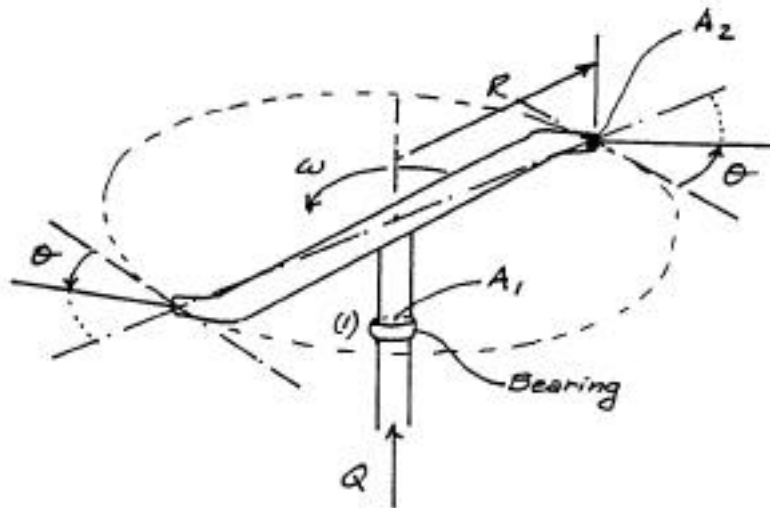


Problem 5.26

Lawn sprinkler



The sketch shows a lawn sprinkler with two horizontal arms of radial length R , at the termination of which are nozzles with exit area A_2 and outward normal vectors in a horizontal plane pointing outward at an angle θ relative to the tangent of a circumferential line, as shown. The sprinkler is free to rotate, but the bearing on which it is mounted exerts a torque $k\omega$ in the direction opposing the rotation, ω being the angular rate of rotation. A constant volume flow rate Q passes through the sprinkler, the water being incompressible at density ρ .

(a) Derive an expression for the steady-state angular velocity ω_∞ of sprinkler in terms of the given quantities R , A_2 , θ , Q , ρ and k . In this steady state, what is the velocity vector of the fluid emerging from the nozzles, as seen by an observer in the non-rotating reference frame? What is the fluid velocity relative to the ground at the nozzle exit planes if the bearing is frictionless ($k=0$)? Comment.

HINT

HINT 2

ANSWER

(b) Now consider the startup of the rotation. Let the rotating arms of the sprinkler have a total mass m per unit length in the radial direction (kg/m), including the solid parts and the water contained therein. Suppose the sprinkler is turned on at $t=0$ in a static state. Derive a differential equation for its angular velocity, making whatever approximations you consider appropriate, and obtain a solution for $\omega(t)$.

HINT

ANSWER