Massachusetts Institute of Technology - Physics Department

Physics - 8.02 **Assignment #1** February 6, 2002.

***Do yourself a favor and prepare for lectures!***
We recommend strongly that you read about the topics before they are covered in lectures. Reading through the new concepts (for half an hour or so) **before the lectures** would be a great help to you! Thorough reading and studying the material, of course, is needed before you start the problems.

| **Lecture Date** | **Topics Covered** | **Reading from Giancoli** |
|---|---|---|
| #1 Wed 2/6 | What holds our world together?<br>Electric charges (historical) - Polarization<br>Electric Force - Coulomb's Law | Chapter 21 through Sect. 21-5 |
| #2 Fri 2/8 | Electric Field - Field Lines - Superposition<br>Inductive Charging - Dipoles - Induced Dipoles | Sect. 21-6 through 21-11 |
| #3 Mon 2/11 | Electric Flux - Gauss's Law - Examples | All of Chapter 22 |
| #4 Wed 2/13 | Electrostatic Potential - Electric Energy - $eV$<br>Conservative Field - Equipotential Surfaces | Chapter 23 through Sect. 23-8 |
| #5 Fri 2/15 | $\vec{E} = -\text{grad } V$<br>More on Equipotential Surfaces - Conductors<br>Electrostatic shielding (Faraday cage) | Sect. 23-5 & 23-7 |

***Due before 4 PM, Friday, February 15 in 4–339B.***

**Problem 1.1**
*Relative strengths of Gravitational and Electrostatic forces.*
The gravitational force between two concentrated ("point-like") masses is very similar in its mathematical structure to the electrostatic force between two concentrated charges. The "strength" of these two forces is, however, vastly different. To illustrate this, consider the following example. Somewhere in outer space are two identical spherical dust grains, 50 $\mu$m in diameter, with mass density 2.5 g/cm$^3$. They are at a distance $d$ meters apart. If the grains were electrically neutral, free of other external forces, and have negligible relative velocity initially, they would eventually collide gravitationally.

Now suppose that both grains are electrically charged, each having $n$ "extra" electrons. Find the minimum value of $n$ that would prevent the gravitational collision. Compare this with the approximate total number of electrons contained in one grain.

*1 $\mu m = 10^{-6} m$; for other necessary information, see the inside of the cover of your book. Recall that the total mass of matter is almost entirely due to the neutrons and protons (roughly of equal mass). Electrically neutral material contains an equal number of protons and electrons. Assume that in the grains the number of protons equals the number of neutrons.*

**Problem 1.2**
*Electric field along the line passing through two point charges.*
A point charge $Q_1 = +3\ \mu$C is placed at the origin, and a point charge $Q_2 = -7\ \mu$C is placed at $x = 0.4$ m on the $x$-axis of a cartesian coordinate system.

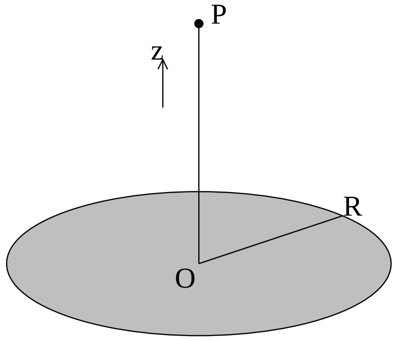
(a) Determine the electric field, $\vec{E}(x) = E(x)\hat{x}$, at all points along the $x$-axis.

(b) Plot $E(x)$ vs. $x$ for $x < 0$.

(c) At what points, if any, (apart from $|x| = \infty$), is $E(x) = 0$?

**Problem 1.3**
*Continuous charge distribution.*
Giancoli 21-49.

**Problem 1.4**
*E-field of a uniformly charged disk.*
A disk of radius $R$ carries a uniform surface charge density $\sigma$ (see sketch). The $z$-axis passes through the center $O$. The total charge, $Q$, on the disk is thus $Q = \pi R^2 \sigma$.




(a) What is the electric field $\vec{E}$ (magnitude and direction) at a point $P$ a distance $z$ above the center of the disk? The procedure to follow is to sum (integrate) the contributions of all infinitesimal charges $dQ = \sigma dA$ of the surface elements $dA$ on the disk. The integration is relatively easy if you introduce rings with radii $r$ (centered on $O$) and width $dr$, and then introduce a new variable $s = \sqrt{r^2 + z^2}$, and observe that $rdr = sds$. Express your answer for $E(z)$ in terms of $Q$, $R$, $\varepsilon_0$ and $z$.

(b) Plot $E(z)$ as a function of $z$ for all positive $z$'s. Use $R$ as your unit on the abscissa; use $Q/(4\pi\varepsilon_0 R^2)$ as your unit for $E(z)$.

(c) Using the binomial expansion (see Appendix A in Giancoli) for $\sqrt{z^2 + R^2}$, find simplified expressions for $E(z)$ in two limiting cases:

  (i) $z^2 \ll R^2$

  (ii) $z^2 \gg R^2$.

  In your calculations retain only the first non-vanishing term of your expansion.

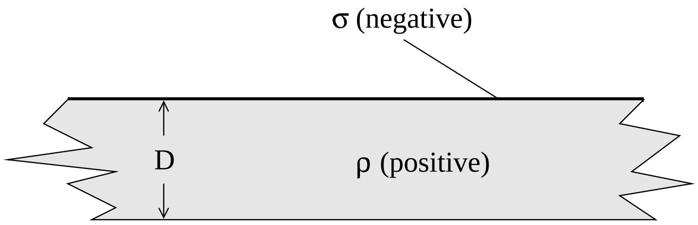
(d) Compare your result in case (ii) with the result you can obtain "effortlessly" by making use of Coulomb's Law for a point-like charge.

(e) Using Gauss's law (choose a proper "pillbox"), calculate $E(z)$ near point $O$ for case (i); compare your answer with (c).

**Gauss's Law** *is a powerful tool in qualitative studies of the electric field for some surmised charge distributions (for example, for surface charge distributions on metals). We will see such an example in the next assignment. Gauss's Law leads also very quickly to quantitative results if the charge distributions exhibit simple symmetry (spheres, cylinders & large planes) problems 1.4(e), 1.6, 1.7 & 1.8 are classic examples of this.*

**Problem 1.5**
*Electric Dipole*
Giancoli 21-65.

**Problem 1.6**
*Gauss's law and the Superposition Principle.*
We have an infinite, non-conducting, sheet of negligible thickness carrying a negative uniform surface charge density $\sigma$ and, next to it, an infinite parallel slab of thickness $D$ with positive uniform *volume* charge density $\rho$ (see sketch). All charges are fixed. Calculate the direction and the magnitude of the electric field.




(a) Above the negatively charged sheet.

(b) Below the slab.

(c) In the slab.

(d) Make a plot of $\vec{E}$ as a function of distance, $z$, from the sheet.

**Problem 1.7**
*Two spherical charged shells.*
Giancoli 22-21.

**Problem 1.8**
*Two concentric charged cylinders.*
Giancoli 22-29.

## Recitations.

There are 28 recitation sections (see the 8.02 Website). If *for any reason* you want to change section, please see Maria Springer in 4-352.