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Exam #2 Solutions

Problem 1 *(Homework problem 6.6; simplified version)*

(a) Before the switch is closed, all currents are zero. Immediately after the switch is closed, I_3 will still be zero, for the inductor prevents instantaneous changes in I_3. That leaves us with $V = R_1 I_1 + R_2 I_2$, and $I_1 = I_2$ because $I_3 = 0$. So $I_1 = I_2 = V/(R_1 + R_2)$.

(b) The ohmic resistance of the self-inductor is zero. So if we wait a long time, until the self-inductor is no longer opposing changes in I_3, we have a wire with zero resistance in parallel with R_2. Thus I_2 will be zero, and $I_1 = I_3$. Applying Ohm's law, we find $I_1 = V/R_1$ $(= I_3)$.

Problem 2

One way: The Lorentz force on the left segment is IbB to the left (with uniform force per unit length), the force on the right segment is IbB to the right, the force on the top segment is IaB upward, and the force on the bottom segment is IaB downward. This combination of forces produces no torque on the loop.

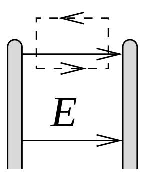
Another way: The loop's magnetic moment $\boldsymbol{\mu}$ has direction perpendicular to the plane surface bounded by the loop. $\mathbf{B}$ is also perpendicular to that surface. So the torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = 0$.

Problem 3 *(Homework problem 4.6.)*

The kinetic energy acquired during acceleration is $Mv^2/2 = (2e)V$, where v is the ion's final speed. In the B-field region, (Lorentz force) = (centripetal force) gives $(2e)vB = Mv^2/d$. Eliminating v and solving for M gives $M = eB^2d^2/V$.

Problem 4 *(Homework problem 5.7.)*

Assuming an abrupt drop to zero, integrating around the dashed path shown gives $\oint \mathbf{E} \cdot d\boldsymbol{l} \neq 0$, for there is an E-field between the plates. Since this is a static situation, we have $d\Phi_B/dt = 0$ for the rate of change of magnetic flux through the open surface bounded by our loop, and thus Faraday's law asserts $\oint \mathbf{E} \cdot d\boldsymbol{l} = 0$. Therefore the E-field *cannot* drop abruptly to zero outside the capacitor.



Problem 5

Tiny elements $d\boldsymbol{l}$ along the AC wire segment are parallel to the unit vector $\hat{\mathbf{r}}$ directed from the element to point P, giving $I\, d\boldsymbol{l} \times \hat{\mathbf{r}} = 0$. So, by the Biot-Savart law, the AC wire segment makes no contribution to the magnetic field at P.

For tiny elements $d\boldsymbol{l}$ along the CD wire segment, $I\,d\boldsymbol{l} \times \hat{\mathbf{r}}$ is purely in the "out-of-the-paper" direction, so the contribution of the CD wire segment to the B-field at P is directed purely out of the paper. By the principle of linear superposition, this contribution will be exactly one-half the field at P due to a *complete* infinite straight wire which extends beyond C. The familiar application of Ampère's law for an "infinitely" long wire gives $B = \mu_0 I/2\pi d$, so our answer is half that value

$$\mathbf{B}_{\mathrm{P}} = \frac{\mu_0 I}{4\pi d} \text{ out of the paper.}$$

Problem 6 *(See the lecture supplement of March 15.)*

A voltmeter will read a negative value if the (small) current that flows through it enters through the negative terminal, so the current in our circuit must be counterclockwise.

(a) Both voltmeters have equal internal resistances and they are wired in series, so both will have the same 0.1 Volt drop across themselves. The induced EMF driving the current must therefore be

$$\mathcal{E} = 0.2\,\text{Volts} \quad \text{(counterclockwise)}.$$

(b) The counterclockwise current enters the *positive* terminal of the left-hand voltmeter, so it will read

$$V_{\text{left}} = +0.1\,\text{Volts}.$$

Problem 7

If θ is the angle between the resistor wire and the moving bar, the area of the plane surface bounded by the closed circuit is

$$(\pi D^2)\frac{\theta}{2\pi} = \frac{1}{2}D^2\theta \quad .$$

The magnetic flux through this surface is then $\Phi_B = \frac{1}{2}BD^2\theta$, and the magnitude of the induced EMF is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{1}{2}BD^2\frac{d\theta}{dt} = \frac{1}{2}BD^2\omega \quad .$$

The induced current will be

$$I = \frac{|\mathcal{E}|}{R} = \frac{BD^2\omega}{2R} \quad .$$

(By Lenz's law, this current will flow in a counterclockwise sense, so as to counter the changing flux due to the motion of the bar.)

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