Massachusetts Institute of Technology - Physics Department

Physics 8.02 February 22, 2002.

Dielectrics & Polarization

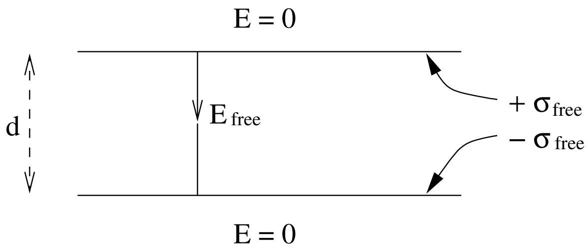
Some notes from Lecture #8 on Friday February 22.

These notes do not stand on their own; they should be "consumed" in combination with my lecture.

We discussed twice before that non-conductors can become *polarized* due to an external electric field. The field can *induce* dipoles in atoms and molecules; the stronger the field, the stronger the degree of polarization. We call these materials *dielectrics*[1]. We exclude all ideal conductors in which charge ***flows*** in response to the field. What I will discuss now is notoriously confusing.

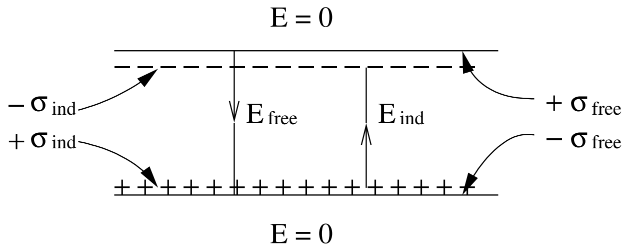
Symbols without arrows represent magnitudes.

Take a plate capacitor (conducting plates). We connect it to a battery (of voltage V) and consequently charge the plates with the same amount of charge but of opposite sign. I will call the charge on ***each*** plate Q_{free}, and the resulting surface charge density σ_{free} (this is Q_{free} divided by the surface area of one plate), and I call the electric field inside the capacitor due to this charge, E_{free}. Notice, Giancoli, on page 625 mentions the words "free charge", but uses the symbols Q and σ (without the subscript "free").



$$E_{free} = \frac{\sigma_{free}}{\varepsilon_0} \tag{1}$$

After the capacitor is charged up, *I disconnect the battery* that supplied the potential difference V between the plates. The charge on the plates is now "trapped". We now shove a dielectric between the plates. It produces two layers of induced charge on the dielectric (remember the "+ -" transparency shown in lectures). I will call this induced surface charge density σ_{ind}; it is the result of polarization of the dielectric under influence of the electric field. The induced charge is often called "bound charge", as opposed to the "free charge".



[1] *The molecules of some dielectrics have an intrinsic dipole moment even in the absence of an external electric field. When they are "exposed" to an E field, they try to line up with the field. The degree of success depends on the strength of the applied field. They are included here.*

The induced charges produce "their own" electric field, $\vec{E}_{ind}$, which ***opposes*** the field $\vec{E}_{free}$.

$$E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0} \tag{2}$$

and the resulting ***net*** field is

$$\overrightarrow{E} = \overrightarrow{E}_{free} + \overrightarrow{E}_{ind} \tag{3}$$

The magnitude of E is thus less than that of E_{free} (the electric field has become weaker).

$$E = E_{free} - E_{ind} \tag{4}$$

Under "normal" circumstances, $E_{ind} \propto E_{free}$, thus $E_{ind} = bE_{free}$, where b is a constant which depends only on the dielectric substance. Thus, $\overrightarrow{E}_{ind} = -b\overrightarrow{E}_{free}$. We substitute this in eq. (3), and find $\overrightarrow{E} = (1-b)\overrightarrow{E}_{free}$. The constant $(1-b)$ is called $1/\kappa$ (κ is the dielectric constant ; it depends only on the material that is inserted between the plates). Thus,

$$\overrightarrow{E} = \frac{\overrightarrow{E}_{free}}{\kappa} \tag{5}$$

This is a key equation.

In our experiment where the free charge was trapped (the battery was disconnected before we inserted the dielectric), the dielectric material ***lowered*** the field strength by a factor κ. Note, the potential difference between the plates, V, must go down by the same factor as d remains unchanged (V ***always*** equals the product of the ***total*** E field (E_{tot}) between the plates and the distance d between the plates). *However, if I keep the battery connected while I stick the dielectric in, the potential difference between the plates remains unchanged; thus E can* ***not*** *go down, therefore Q_{free}, and thus E_{free}, must go* ***up***, consistent with eq. (5); the additional charge that will then flow to the plates will, of course, be delivered by the battery.

As an example, if I use glass, with $\kappa \approx 5$ (see Table 24.1, page 622 of Giancoli), the E field will be reduced by a factor of ≈ 5. For water, $\kappa \approx 80$ (this is enormous), the E field will be reduced by a factor of 80 (for ice at $-40°C$, $\kappa \approx 100$).

In vacuum, per definition, $\kappa = 1.0000000$ exactly. For most gases, κ is only a "hair" larger than 1.000 which means that b is very small, and for most applications in air, unless stated otherwise, we use for κ exactly 1.

It follows from the above relation between κ and b that

$$E_{ind} = \left(1 - \frac{1}{\kappa}\right) E_{free} \tag{6}$$

Since $E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$, and $E_{free} = \frac{\sigma_{free}}{\varepsilon_0}$ (see eqs. 1 and 2), it follows that

$$\sigma_{ind} = b\sigma_{free} = \left(1 - \frac{1}{\kappa}\right)\sigma_{free} \tag{7}$$

Eq. 7 gives us the magnitude of the induced surface charge density, σ_{ind}, if σ_{free}, and κ are known. Keep in mind that the induced charge is negative (*positive*) near the plate where the free charge is positive (*negative*); see the figure above and below. If you feel the need to express the difference in sign, you could add a minus sign to eq. (7), but I don't advise that. In all my equations σ_{ind} and σ_{free} represent the magnitude of the charge; that is why my figures indicate $-\sigma_{ind}$ and $-\sigma_{free}$ where needed.

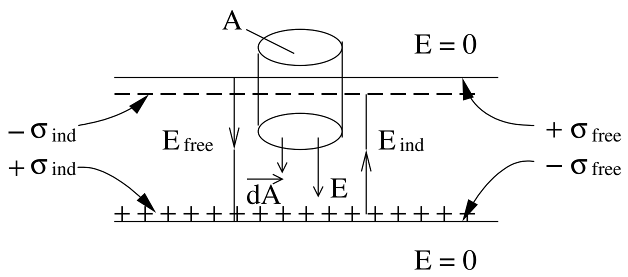
Notice that for very high values of κ ($b \approx 1$), $\sigma_{ind} \approx \sigma_{free}$, and the *net* surface charge density at either plate is then ≈ 0, and there is ***almost no field between the plates!***

Does Gauss's law still hold? Of course! Observe the pill box below; the flat top and bottom (each with area A) are parallel to the plates.

The integral of the electric flux over the entire surface of the pill box (***closed surface integral***):

$$\oint \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \sum Q_{enclosed} \tag{8}$$

and $Q_{encl} = A(+\sigma_{free} - \sigma_{ind})$. The pill box contains the "free" charge on the conducting plate and the "induced" charge on the surface of the dielectric; both have to be taken into account.



The electric field is assumed to be zero outside the capacitor, and uniform in the dielectric. At the curved cylindrical surface, $\vec{E}$ and $\vec{dA}$ are perpendicular to each other, but at the bottom flat surface of the pill box (area A), $\vec{E}$ and $\vec{dA}$ have the same direction. Thus, eq. (8) becomes:

$$EA = \frac{1}{\varepsilon_0}\left(\sigma_{free} - \sigma_{ind}\right)A \tag{9}$$

Using eq. (7), I can eliminate σ_{ind}, and find:

$$E = \frac{\sigma_{free}}{\kappa\varepsilon_0} = \frac{E_{free}}{\kappa} \tag{10}$$

This is exactly what we found above (eq. 5).

It simplifies matters to modify eq. (8) and replace the ***enclosed*** charge (which includes both the induced and the free charge) by the ***free charge*** only:

$$\oint \vec{E} \cdot \vec{dA} = \frac{1}{\kappa \varepsilon_0} \sum Q_{enclosed,free} \tag{11}$$

which is **Gauss's law for dielectrics**.

Notice that eqs. (5), (10) and (11) do not explicitly contain the induced surface charge density σ_{ind} (i.e. the polarization charge); they only contain the surface charge density on the conducting plate (i.e., the "free" charge).

The κ takes the induced charge *automatically* into account.

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