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Exam #3 Solutions

Problem 1

The form will be $x(z,t) = x_0 \sin[k(z - vt)]$. x_0 and v are given. k is obtained from $\lambda = v/f = (100)/(400) = \frac{1}{4}\,\mathrm{m}$ and $k = 2\pi/\lambda = 8\pi\,\mathrm{m}^{-1}$. The desired equation is then

$$x(z,t) = (0.005)\sin[8\pi(z - 100t)] \quad ,$$

with x and z in meters and t in seconds.

Problem 2

(a) $\lambda = 2\pi/k = 2\pi/(\frac{\pi}{2}) = 4\,\mathrm{m}$ is the wavelength. The wave speed in the medium is $v = \omega/k = (10^8\pi)/(\frac{\pi}{2}) = 2 \times 10^8\,\mathrm{m/s}$, so the index of refraction is $n = c/v = (3 \times 10^8)/(2 \times 10^8) = 1.5$.

(b) At $y = 0.5\,\mathrm{m}$, $\cos(\frac{\pi}{2}y) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$. The maximum value of $\sin(10^8\pi t)$ is 1. The particular x and z values are irrelevant to the question, as this is a plane wave. The maximum E value of interest is thus $E_{\max} = \frac{3}{\sqrt{2}}\,\mathrm{V/m}$.

Problem 3

(a) At resonance, the reactance $X = (\omega L - \frac{1}{\omega C}) = 0$, so the impedance $Z = R$, and the phase angle ϕ is given by $\tan\phi = X/R = 0 \Rightarrow \phi = 0$. The time-averaged power generated by the power supply is then

$$\overline{P} = \overline{VI} = \frac{V_0^2}{2Z}\cos\phi = \frac{V_0^2}{2R}\cos(0) = \frac{(10)^2}{2(5)}(1) = 10\,\mathrm{W} \quad .$$

(b) With $(\frac{1}{\omega C} - \omega L) = 5\,\Omega = R$, the impedance is now $Z = \sqrt{R^2 + R^2} = \sqrt{2}R$. We have $X = (\omega L - \frac{1}{\omega C}) = -5\Omega$, so $\tan\phi = X/R = -1 \Rightarrow \phi = -45°$. The time-averaged power generated is now

$$\overline{P} = \overline{VI} = \frac{V_0^2}{2Z}\cos\phi = \frac{V_0^2}{2\sqrt{2}R}\cos(-45°) = \frac{(10)^2}{2\sqrt{2}(5)}\left(\frac{1}{\sqrt{2}}\right) = 5\,\mathrm{W} \quad .$$

Notice that $\cos(-45°) = \cos(45°)$.

(**Note:** This problem can also be solved by calculating the average power dissipated in the resistor (I^2R), for energy conservation dictates that this be equal to the time-averaged power delivered by the power supply.)

Problem 4 *(See homework problem 7.5.)*

The magnetic field lines will form closed loops, so the magnetic field strength will be approximately the same in the gap as in the bar. Taking this field strength to be roughly constant, we have

$$\oint \frac{1}{\kappa_M}\mathbf{B}\cdot d\boldsymbol{l} = \mu_0 I_{\substack{\text{piercing}\\ \text{surface}}} \quad\Longrightarrow\quad B\left(\frac{L}{\kappa_M}+d\right) \approx \mu_0 N I \quad ,$$

where $L = 80\,\text{cm}$, $d = 4\,\text{mm}$, $N = 1000$, and $I = 2\,\text{A}$. Solving for B:

$$B \approx \frac{\mu_0 N I}{L/\kappa_m + d} = \frac{(4\pi\times 10^{-7})(1000)(2)}{(0.8)/(200)+0.004} = \frac{\pi}{10}\,\text{T} \simeq 0.314\,\text{T} \quad .$$

Problem 5

(a) TRUE: The index of refraction is greater in water than in air, so the speed of light v is slower in water than in air. The wavelength is $\lambda = v/f$, so $v\downarrow \;\Rightarrow\; \lambda\downarrow$, as the frequency will not change.

(b) TRUE: The Poynting vector for a standing EM wave may be non-zero at an instant, but it will average to zero over one full period. Standing waves transmit no energy through space (See homework problem 9.4).

(c) TRUE: We have propagation in $-\hat{x}$ direction, with the maxima of E_y and E_z in phase with one another $\Rightarrow$ linear polarization (See homework problem 9.5).

(d) TRUE: Total internal reflection will occur when the angle of incidence $\theta_1 > \theta_c$, where θ_c is the angle of incidence for which the angle of refraction $\theta_2 = 90°$. From Snell's law, we find that $\sin\theta_c = n_2/n_1$. Thus θ_c depends on both n_1 and n_2 (but note that θ_c only exists if $n_1 > n_2$).

END