Adam S. Bolton
bolton@mit.edu

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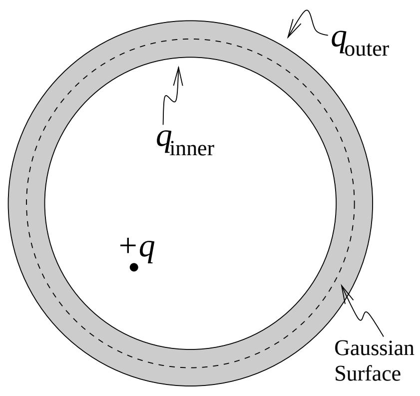
Assignment #2 Solutions

Problem 2.1

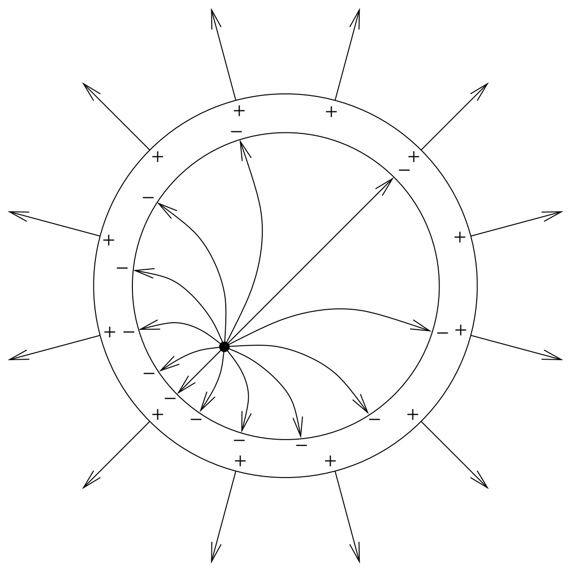
Electric field of a point charge inside a hollow metal sphere: Gauss's law in action.

Before attempting this problem, you should review Section 21-9 (pp. 562-63) and Example 22-7 (p. 584) of Giancoli. From these discussions, we know that: (1) the electric field is zero inside the metal; (2) any charge on the metal must reside on the surface of the metal; (3) the electric field just outside the metal must be normal to the surface, and the local surface charge density satisfies $\sigma = \epsilon_0 E$. Let us apply these general principles to the problem at hand.



(a) the $+q$ charge in the interior of the spherical metal shell will induce a charge separation on the metal. Let q_{inner} be the total charge induced on the inner surface, and q_{outer} the total charge induced on the outer surface. Since the sphere started out uncharged, charge conservation dictates that we must have $q_{\text{inner}} + q_{\text{outer}} = 0$. What is q_{inner}? Take a Gaussian surface lying entirely inside the metal of the shell as shown in the diagram at right: the total charge inside this surface is $(q_{\text{inner}} + q)$. But since $\mathbf{E}$ is zero inside the metal,

$$\oint \mathbf{E} \cdot d\mathbf{A} = (q_{\text{inner}} + q)/\epsilon_0 \equiv 0 \quad .$$

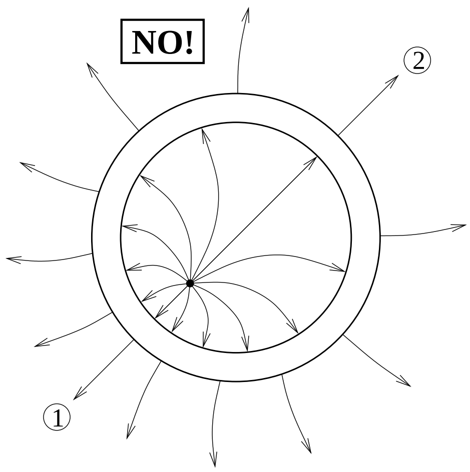


Thus $q_{\text{inner}} = -q$, and therefore $q_{\text{outer}} = +q$. The distribution of q_{inner} and q_{outer}, and the associated electric field lines, are as shown in the sketch at left. The negative charge on the inner surface is concentrated toward that part of the sphere closest to the $+q$ interior charge. In contrast, the positive charge on the outer surface is <u>uniformly</u> distributed over the outer surface. Why is it uniform there? Because the metal is an equipotential, and there is no charge outside (as there is inside), so that we must have spherical symmetry outside.

The only allowable solution for the electric field outside is thus

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r} \tag{1}$$

as we have sketched above. Any other solution for the $\mathbf{E}$ field outside (for example, see sketch at right) is <u>not</u> permissible because the surface of the metal will not be an equipotential for such topologies. For example, the work required to bring a test charge in from ∞ along field line 1 in the sketch would be greater than the work required to bring a test charge in along field line 2, and thus the metal would not be an equipotential for this field. (Notice that the electric field strength near 1 is larger than near 2, as the density of field lines is greater.)



To argue rigorously (as opposed to hand waving) that equation (1) is the only solution for $\mathbf{E}$ outside, we need to appeal to something called the Uniqueness Theorem in electrostatics: if you have one solution that satisfies all of your boundary and other conditions (metal is an equipotential, no free charge outside the sphere), then it is the <u>only</u> solution (i.e. it is unique). The proof of this is beyond the level of this course, so we must hand-wave.

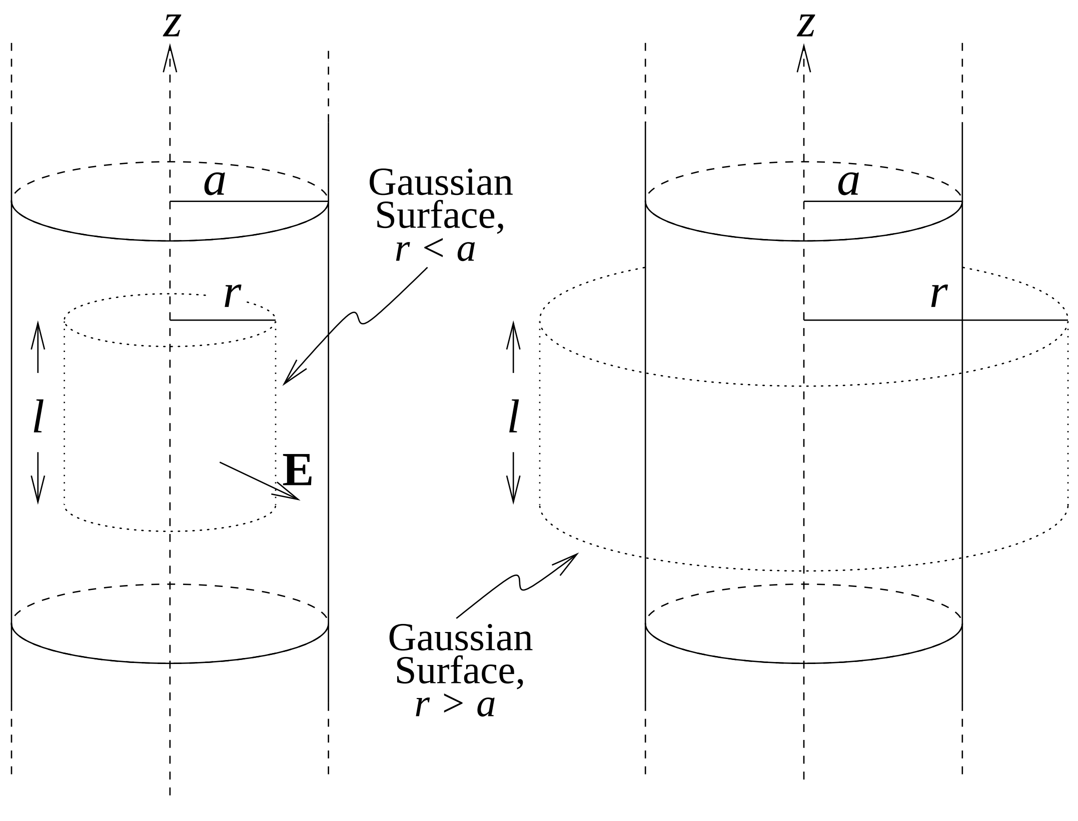
(b) The charge distribution on the outside <u>does</u> <u>not</u> change as we move the $+q$ inside around, for the reasons given above. Of course, the charge induced on the inside surface <u>does</u> redistribute itself as we move $+q$, to insure that we maintain $\mathbf{E} = 0$ everywhere inside the metal.

(c) When the $+q$ touches the inner surface, the induced $-q$ is concentrated entirely at the point of contact and cancels the $+q$. We are left only with the $+q$ on the outer surface and its associated $\mathbf{E}$ as given by equation (1).

Problem 2.2

Electric field and potential of a charged cylinder.

(a) **E** will be in the cylindrical radial direction because **E** mirrors the symmetry of the charge distribution that generates it. To illustrate this, suppose you asserted that **E** had a positive z-component at some point in space. Your recitation instructor would counter that this could not be so, for **E** has no reason to prefer $+\hat{z}$ more than $-\hat{z}$: the (infinitely long) charge distribution contains no feature that distinguishes $+\hat{z}$ from $-\hat{z}$. The same argument would hold if you asserted that **E** had a positive azimuthal component. Therefore, **E** can only be radial, since clearly there is radial structure in the charge distribution.



(b) Based on the arguments presented in (a) and the fact that the magnitude of **E** may only depend upon r, we assume $\mathbf{E} = E(r)\,\hat{r}$ and first consider the case $0 \le r \le a$. Take the Gaussian surface shown at left in the diagram above. The total charge inside this cylinder of height l and radius r is its volume, $\pi r^2 l$, times the charge per unit volume, ρ. For the "left-hand side" of Gauss's law, we have

$$\begin{aligned}\oint \mathbf{E}\cdot d\mathbf{A} &= \int_{\text{ends}} \mathbf{E}\cdot d\mathbf{A} + \int_{\text{sides}} \mathbf{E}\cdot d\mathbf{A} \\ &= \int_{\text{ends}} 0 + \int_{\text{sides}} E(r)\,dA \\ &= E(r)\int_{\text{sides}} dA \\ &= E(r)\,2\pi r l \quad . \end{aligned} \tag{2}$$

Let's spell out what is involved in equation (2). First, we may break the surface integral up into two integrals: one over the ends and one over the sides. Everywhere on the ends, the electric field is by assumption perpendicular to the normal to the Gaussian surface, and thus $\mathbf{E} \cdot d\mathbf{A} = 0$ there. So the original surface integral in fact contains only contributions from the sides of the cylinder. On the sides of the cylinder, $\mathbf{E} = E\hat{r}$, and $\hat{r}$ is the outward normal to the Gaussian surface. So $\mathbf{E} \cdot d\mathbf{A} = E\,dA$ on the sides. Next, the assumption that E depends only on r allows us to move $E(r)$ outside of the integral over the side surface, for r does not vary on the surface over which we are integrating. The remaining integral is just the area of the side surface of the cylinder, or $2\pi rl$. Thus, from Gauss's law, we have

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{A} &= Q_{\text{encl}}/\epsilon_0 \\ \Longrightarrow 2\pi rlE(r) &= = \rho\pi r^2 l/\epsilon_0 \quad , \end{aligned}$$

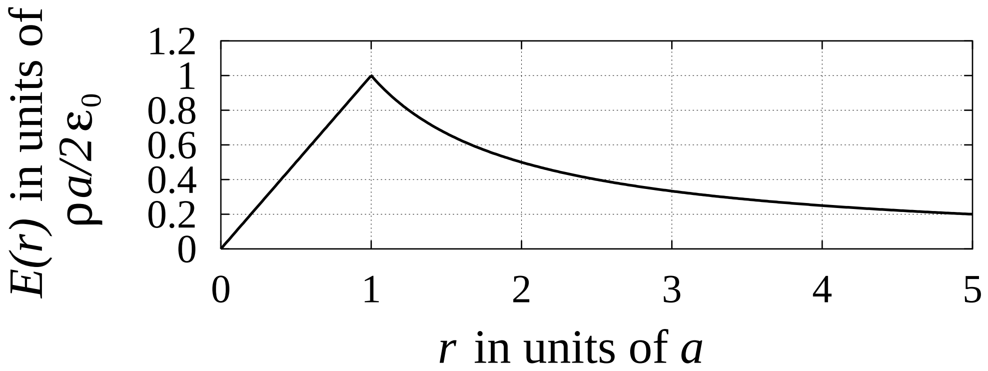
or

$$E(r) = \frac{r\rho}{2\epsilon_0}, \quad 0 \le r \le a \quad . \tag{3}$$

Now, for $r > a$, consider the Gaussian surface on the right. Everything proceeds as above, except now the total charge inside the Gaussian surface is fixed at $\rho\pi a^2 l$, because our continuous charge distribution ends at $r = a$. Thus we now have

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{A} &= Q_{\text{encl}}/\epsilon_0 \\ \Longrightarrow 2\pi rlE(r) &= = \rho\pi a^2 l/\epsilon_0 \\ \Longrightarrow E(r) &= \frac{a^2\rho}{2\epsilon_0 r}, \quad r > a \quad . \end{aligned} \tag{4}$$

Using (3) and (4), we can plot what $E(r)$ looks like:



Note that there is a "kink" in this curve at $r = a$, but no discontinuity; both forms for $E(r)$ give the same value at $r = a$. You will find discontinuities in $\mathbf{E}$ only when you have point charges, line charges, or sheets of charge, not when you have continuous volume charge distributions as in this case. Compare this curve to Figure 22-13 of Giancoli (p. 582) for a sphere of charge. They are very similar, except that Giancoli's solution for the sphere falls off as $1/r^2$ for $r > r_0$, whereas we have E falling off as $1/r$ for (cylindrical) $r > a$. Our less steep fall off for the cylinder occurs because in this case we have an infinite amount of charge (for an infinitely long cylinder), whereas for the sphere there is a finite amount of charge.

(c) The potential difference between 0 and r is

$$\Delta V = -\int_0^r \mathbf{E} \cdot d\mathbf{l}$$

(Note: this "$d\mathbf{l}$" is no relation to the "l" that previously specified the height of our Gaussian cylinder.) Here we have $\mathbf{E} = E(r)\,\hat{r}$ and $d\mathbf{l} = \hat{r}\,dr$, so

$$\Delta V = -\int_0^r E(r')\,\hat{r}\cdot\hat{r}\,dr' = -\int_0^r E(r')\,dr' \quad .$$

For $0 \le r \le a$, this is easy because we only have one functional form for $E(r)$:

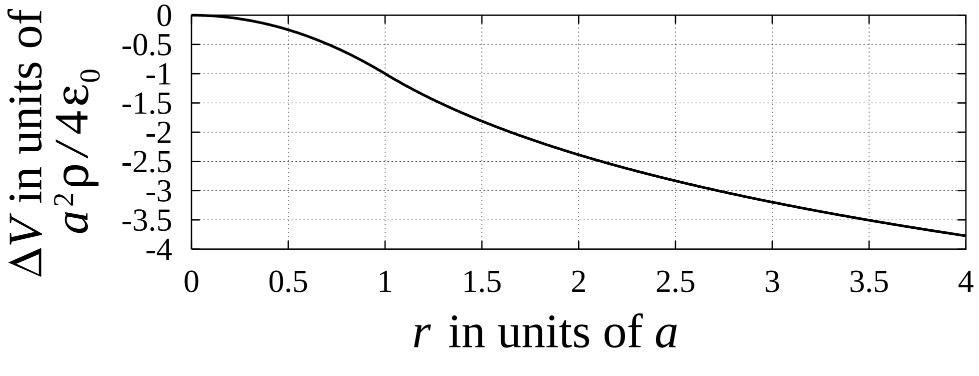
$$\Delta V = -\int_0^r \frac{r'\rho}{2\epsilon_0}\,dr' = -\frac{1}{2}\left.\frac{(r')^2\rho}{2\epsilon_0}\right|_0^r$$

$$\Delta V = -\frac{r^2\rho}{4\epsilon_0} \qquad 0 \le r \le a \quad .$$

For $r > a$, we have to break our integral up into two parts because the expression for $\mathbf{E}$ changes at $r = a$:

$$\begin{aligned}
\Delta V &= -\int_0^a \frac{r'\rho}{2\epsilon_0}\,dr' - \int_a^r \frac{a^2\rho}{2\epsilon_0}\frac{dr'}{r'} \\
&= -\frac{1}{2}\left.\frac{(r')^2\rho}{2\epsilon_0}\right|_0^a - \left.\frac{a^2\rho}{2\epsilon_0}\ln r'\right|_a^r \\
\Delta V &= -\frac{a^2\rho}{4\epsilon_0}\left(1 + 2\ln(r/a)\right) \qquad r > a \quad .
\end{aligned}$$

A plot of ΔV vs. r follows:



Note that $\Delta V \to -\infty$ as $r \to \infty$. Thus one cannot in this case normalize ΔV to be zero at ∞. This is because for an infinitely long cylinder there is an infinite amount of charge, so that it takes an infinite amount of work to move a test charge in from $r = \infty$. Note also that ΔV is everywhere negative. This is because ΔV represents the amount of work it takes you per unit positive charge to move such a charge <u>from</u> 0 <u>to</u> r. Since the $\mathbf{E}$ field is always positive outward, you do negative work in this process: the field is pushing the test charge outward (the direction in which you are moving it), and if you are holding onto it you can get useful energy out of the ride. Thus you are doing "negative" work.

Problem 2.3

Electrostatic potential and potential energy. (Giancoli 23-66.)

(a) The coordinates of the cube's center are $(x, y, z) = (l/2, l/2, l/2)$. Thus the distance d from the center of the cube to the point charge at the origin (or to any other charge, since all charges are equidistant from the cube's center) is given by

$$d = \sqrt{(l/2)^2 + (l/2)^2 + (l/2)^2} = \frac{\sqrt{3}}{2} l \quad .$$

Electrostatic potential obeys the superposition principle, so the total potential at the center of the cube is simply equal to 8 times the potential there due to any single charge. Taking $V = 0$ at ∞ and making use of equation (23-6a) of Giancoli (p. 599), we have:

$$V_{\text{center}} = \frac{1}{4\pi\epsilon_0} \frac{8Q}{d} = \frac{1}{4\pi\epsilon_0} \frac{16Q}{\sqrt{3}l} \quad .$$

(b) At any given corner of the cube, there are 3 charges located a distance $d_1 = l$ away, three charges a distance $d_2 = \sqrt{2}l$ away, and one charge at the opposite corner, $d_3 = \sqrt{3}l$ away. Again invoking the superposition principle and taking $V = 0$ at ∞, we have

$$V_{\text{corner}} = \frac{1}{4\pi\epsilon_0} \left(\frac{3Q}{d_1} + \frac{3Q}{d_2} + \frac{Q}{d_3} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{l} \left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}} \right) \simeq (5.7) \frac{1}{4\pi\epsilon_0} \frac{Q}{l} \quad .$$

(This does <u>not</u> include the contribution from the point charge located <u>at</u> the corner under consideration. If this point charge had zero size, its contribution to the potential right at the corner would be infinite!)

(c) (Also see Giancoli Section 23-8 for a discussion of the electrostatic potential energy of a configuration of charges.)

The electrostatic potential energy of any collection of point charges is equal to the work we would need to do to assemble the collection if all the charges were initially infinitely far apart from one another. Bringing the first charge into place requires no work. Bringing in the second, we do work W_{12}, resulting from the force of 1 on 2 as we move 2 into place. Bringing in the third charge, we do work $W_{13} + W_{23}$, as 3 experiences a force from both 1 and 2. Bringing in the fourth charge costs us $W_{14} + W_{24} + W_{34}$, and so on until the configuration is assembled. From this pattern, we can see that the total electrostatic potential energy of a collection of point charges will be a sum of many terms, *one term for each possible pairing of charges in the system:*

$$U_{\text{total}} = W_{12} + W_{13} + W_{23} + W_{14} + W_{24} + W_{34} + \cdots$$

We know how to express these W's in terms of charges and charge separations. Consider the pairing of charge i with charge j, separated by a distance d_{ij}:

$$W_{ij} = \frac{1}{4\pi\epsilon_0}\frac{Q_iQ_j}{d_{ij}} \quad .$$

Note that although we have imagined constructing the system in a particular charge-by-charge order, the result for the potential energy of the configuration depends only on the final configuration and not on the order in which the charges were brought into place.

For our cube, the quantity QV_{corner} gives the sum of the potential energy terms for 7 pairings in all: all of those pairings that involve one particular charge. The quantity $8QV_{\text{corner}}$ thus adds up the total potential energy of the configuration, but counts all pairs *twice* (convince yourself!) The correct value for the total potential energy of the cubic configuration is then

$$U_{\text{total}} = 4QV_{\text{corner}} \simeq (22.8)\frac{1}{4\pi\epsilon_0}\frac{Q^2}{l} \quad .$$

Problem 2.4

Electric field, potential, and electrostatic potential energy.

From equation (23-6a) of Giancoli (p. 599), we have at any position $\mathbf{r}$ the electrostatic potential as

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{|\mathbf{r}_1 - \mathbf{r}|} + \frac{Q_2}{|\mathbf{r}_2 - \mathbf{r}|} + \frac{Q_3}{|\mathbf{r}_3 - \mathbf{r}|}\right) \quad ,$$

or

$$V(\mathbf{r}) = \frac{10^{-6}}{4\pi\epsilon_0}\left(\frac{5}{|\mathbf{r}_1 - \mathbf{r}|} - \frac{1}{|\mathbf{r}_2 - \mathbf{r}|} + \frac{2}{|\mathbf{r}_3 - \mathbf{r}|}\right) \qquad \text{(SI units).}$$

(Here, $\mathbf{r}$, $\mathbf{r}_1$, etc. are position *vectors*, and thus $|\mathbf{r}_1 - \mathbf{r}|$ and the like are *magnitudes* of *vector differences.*)

<u>At P_1:</u>

$$|\mathbf{r}_1 - \mathbf{r}_{\mathrm{P}_1}| = |\mathbf{r}_3 - \mathbf{r}_{\mathrm{P}_1}| = 1\ \text{m}, \quad |\mathbf{r}_2 - \mathbf{r}_{\mathrm{P}_1}| = \sqrt{2}\ \text{m}$$
$$\Rightarrow V = 5.7 \times 10^4\ \text{Volts.}$$

<u>At P_2:</u>

$$|\mathbf{r}_1 - \mathbf{r}_{\mathrm{P}_2}| = |\mathbf{r}_2 - \mathbf{r}_{\mathrm{P}_2}| = |\mathbf{r}_3 - \mathbf{r}_{\mathrm{P}_2}| = \frac{1}{\sqrt{2}}\ \text{m}$$
$$\Rightarrow V = 7.6 \times 10^4\ \text{Volts.}$$

At P_3:

$$|\mathbf{r}_1 - \mathbf{r}_{\mathrm{P}_3}| = 3 \text{ m}, \quad |\mathbf{r}_2 - \mathbf{r}_{\mathrm{P}_3}| = 2 \text{ m}, \quad |\mathbf{r}_3 - \mathbf{r}_{\mathrm{P}_3}| = \sqrt{5} \text{ m}$$
$$\Rightarrow V = 1.86 \times 10^4 \text{ Volts.}$$

(b) Our expression for V is the sum of three terms, two positive and one negative. One can see that if we get close enough to the negative charge (Q_2), the negative term can be large enough to cancel the two positive terms. Thus there exists some "egg-shaped" surface surrounding Q_2 where the potential is zero (see sketch below).

(c) There are two points where $\mathbf{E}$ is zero, as shown in the sketch below. One is "sort of" between the two positive charges, where the two fields balance out (this is modified by the presence of the negative charge). The other is to the upper right of the negative charge, and is where the repulsion of the two positive charges is just balanced by the attraction of the negative charge (for a positive test charge).

(d) See sketch of field lines below. The ratio of the number of field lines for the three charges is 5:2:1 in 3-D space. The sketch is 2-dimensional; it is only meant to be illustrative.

(e) The electrostatic potential energy of the system is (see Section 23-8 of Giancoli)

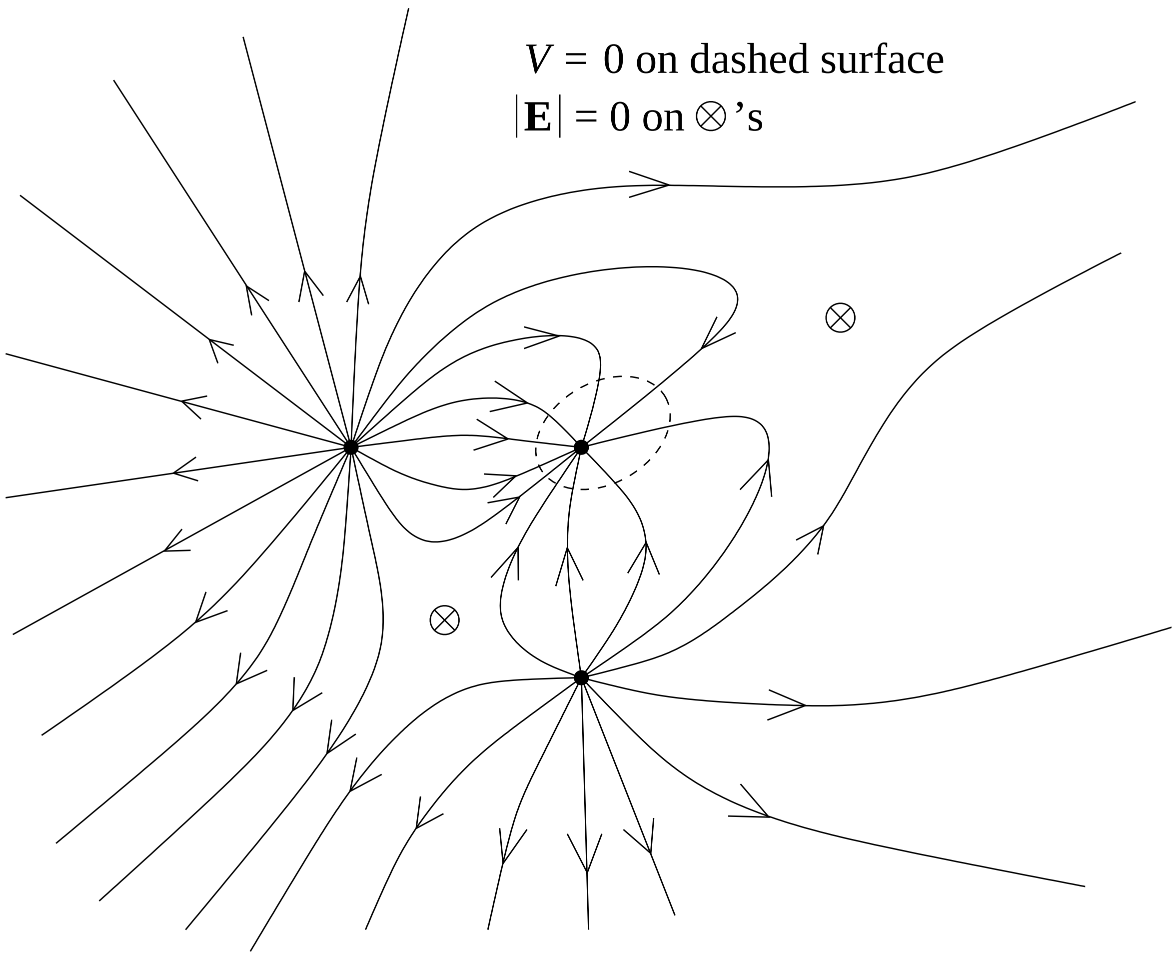
$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1Q_2}{|\mathbf{r}_1 - \mathbf{r}_2|} + \frac{Q_2Q_3}{|\mathbf{r}_2 - \mathbf{r}_3|} + \frac{Q_3Q_1}{|\mathbf{r}_3 - \mathbf{r}_1|}\right) \\ &= \frac{10^{-12}}{4\pi\epsilon_0}\left(-\frac{5}{1} - \frac{2}{1} + \frac{10}{\sqrt{2}}\right) \quad \text{(SI units)} \\ &= +6.4 \times 10^{-4} \text{ joules.} \end{aligned}$$

(f) The energy above is the amount of energy it takes to bring the three charges in from infinity to their locations as shown. If we could release these charges in such a way that they all flew off to ∞, all infinitely far away from each other, then the energy $U > 0$ above would be converted to an equal amount of kinetic energy of the three charges at infinity. However, because one of the charges is negative, the charges will not all fly off to ∞ when released, and the amount of kinetic energy they gain upon release can vary enormously. For example, suppose you release Q_3 first. It will start to move in the direction of the net E-field due to the other two charges. So it will take off in the direction a little South of due East. What happens thereafter is impossible to evaluate based solely upon the given information. It is quite possible that Q_3 will make it all the way out to ∞, but we cannot exclude the possibility that it will end up on Q_2. The trajectory of Q_3 will not only depend on the E-field configuration due to the other two charges, but also on Q_3's mass. Different masses will experience different accelerations and will therefore have different trajectories. If the mass of Q_3 were immensely large, Q_3 would closely follow the field line going through its point of origin. However, if its mass were small, it would immediately leave that field line.

Now release Q_2 (suppose that Q_3 has in fact flown off to ∞). This charge will be attracted to Q_1 and will smash directly into it. If these were truly point charges, the kinetic energy of

Q_2 would become infinite as $|\mathbf{r}_1 - \mathbf{r}_2| \to 0$.

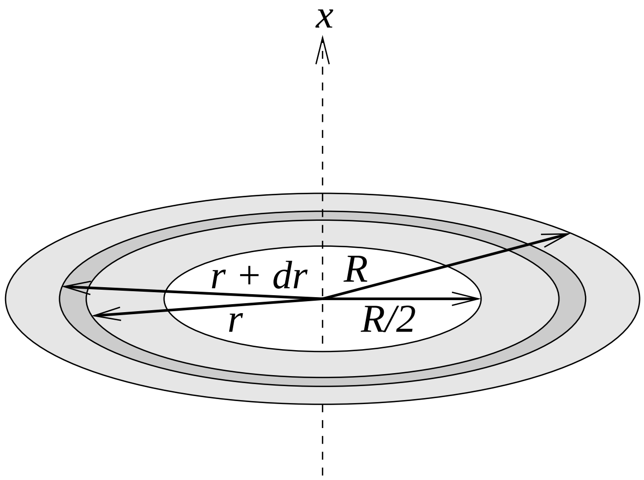
In contrast, if you release Q_2 first, it will become bound to Q_1, performing a complicated orbit about Q_1. If Q_3 is released at the right time in the orbit of Q_2 about Q_1, Q_3 might fly off to ∞, leaving Q_1 and Q_2 to orbit one another. Thus this question has an infinite number of answers—all greater than zero—but is otherwise ill-defined.



Problem 2.5

Electric potential of flat ring with hole in its center. (Giancoli 23-78.)

This problem is very similar to Giancoli Example 23-9 (p. 599). Taking $x = 0$ to be in the plane of the disk, we proceed by dividing up the disk (with hole) into many infinitesimally thin rings of charge and adding up the contribution of all rings to the potential on the symmetry axis (see diagram at right).



The disk carries a charge Q and has surface area $\pi R^2 - \pi(R/2)^2 = 3\pi R^2/4$, so its surface charge density is $\sigma = 4Q/3\pi R^2$. A single infinitesimal ring of radius r and thickness dr (see diagram) has area $2\pi r\, dr$ and thus carries a charge $dq = (4Q/3\pi R^2)(2\pi r\, dr) = 8Qr\, dr/3R^2$. All of the charge of this ring is at a distance $\sqrt{x^2 + r^2}$ from a given point on the x-axis, and thus the contribution of the ring to the potential at that point is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2 + r^2}} = \frac{1}{4\pi\epsilon_0} \frac{8Q}{3R^2} \frac{r\, dr}{\sqrt{x^2 + r^2}} \quad .$$

To find the total potential, we integrate this expression over the entire disk, from $r = R/2$ to $r = R$:

$$\begin{aligned} V = \int dV &= \frac{1}{4\pi\epsilon_0} \frac{8Q}{3R^2} \int_{R/2}^{R} \frac{r\, dr}{\sqrt{x^2 + r^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{8Q}{3R^2} \int_{\sqrt{x^2+R^2/4}}^{\sqrt{x^2+R^2}} du \qquad \text{(after substitution } u = \sqrt{x^2 + r^2}.\text{)} \\ = V(x) &= \frac{2Q}{3\pi\epsilon_0 R^2} \left(\sqrt{x^2 + R^2} - \sqrt{x^2 + R^2/4} \right) \quad . \end{aligned}$$

Problem 2.6

Electric breakdown fields.

(a) Let a be the radius of our sphere. We know from our previous dealings with charged conducting spheres that just outside the surface the electric field is given by

$$E(r=a) = \frac{Q}{4\pi\epsilon_0 a^2}$$

and the potential (relative to ∞) at its surface is

$$V(r=a) = \frac{Q}{4\pi\epsilon_0 a} \quad .$$

From these two equations, we easily deduce the relation between E and V at $r = a$ to be

$$E(r=a) = \frac{V(r=a)}{a} \quad .$$

For a fixed voltage of -4000 V, E on the surface becomes larger and larger as a becomes smaller and smaller. For E to have a magnitude less than 3×10^6 V/m, we need

$$a > \frac{4000}{3 \times 10^6} = 1.33 \times 10^{-3} \text{ m}$$
$$\Rightarrow a > 1.33 \text{ mm} \quad \text{for no breakdown.}$$

(b) From above, $Q = 4\pi\epsilon_0 aV$, or for our numbers, $Q = -6 \times 10^{-10}$ Coulomb.

(c) An electron in the air just outside the sphere will be accelerated radially away from the sphere by the electric field. If it were moving in a constant electric field of magnitude 3×10^6 V/m, it would pick up an energy of 10 eV in a distance l satisfying

$$e(3 \times 10^6)l = (10)e \quad .$$

This equation is simply force (qE) times distance is equal to energy, where we have used the fact that 1 eV (in Joules) is the charge of an electron (in Coulombs) times 1 Volt. Solving the above equation for l gives

$$l = \frac{10}{3 \times 10^6} = 3.3 \times 10^{-6} \text{ m} \quad .$$

Now in fact the E field is not constant outside the sphere: it falls off as $1/r^2$, of course. But in 3.3×10^{-6} m it falls off only a very tiny amount, so that the assumption of a constant field over the distance l is a very good one.

(d) We may conclude from the result of part (c) that an electron can gain 10 eV within three mean free paths or so, if one mean free path is 10^{-6} m.

We can understand "breakdown" as follows. The energy required to ionize an air molecule is ~ 10 eV. If $E \ll 3 \times 10^6$ V/m, a "stray" electron in the air will pick up energy between collisions, but not enough to ionize an air molecule when it collides. If $E > 3 \times 10^6$ V/m, the "stray" electron will pick up enough energy to ionize an air molecule. This gives us two free electrons which will then become 4, and then 8, etc. This cascade, or chain reaction, will eventually reach macroscopic scales as a spark, and the sphere will partially discharge through the air.

Problem 2.7

5.2 keV protons. (Giancoli 23-72.)

(a) Denote the electric field acting upon the particles by $\mathbf{E}(\mathbf{r})$. The force exerted on the proton at any position $\mathbf{r}$ will be $e\mathbf{E}(\mathbf{r})$ (here, $e > 0$ is the magnitude of the electron or proton charge). The energy acquired by the proton during its trip from P to Q is thus

$$(KE)_p = \int_{\mathrm{P}}^{\mathrm{Q}} e\mathbf{E}(\mathbf{r}) \cdot d\mathbf{l} = 5.2 \text{ keV} \quad .$$

The force on the electron at any given $\mathbf{r}$ will be $-e\mathbf{E}(\mathbf{r})$, and hence the energy it would gain in traveling from Q to P is

$$(KE)_e = \int_{\mathrm{Q}}^{\mathrm{P}} [-e\mathbf{E}(\mathbf{r})] \cdot d\mathbf{l} = \int_{\mathrm{P}}^{\mathrm{Q}} [-e\mathbf{E}(\mathbf{r})] \cdot (-d\mathbf{l}) = \int_{\mathrm{P}}^{\mathrm{Q}} e\mathbf{E}(\mathbf{r}) \cdot d\mathbf{l} \quad .$$

So we see that

$$(KE)_e = (KE)_p = 5.2 \text{ keV} \quad .$$

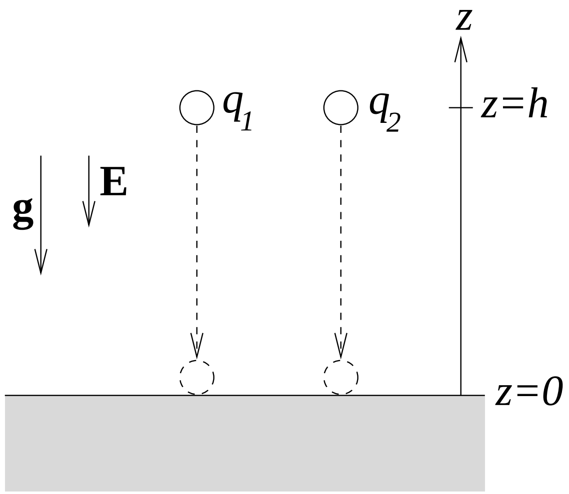
(b) We can find the ratio of particle speeds from

$$1 = \frac{(KE)_e}{(KE)_p} = \frac{m_e v_e^2/2}{m_p v_p^2/2}$$

$$\Rightarrow \frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67 \times 10^{-27}}{9.11 \times 10^{-31}}} \simeq 43 \quad .$$

Problem 2.8

Dropping charged objects in the Earth's electric field. (Giancoli 23-74.)



We'll take the z-direction to be vertically upward, with the ground at $z = 0$ (see diagram). The gravitational potential energy of either ball is then

$$U_{\mathrm{grav}}(z) = mgz$$

(taking $U_{\mathrm{grav}} = 0$ at $z = 0$).

From the given electric field we may construct an electric potential $V(z) = Ez$, where $E = 150$ V/m (we take $V = 0$ at $z = 0$). The electrostatic potential energies of the two balls are then given by

$$U_{\mathrm{elec},1}(z) = q_1 Ez \quad \text{and} \quad U_{\mathrm{elec},2}(z) = q_2 Ez$$

Conservation of energy (mechanical *and* electrostatic) from the initial state at $z = h = 2.00$ m to the final state at $z = 0$ for the two balls gives

$$\begin{aligned} mv_1^2/2 &= mgh + q_1 Eh \\ mv_2^2/2 &= mgh + q_2 Eh \quad , \end{aligned}$$

which solves to

$$\begin{aligned} v_1 &= \sqrt{2gh + 2q_1 Eh/m} \\ v_2 &= \sqrt{2gh + 2q_2 Eh/m} \quad . \end{aligned}$$

For the numbers given in this problem, we have

$$\begin{aligned} v_1 &= \sqrt{39.24 + 0.61} \text{ m/s} \\ v_2 &= \sqrt{39.24 - 0.61} \text{ m/s} \quad , \end{aligned}$$

giving

$$\Delta v = v_1 - v_2 = 0.097 \text{ m/s} = 9.7 \text{ cm/s} \quad .$$

It is appropriate that $v_1 > v_2$, because the electric field will force the positively charged first ball downward, along with gravity, while forcing the negatively charged second ball upward, opposing gravity.

END