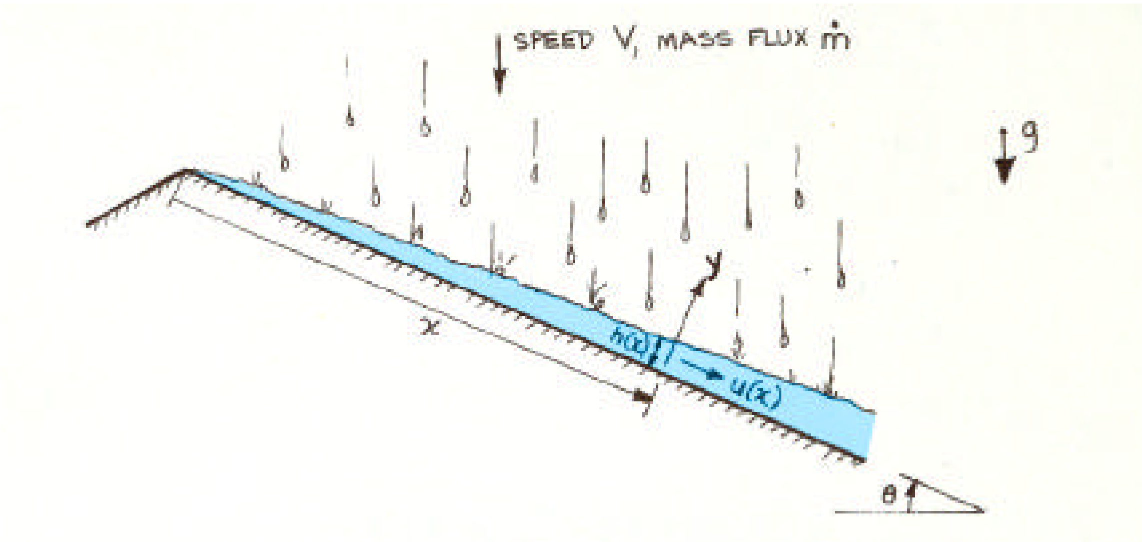
Problem 5.13

Rain on roof



A violent rainstorm hits a roof inclined at an angle θ from the horizontal. The rain pours down at a mass flow rate $\dot{m}$ per unit horizontal area, each drop falling vertically at a speed V. Soon a steady state is established where the water (density ρ) flows down the roof while the raindrops splatter violently into the layer from above and mix into it. We are interested in estimating the water layer thickness distribution h(x) on the roof.

The roof inclination angle θ is not small (think of order 45 degrees). The slope of the water surface relative to the roof is small, however ($dh/dx \ll 1$); the water flow is almost (but not quite) parallel to the roof. The principal driving forces for the water flow are the downstream component of gravitaty force and the x-momentum carried by the impacting raindrops. We shall (for simplicity) assume that the frictional resistance at the roof is negligible.

Given: V, ρ, θ, $\dot{m}$, g.

(a) Neglecting the effect of the water's weight (the layer is thin), what is the gage pressure inside the water layer and on the roof?

HINT ANSWER

(b) Continuing with the assumption that the water's weight is insignificant, derive, using a control volume between x and x+dx (x being the distance from the crest of the roof), a differential equation for the water layer thickness *h(x)*.

ANSWER

(c) Derive a criterion for when the rain is so violent that the shape of the water layer on the roof is independent of gravity. Obtain an explicit solution for $h(x)$ for this case. What is the velocity $u(x)$ of the water stream in this limiting case?

ANSWER