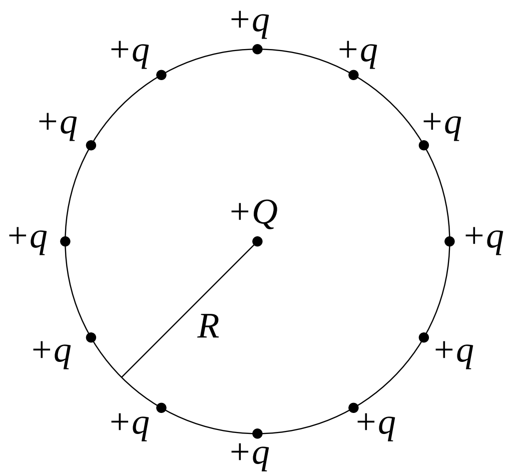
Problem 1 (5 points)

Twelve equal charges $+q$ are situated in a circle with radius R, and they are equally spaced (see the figure).



a. (2) What is the net force (magnitude and direction) on a charge $+Q$ at the center of the circle?

We remove **only** the $+q$ charge which is located at “3-o’clock.”

b. (3) What now is the force (magnitude and direction) on the charge $+Q$ at the center of the circle?

Problem 2 (6 points)

Two very thin concentric hollow conducting cylinders with radii a and $3a$ carry uniformly distributed currents in **opposite directions**. The magnitude of the current through the inner cylinder with radius a is I, the magnitude of the current through the outer cylinder with radius $3a$ is $3I$.

Find the magnetic field at distance r from the axis of symmetry in the three regions

(i) inside the inner cylinder

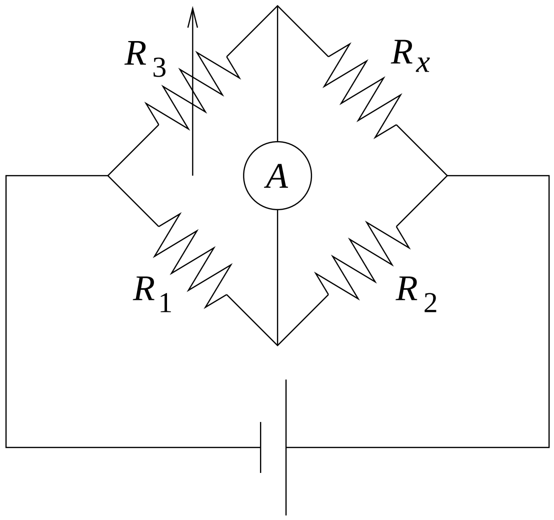
(ii) between the two cylinders, and

(iii) outside the outer cylinder.

Show a sketch of the two cylinders which clearly indicates the direction of the currents **as chosen by you**. Also indicate the direction of the associated magnetic fields as you found for the three regions.

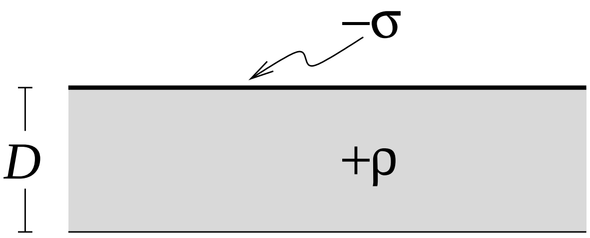
Problem 3 (5 points)

The circuit below (called a Wheatstone Bridge) is designed to measure the unknown resistance of a resistor R_x. The battery provides 10.0 Volts. The resistances of R_1 and R_2 are well known. The variable resistor R_3 is adjusted so that the current through the meter (called A in the figure) becomes zero. In this zero-current situation, we carefully read the dial of the variable resistor R_3. So we now know very accurately the values of the resistance of R_1, R_2 and R_3. What is the resistance of R_x in terms of R_1, R_2 and R_3?



Problem 4 (7 points)

We have an infinite, non-conducting sheet of negligible thickness carrying a uniform surface charge density $-\sigma$ and, next to it, an infinite parallel slab of thickness D with uniform *volume* charge density $+\rho$ (see sketch). All charges are fixed.



Calculate the direction and the magnitude of the electric field.

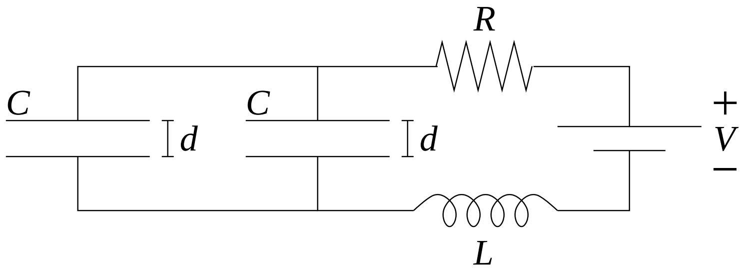
a. (2) a distance h above the negatively charged sheet.

b. (3) inside the slab at a distance d below the negatively charged sheet ($d<D$).

c. (2) a distance H below the bottom of the slab.

Problem 5 (8 points)

A resistor, R, an ideal self-inductor L, and two identical parallel plate capacitors in air, each with capacitance C, have been connected for a long time to a battery of voltage V (see the circuit below). The capacitor plates are separated by a distance d. Express all your answers in terms of C, V, R, L and d (and also κ in the case of questions b and c).



a. (2) What is the electric field (direction and magnitude) between the plates of each capacitor?

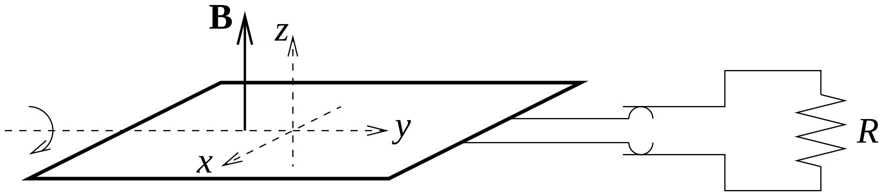
We fill the entire air gap of the capacitor **on the left** with a dielectric ($\kappa = 2$); **we keep the battery connected**, and again we wait a long time.

b. (3) What now is the electric field (direction and magnitude) between the plates of each capacitor?

c. (3) Suppose that we had **disconnected the battery** before inserting the dielectric in the capacitor on the left, what then would the electric field have been between the plates of each capacitor (direction and magnitude)?

Problem 6 (7 points)

A simple electric generator (as shown below) is rotating about the y-axis with a frequency of f Hz. There is a uniform magnetic field B = 0.5 Tesla in the +z-direction. The rotor consists of a coil of n windings each with an area of $S\,\mathrm{m}^2$. The generator, through slipping contacts, is powering a light bulb whose resistance is $R\ \Omega$ (see the figure). The ohmic resistance of the coil is negligibly small compared to that of the light bulb. You may also assume here, for simplicity, that the self-inductance of the coil is negligibly small.



a. (3) What is the maximum value (I_{max}) of the induced alternating current? Also indicate in the figure one of the two positions of the coil when this maximum current occurs.

b. (4) What is the time-averaged mechanical power (in Watts) that must be supplied to maintain the rotation (neglect friction in the bearings)?

Problem 7 (7 points)

A plane electromagnetic wave is given by:

$\vec{E} = 2\hat{z}\ \sin(\frac{2\pi}{\lambda}\mathrm{y} + 4\pi{\times}10^{15}\mathrm{t})$ V/m
$\vec{B} = \vec{B_o}\ \sin(\frac{2\pi}{\lambda}\mathrm{y} + 4\pi{\times}10^{15}\mathrm{t})$ Tesla

The index of refraction of the medium is 1.5.

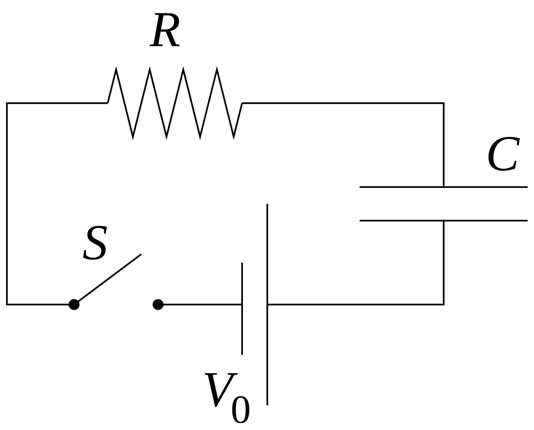
a. (2) In what direction does the wave propagate?

b. (3) What is $\vec{B_o}$ (magnitude and direction)?

c. (2) What is the wavelength of the wave (in meters)?

Problem 8 (8 points)

A resistor, R, a capacitor C, a switch S and a battery (with EMF V_o) are in series as shown. The switch is closed at $t = 0$. At that time, the capacitor holds no charge.



For $t > 0$:

a. (2) Write down the differential equation for the charge Q on the lower plate of the capacitor.

b. (1) Show, by substitution into your equation above, that $Q = CV_o(1 - e^{-\frac{t}{\tau}})$ is the correct solution, provided that τ is chosen properly. Determine τ.

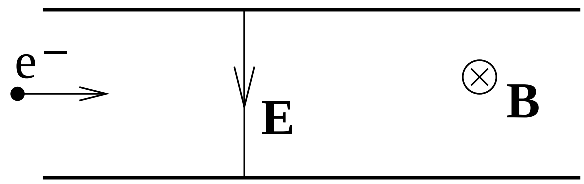
c. (2) What is the current in the circuit at time t_1 ($t_1 > 0$)?

d. (1) How much energy is stored in the capacitor at time t_1?

e. (2) How much heat has been generated in the resistor between $t = 0$ and t_1?

Problem 9 (5 points)

A parallel plate capacitor (in vacuum) is placed in a uniform magnetic field $\vec{B}$ of strength 0.1 T. The direction of the magnetic field is parallel to the plates as indicated in the figure. The capacitor is charged to provide a uniform electric field $\vec{E}$ of 10^5 V/m in the direction shown. An electron enters the capacitor from the left with a speed of 3×10^6 m/sec; its direction of motion is at right angles to both the magnetic field and the electric field as shown.



a. (3) What is the net force in Newtons (magnitude and direction) that the electron experiences as it enters the capacitor?

b. (2) If we could change the strength of the electric and/or magnetic field, would it then be possible for this electron to pass through the capacitor in a straight line (thus undeflected)? Explain your answer.

Problem 10 (7 points)

A beam of light with a cross-sectional area of 1 cm^2 and with energy flux 1 kW/m^2 falls for 30 seconds on a plane surface. This surface is perpendicular to the direction of the light beam, and it absorbs the radiation completely.

a. (2) What is the energy (in Joules) absorbed by the surface?

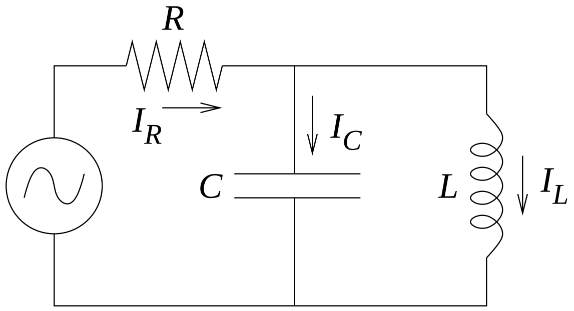
b. (3) What is the force (in N) exerted on the surface by the light during the 30 seconds?

The same beam of light is now 100% reflected by a mirror whose surface is also perpendicular to the light beam.

c. (2) What now is the energy (in Joules) absorbed by the mirror?

Problem 11 (8 points)

The LRC circuit as shown is driven by a power supply whose EMF = $V_o\cos(\omega t)$. In steady state, the current through the ideal self-inductor is I_L, the current through the ideal capacitor is I_C, and the current through the resistor is I_R. **Steady state means that you wait a long time so that all transient phenomena have died out**. **Don't even THINK of writing down a differential equation. This problem is designed to see whether you have an appreciation for how a capacitor and a self inductor behave in extreme situations. No fancy math is needed**. Express all your answers in terms of L, R, C and V_o.



a. (2) What are the maximum values of I_L, I_C, and I_R in case $\omega = 0$ (zero frequency means that the power supply is now a simple battery with zero internal resistance). **We are asking you for steady state solutions, NOT transient solutions.**

b. (3) Answer the same question as under "a", for the other extreme when ω approaches a value which is infinitely high.

c. (2) Do you expect the maximum value of the current I_R to be higher or lower than the value you found under "a" in the case that the frequency is somewhere in between the above two extremes. **Give your reasons**.

d. (1) There is one frequency (**in steady state**) for which I_R is zero. This is not so intuitive, but given the fact that this is so, what do you think that frequency is? **Please, do not try to calculate this frequency**.

Problem 12 (6 points)

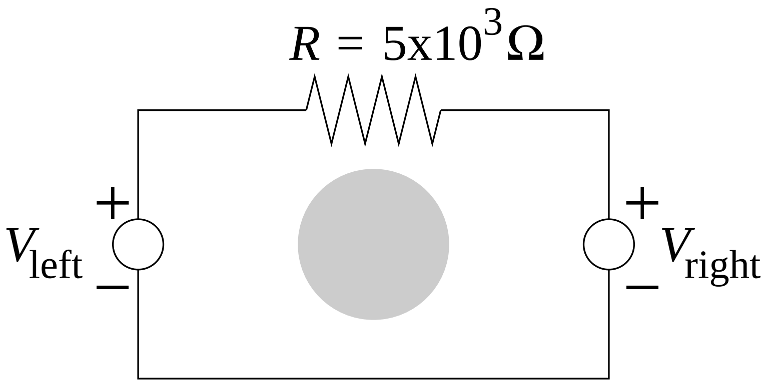
Apply Faraday's law to show that a static electric field between the plates of a parallel-plate capacitor cannot drop abruptly to zero at the edges of the capacitor.

Problem 13 (6 points)

Two voltmeters, V_{right} and V_{left}, each with an internal resistance of $10^4\ \Omega$ are in series with a resistor R of $5\times10^3\ \Omega$. They are connected through wires of negligible resistance (see the circuit below). The "+" side of both voltmeters is up as shown. A changing magnetic field is present in the shaded area. At a particular moment in time V_{left} reads $+0.1$ Volt.

a. (3) At that moment in time, what is the reading of V_{right}?

b. (3) What, at that moment, is the current (in A) in the circuit (magnitude and direction), and what is the induced EMF?



Problem 14 (15 points)

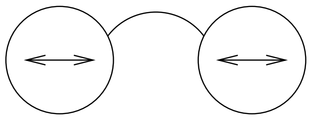
Here follow 15 "True-False" questions. **For each correct answer you get 1 point, for each incorrect answer, you lose 1 point. You have the option of leaving one or more questions unanswered in which case you will neither gain nor lose points. Your total score for this problem will not be less than zero.** Please write only a "T" or an "F" in each box, or leave it blank.

a. The Brewster angle for red light (reflecting off glass) is different from that of blue light. Therefore there is no angle of incidence for which we can produce exactly 100% linearly polarized white light by reflection off glass.

b. Spectra observed from stars that are approaching us show lines that have been Doppler-shifted towards longer wavelengths.

c. The secondary rainbow is always blue on the outside and its light is strongly polarized.

d. Polaroid sunglasses with a direction of polarization as indicated below can greatly suppress the glare from puddles on the road.



e. NASA's tether experiment generated electric power by moving a very long conducting wire (attached to the shuttle) through the Earth's magnetic field. The electric energy that was generated was at the expense of the kinetic energy (thus speed) of the shuttle.

f. Two conducting rings, A and B, have identical dimensions. The self-inductance of ring A will be the same as that of ring B even when the rings are made of very different metals.

g. In comparing two capacitors which each are 1μF but which differ significantly in size, the largest of the two can probably be charged up to a higher potential difference than the smaller one. It will thus be able to hold more energy than the smaller one.

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h. The plastic transmission grating in your 8.02 optics kit can decompose white light into a spectrum of colors. Such a spectrum would not appear if the index of refraction of the plastic were the same for all colors.

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i. The Hubble Space Telescope (HST) is diffraction limited. Its angular resolution is therefore better at short wavelengths (e.g. blue light) than at long wavelengths (e.g. red light).

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j. The wave character of light is well demonstrated with Young's double-slit interference pattern. We can expose a photographic plate to this pattern which will then show the dark and bright regions of destructive and constructive interference. Even if we reduced the intensity of the light source such that **only one photon** would be emitted at a time, for instance only 1 every 10 seconds, the interference pattern would still show up, provided we expose the photographic plate long enough.

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k. The angular size of glories *increases* with *decreasing* size of the water drops in the clouds. This is not surprising as diffraction plays a key role in the formation of the glories.

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l. The speed of sound in air depends on the air temperature. The musical tones produced by a flute of given length are therefore different when the instrument is played outdoors in the winter (28 F) or indoors in a warm room (70 F).

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m. The 22 degree halo which can been seen around the sun as well as the moon is the result of dispersion in ice crystals.

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n. Charge particles in a magnetic field can not experience an acceleration due to the $\vec{v} \times \vec{B}$ force.

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o. When the magnetic flux through an open surface attached to a conducting loop is zero, there cannot be an *induced* EMF in that loop.

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