Adam S. Bolton
bolton@mit.edu

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Assignment #10 Solutions

Problem 10.1

Linear polarizers – Home experiment.

(a) Suppose you are viewing the glare reflected from a glass surface. Glass has an index of refraction of about 1.5 and air has an index of refraction of about 1. The "Brewster angle"—the angle of incidence for which reflected light is totally polarized—is then given by (Giancoli Eq. (36-22), p. 910)

$$\tan\theta_{\mathrm{p}} \simeq \frac{1.5}{1} = 1.5 \quad \Longrightarrow \quad \theta_{\mathrm{p}} \simeq \arctan(1.5) \simeq 56^\circ \quad .$$

For this reason, glare observed at $\sim 55^\circ$ or so will be strongly linearly polarized. The direction of polarization will be parallel to the reflecting surface and (of course) perpendicular to the direction of propagation).

(c) The purpose of polarized sunglasses is to cut down on reflected glare (from the air-water interface at the surface of a lake, for instance). This glare will be polarized to some degree in the plane parallel to the reflecting surface (generally the horizontal in the case of water). Therefore a polarized lens will most effectively block glare if *its own* polarization is *perpendicular* to the plane of the reflecting surface. For horizontal reflecting surfaces, this amounts to vertical lens polarization: configuration "b". (Of course, this design will not function properly if you fish while lying on your side. . .)

(d) For an ideal (HN-50) polarizer, unpolarized light will be reduced in intensity by 50% at the first polarizer. At subsequent polarizers, the intensity will drop further by a factor of $\cos^2\theta$, where θ is the angle between the $\mathbf{E}$ of the (now linearly polarized) light and the polarization of the polarizer (see Giancoli section 36-11, pp. 907–909). This angle takes on the values 0°, 45°, and 90° in our particular polarizer configurations. The various "regional intensities" of interest, normalized to the unpolarized incident intensity I_0, are given in the table below (following part (e)).

(e) First of all, let's examine the meaning of the "50" and "38" in HN-50 and HN-38.

The intensity of any light wave propagating in the z-direction can be regarded as the sum of an x-polarized intensity and a y-polarized intensity. This additive decomposition is possible because total intensity is proportional to the *square* of the electric field vector:

$$I = \alpha|\mathbf{E}|^2 = \alpha E_x^2 + \alpha E_y^2 = I_x + I_y$$

for some proportionality constant α. (Review Giancoli section 32-7 (pp. 800–802) for the details of the electric field/light intensity connection.)

Unpolarized light propagating in the z-direction may be regarded on average as being 50% x-polarized and 50% y-polarized. An HN-50 polarizer with its polarization in the y-direction transmits *all* of the incident y-polarized intensity and *none* of the incident x-polarized intensity. 50% of the total incident intensity gets through, hence the "50" in HN-50. A similarly-oriented HN-38 polarizer will transmit 76% of the incident y-polarized intensity (why 76%. . ?) and none of the x-polarized intensity. Thus (76% of 50% + 0% of 50%) = 38% of the total incident intensity is transmitted. *That's* where the "38" in HN-38 comes from.

For the purposes of this problem, we note that an HN-38 polarizer will transmit 76% of whatever intensity *would be* transmitted by a similarly-placed HN-50 polarizer. So to recalculate our regional intensities for the case of HN-38 polarizers, we simply multiply our HN-50 results by $(0.76)^N$, where N is the number of polarizers through which the light passes.

Region	HN-50 Intensity	N	$(0.76)^N$	HN-38 Intensity
A	0.5	1	0.76	0.38
B	0.5	1	0.76	0.38
C	0.5	1	0.76	0.38
D	0.5	1	0.76	0.38
E	$(0.5)(\cos^2 90^\circ) = 0$	2	0.58	0
F	0.5	1	0.76	0.38
G	$(0.5)(\cos^2 90^\circ) = 0$	2	0.58	0
H	$(0.5)(\cos^2 45^\circ) = 0.25$	2	0.58	0.144
K	$(0.5)(\cos^2 45^\circ)(\cos^2 45^\circ) = 0.125$	3	0.44	0.055
L	$(0.5)(\cos^2 45^\circ) = 0.25$	2	0.58	0.144
M	$(0.5)(\cos^2 45^\circ) = 0.25$	2	0.58	0.144
N	0.5	1	0.76	0.38
O	0.5	1	0.76	0.38
P	$(0.5)(\cos^2 0^\circ) = 0.5$	2	0.58	0.29
Q	0.5	1	0.76	0.38

Problem 10.2

Brewster Angle I. (Giancoli 36-49.)

Taking $n = 1.00$ for air and $n = 1.33$ for water, Brewster's angle for reflections off of the air-water interface for light coming from beneath the surface is given by

$$\tan\theta_{\rm p} = \frac{1.00}{1.33} = 0.75 \quad \Longrightarrow \quad \theta_{\rm p} = 40^\circ \quad .$$

For comparison, the critical angle for total internal reflection is given by

$$\sin\theta_{\rm C} = \frac{1.00}{1.33} = 0.75 \quad \Longrightarrow \quad \theta_{\rm C} = 49^\circ \quad ,$$

and Brewster's angle for above-surface reflections by

$$\tan\theta_{\rm p} = \frac{1.33}{1.00} \quad \Longrightarrow \quad \theta_{\rm p} = 53^\circ \quad .$$

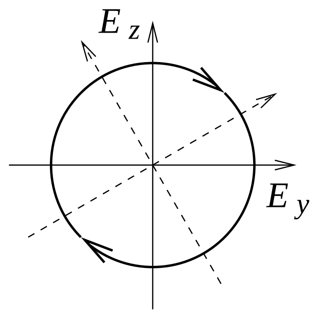
Problem 10.3

Brewster Angle II. (Giancoli 36-66.)

Sunlight reflected off the surface of a still lake will be 100% polarized when the Sun makes an angle equal to the Brewster angle $\theta_{\rm p} = 53^\circ$ (from the previous problem) with the *vertical*. So the angle of the Sun above the horizon should be $90^\circ - 53^\circ = 37^\circ$.

Problem 10.4

Circularly polarized light.



Recall the circularly polarized electromagnetic wave E-field from problem 9.5, evaluated at $x = 0$:

$$E_y = -E_0\cos(\omega t) \ , \quad E_z = E_0\sin(\omega t) \quad .$$

The electric field vector has a constant magnitude and describes a circle in the y-z plane, as shown in the diagram. So the time-averaged values of E_y^2 and E_z^2 will be equal. Importantly, this holds true even if we *rotate* our y and z axes to some other orientation in the y-z plane, such as that shown by the dashed axes. So we are free to take our z-axis to be the axis of our HN-50 polarizer. Half the incident circular-polarized intensity (associated with E_y^2) will be blocked and half (associated with E_z^2) will be transmitted. So, $\frac{1}{2}$ of the light gets through. This transmitted light will be linearly polarized in the z-direction.

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