001	

8PLUS was used to determine information for the "in the Extreme" portion of the table.



If *n* is sufficiently large, we know the distribution of sample means/coeffs. will obey the normal curve



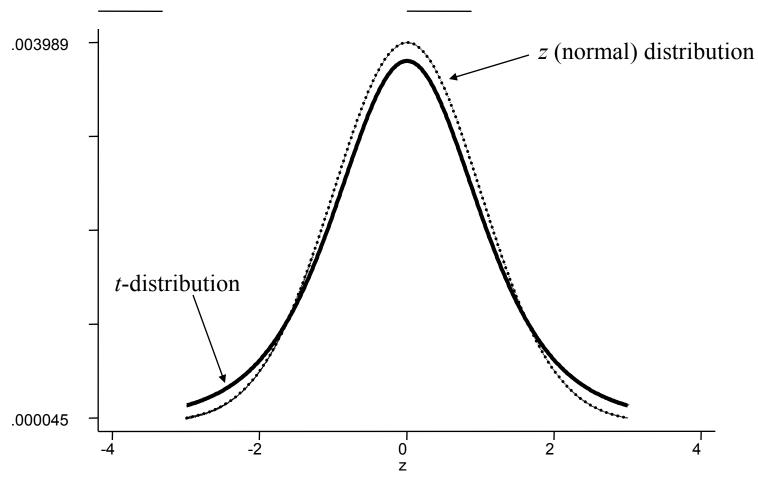


When the sample size is large (i.e., > 150), convert the difference into z units and consult a z table

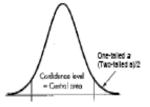
$$Z = (H_1 - H_0) / s.e.$$



t (when the sample is small)







Confidence Level								
4	.89	.90	.95	.98	.59	.998	.599	
1	3.08	6.31	12.71	31.82	63.66	318.31	636.62	
2	1.89	2.92	4.30	6.96	9.92	22.33	31.60	
3	1.64	2.35	3.18	4.54	5.84	10/21	12.92	
4	1.53	2.13	2.78	3.75	4.60	7.17	8.51	
5	1.48	2.02	2.57	3.35	4.03	5.89	6.87	
6	1.44	1.94	2.45	3.14	3.71	521	5.96	
7	1.41	1.99	2.35	3.00	3.50	4.79	5.41	
8	1.40	1.55	2.31	2.90	3.36	4.50	5.04	
9	1.38	1.83	2.25	2.82	3.25	430	4.78	
10	1.37	1.81	2.23	2.76	3.17	4.14	4.50	
11	1.35	1.80	2.20	2.72	3.11	4.02	4.44	
12	1.35	1.78	2.18	2.58	3.05	3.93	4.32	
13	1.35	1.77	2.15	2.65	3.01	3.85	4.22	
14	1.35	1.75	2.14	2.62	2.98	3.79	4.14	
15	134	1.75	2.13	2.60	2.95	3.73	4.07	
16	134	1.75	2.12	2.58	2.92	3.69	4.01	
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97	
18	1.33	1.73	2.10	2.55	2.88	351	3.92	
19	1.33	1.73	2.09	254	2.85	3.58	3.88	
20	1.33	1.72	2.09	253	2.85	3.55	3.85	
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82	
22	1.32	1.72	2.07	2.51	2.82	3.50	3.79	
23	1.32	171	2.07	2.50	2.81	3.48	3.77	
24	1.32	171	2.05	2.49	2.80	3.47	3.75	
25	1.32	171	2.05	2.49	2/9	3.45	3.73	
25	131	1.71	2.05	2.48	2.78	3.43	3.71	
27	131	1.70	2.05	2.47	2.77	3.42	3.00	
28	131	1.70	2.05	2.47	2.76	3.41	3.67	
29	131	1.70	2.05	2.45	215	3.40	3.66	
30	131	1.70	2.04	2.45	2.75	3.39	3.65	
40	1.30	1.58	2.02	2.42	2.70	331	3.55	
50	1.30	1.58	201	2.40	2.58	326	3.50	
60	1.30	1.67	2.00	239	2.55	323	3.45	
70	129	1.67	1.99	2.38	2.65	321	3.44	
80	1.29	1.55	1.99	2.37	2.54	3.20	3.42	
90	1.29	1.55	1.99	237	2.53	3.18	3.40	
100	1.29	1485	1.98	236	2.63	3.17	3.39	
1000	1.282	1.646	1,962	2.330	2 581	3.098	3.300	
infinite	1.281	1.645	1,960	2.325	2.576	3.000	3291	
wo-failed a	.20	.10	.05	.02	.01	.002	.001	_
inn-fuiled a	.10	.05	.025	.01	.005	.001	.0005	

Note that the t-distribution with infinite of is the standard normal distribution.



When the sample size is small (i.e., <150), convert the difference into t units and consult a t table

$$t = (H_1 - H_0) / s.e.$$



A word about standard errors and collinearity

■ The problem: if X₁ and X₂ are highly correlated, then it will be difficult to precisely estimate the effect of either one of these variables on Y



Example: Effect of party, ideology, and religiosity on feelings toward Bush

	Bush Feelings	Conserv.	Repub.	Religious
Bush Feelings	1.0	.39	.57	.16
Conserv.		1.0	.46	.18
Repub.			1.0	.06
Relig.				1.0



Regression table

	(1)	(2)	(3)	(4)
Intercept	32.7	32.9	32.6	29.3
	(0.85)	(1.08)	(1.20)	(1.31)
Repub.	6.73	5.86	6.64	5.88
	(0.244)	(0.27)	(0.241)	(0.27)
Conserv.		2.11		1.87
		(0.30)		(0.30)
Relig.			7.92	5.78
			(1.18)	(1.19)
N	1575	1575	1575	1575
R ²	.32	.35	.35	.36

How does having another *collinear* independent variable affect standard errors?

s.e.
$$(\hat{\beta}_1) = \sqrt{\frac{1}{N-n-1} \frac{S_Y^2}{S_{X_1}^2} \frac{1-R_Y^2}{1-R_{X_1}^2}}$$



 R^2 of the "auxiliary regression" of X_1 on all the other independent variables

Pathologies of statistical significance



Understanding "significance"

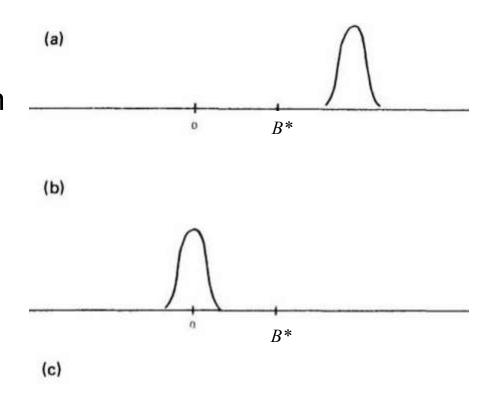
- Which variable is more statistically significant?
- X₁
- Which variable is more important?
- X₂
- Importance is often more relevant

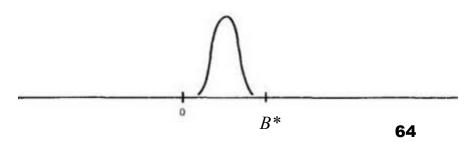
	(1)	(2)
Intercept	0.002	0.003
	(0.005)	(0.008)
X_1	0.500*	0.055**
	(0.244)	(0.001)
X_2	0.600	0.600
	(0.305)	(0.305)
N	1000	1000
R ²	.32	.20
*p<.05, **p <.01		



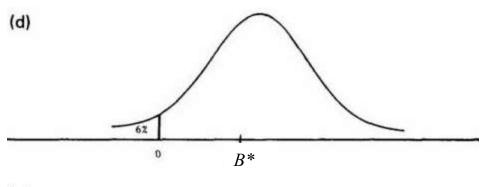
Substantive versus statistical significance

- Think about point estimates, such as means or regression coefficients, as the center of distributions
- Let B* be of value of a regression coefficient that is large enough for substantive significance
- Which is significant?
- (a)

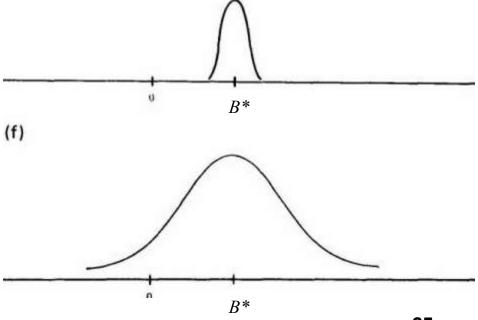




- M
 - Which is more substantively significant?
 - Answer: depends, but probably (d)

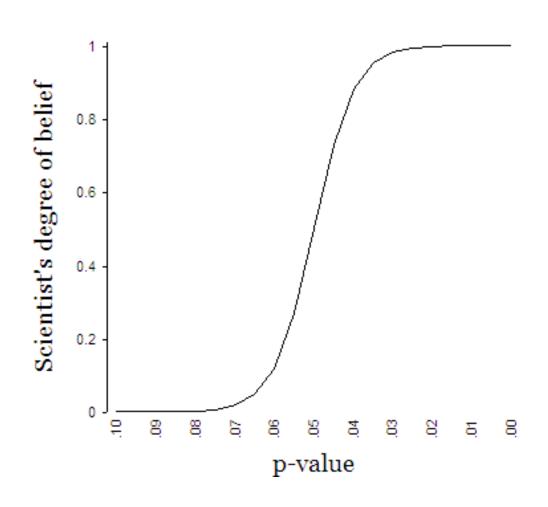


(e)





Don't make this mistake





What to report

- Standard error
- t-value
- p-value
- Stars
- Combinations?

TABLE 1. Explaining Democratic Lower House Seat Shares, 1946-90: Is the Nation Homogeneous?

	Main E	ffects	Southern Int	teractions	Border Interactions	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
South	242.120	2.14	==		_	
Border	262.120	1.32			_	
Year	.032	.91	151	-2.28*	142	-1.31
Democrats (t - 1)	.508	20.10**	.435	7.70**	067	52
Compensation	.007	2.08*	005	-1.03	007	59
Presidential year	-27.079	-11.10**	22.778	6.19**	11.520	1.35
Presidential vote	.498	9.40**	452	5.85**	234	-1.39
Gubernatorial year	18.540	-7.86**	15.111	4.08**	440	04
Gubernatorial vote	.364	8.10**	328	-5.52**	044	20
Off year	-10.042	-10.62**	6.800	3.80**	2.818	.82
GNP growth	.149	2.74**	157	-1.49	148	79

Note: Thirty-one state intercepts not shown. Coefficients are unstandardized. n=1,035, adjusted $R^2=.89$, SEE=7.94

^{*}p < .05; **p < .01



Specification searches

(tricks to get p <.05)

- Reporting one of many dependent variables or dependent variable scales
 - ☐ Healing-with-prayer studies
 - □ Psychology lab studies
- Repeating an experiment until, by chance, the result is significant
 - □ Drug trials
 - □ Called file-drawer problem



Specification searches (tricks to get n < 05)

(tricks to get p <.05)

- Adding and removing control variables until, by chance, the result is significant
 - Exceedingly common



Solutions

- With many dependent variables, use a simple unweighted average
- Bonferroni correction
 - □ If testing n independent hypotheses adjusts the significance level by 1/n times what it would be if only one hypothesis were tested
- Show bivariate results
- Show many specifications
- Model averaging