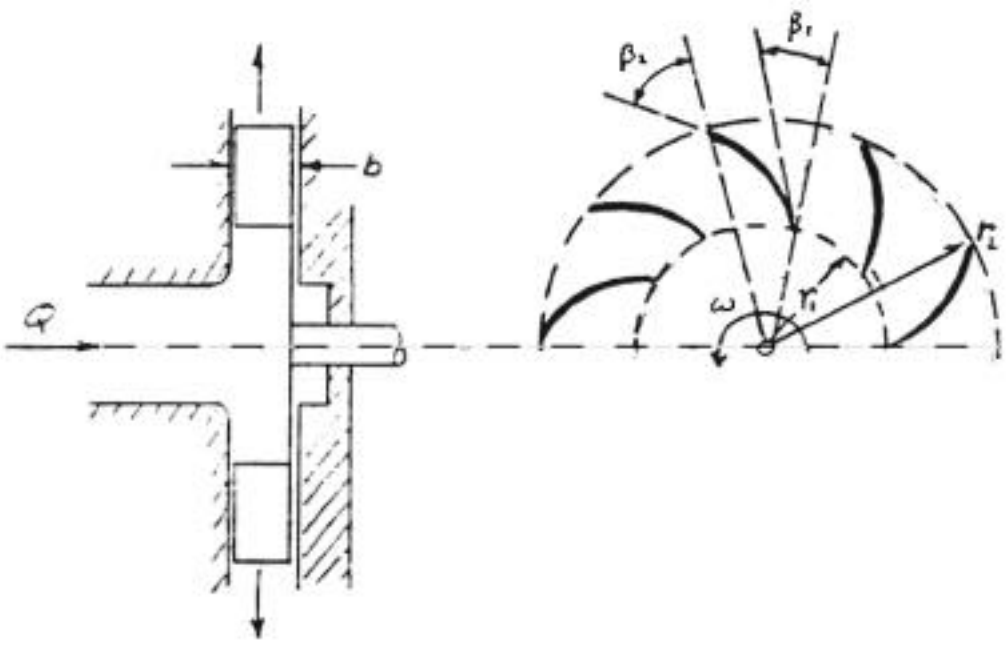
# Problem 5.27

*Centrifugal pump/turbine*




The sketch shows a centrifugal turbine or pump with an impeller in a flat-sided casing of width $b$. A liquid of density $\rho$ flows through the pump at a volume flow rate $Q$ as shown at left. The figure at right shows the blades as seen from the direction of the flow arrow in the left hand figure (you would see this if the casing on the inlet side were removed). The blades are mounted on a thin disc that is attached to the shaft.

Assuming that (i) the liquid enters with a purely axial velocity and (ii) exits at radius $r_2$ with a velocity that follows the (moving) blade surfaces, and (iii) neglecting frictional effects,

(a) derive an expression for the *clockwise* external torque (see the figure at right), $T_S$, exerted by the fluid on the shaft at a clockwise angular speed $\omega$. Express it in terms of $Q$, $\rho$, $b$, $r_1$, $r_2$, $\beta_1$,$\beta_2$ and $\omega$.

(b) What is the system's run-away speed $\omega_0$, that is, the (clockwise) speed of the shaft running unrestrained? What is the torque $T_{S0}$ if the shaft is held at zero speed?

(c) Show that your result in (a) reduces to the universal form

$$T_S/T_{S0} = f(\omega/\omega_0)$$

and plot this function.

(d) Derive the power output to the shaft,

$$\dot{W}_S = T_S\omega \ ,$$

as a function of $\omega$. If $\beta>0$,over what values of $\beta$ will the power output be positive (that is, the system be a pump)?At what value(s) of $\omega/\omega_0$ is the power output maximized? Provide expressions for the maximum power and the angular frequency at which it is attained, expressed in terms of $Q$, $\rho$, $b$, $r_1$, $r_2$, $\beta_1$, and $\beta_2$.