Precise Predictions for W + 3 Jets

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BlackHat and Sherpa

BlackHat:

CFB, Zvi Bern, Lance Dixon, Fernando Febres Cordero, Darren Forde, Harald Ita, David Kosower, Daniel Maitre

BlackHat: arXiv:0902.2760, PRD78 (2008) 036003. Badger: JHEP 0901 (2009) 049. Forde: PRD75 (2007) 125019. CFB, Bern, Dixon, Forde, Kosower: PRD74 (2006) 036009.

Sherpa liaison (real emissions):

Tanju Gleisberg

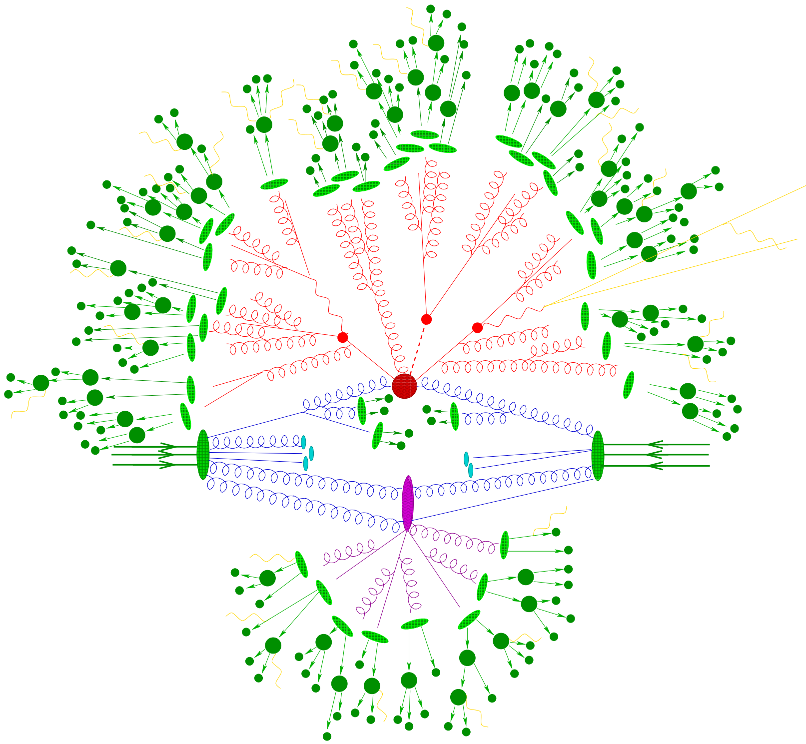
Gleisberg et al, JHEP 0902 (2009) 007. Gleisberg, Krauss, Eur. Phys. J C53 (2008) 501.

Outline

- Introduction
- What is BlackHat?
  - Terms with logarithms (dilogs, ...) from generalized unitarity
  - Rational terms
  - Cross sections - Sherpa
- Physics Results
  - W + 3 jets at the Tevatron and the LHC at NLO
- Current Status and Outlook

Precision Calculations

Precision Calculations



$$N = \mathcal{L} \sum_{i,j} \left(\int f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2) \right)$$

$$\sigma_{ij}(x_1, x_2) = \int d\text{PS} \, |\mathcal{M}_{ij}|^2 \otimes \mathcal{F}_{\text{non-pert}}$$

$$= (c_0 + c_1(\ln^2, \ln, \#)\alpha_s + \ldots) \otimes \mathcal{F}_{\text{non-pert}}$$

Why Not Yet NLO?

General strategy of "conventional" loop calculation:

1. draw all possible Feynman diagrams (topological task)
2. particles in given diagram (combinatorical task)
3. translate diagrams into formulae via Feynman rules (database look-up)
4. contract Lorentz indices, take traces (algebraic manipulation)
5. reduce to known Master integrals (algebraic manipulation)
6. cancel IR and UV singularities (algebraic manipulation)
7. translate output into computer code (programming)
8. run program (wait, drink coffee)

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General strategy of “conventional” loop calculation:

1. draw all possible Feynman diagrams (topological task) [1-loop 6-gluon amplitude: 1,034 graphs, 8-gluon: 3,017,490]
2. particles in given diagram (combinatorical task)
3. translate diagrams into formulae via Feynman rules (database look-up) [Lorentz-structure, e.g. 3-gluon vertex has 6 terms]
4. contract Lorentz indices, take traces (algebraic manipulation) [more proliferation of terms]
5. reduce to known Master integrals (algebraic manipulation) [many terms with spurious singularities that should cancel]
6. cancel IR and UV singularities (algebraic manipulation) [if done numerically - unstable]
7. translate output into computer code (programming) [ouch]
8. run program (wait, drink coffee) [not enough coffee in the universe]

NLO Corrections to LHC Processes

- **Relevant processes all $2 \to n \geq 3$** see e.g. the infamous Les Houches wishlist

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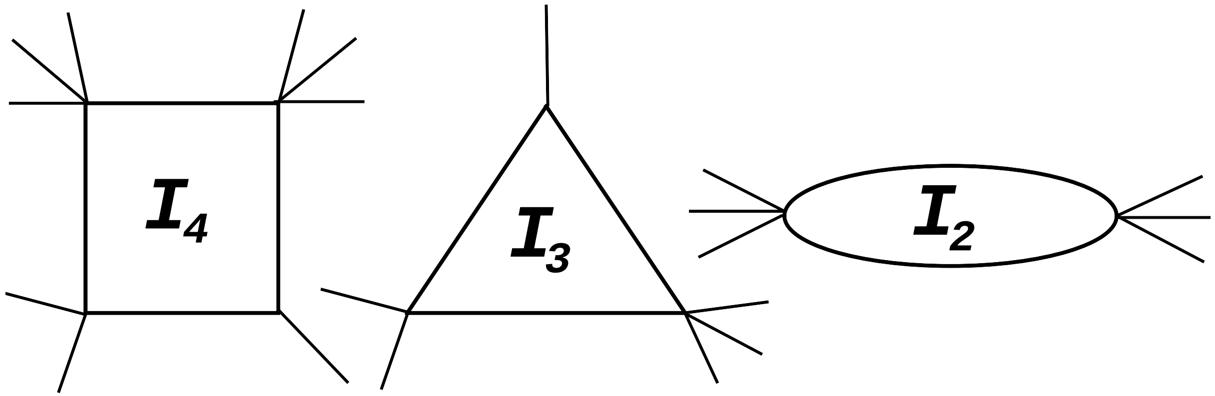
- **To this date there is no complete $2 \to 4$ calculation based on Feynman diagrams**
- **New methods based on (generalized) unitarity and recursion $\Rightarrow$ new codes: BlackHat, Rocket (D-dim unitarity), CutTools (D-dim unitarity at integrand level, not automated)** Rocket: Ellis, Giele, Kunszt, Melnikov, Zanderighi. CutTools: Ossola, Papadopoulos, Pittau

One-Loop Decomposition



Any n-leg (massless) one-loop amplitude expressible in terms of scalar box, triangle and bubble integrals:

$$\mathcal{A} = c_4 I_4 + c_3 I_3 + c_2 I_2 + \text{ rational}$$

With massive partons there are additionally I_1 (tadpoles)

We know the integrals, the task is to determine the coefficients

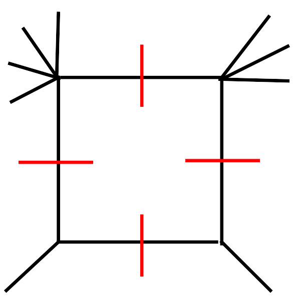
Bern, Dixon, Dunbar, Kosower

Generalized Unitarity

$$c_4 I_4 = c_4 \int d^4 l \frac{1}{l^2 (l-K_1)^2 (l-K_2)^2 (l-K_3)^2}$$

$$\frac{1}{P^2 + i\varepsilon} = \frac{1}{P^2} + i\delta^+(P^2)$$

Box integrals have unique leading singularity $\Rightarrow$ generalized unitarity



$$c_4 \Delta_{LS} I_4 = \int d^4 l \delta^+(l^2) \delta^+((l-K_1)^2) \times \delta^+((l-K_2)^2) \delta^+((l-K_3)^2) \times A_1^{\text{tree}}(l) \times A_2^{\text{tree}}(l) \times A_3^{\text{tree}}(l) \times A_4^{\text{tree}}(l)$$

$$c_4 = A_1^{\text{tree}}(l_{\text{sol}}) \times A_2^{\text{tree}}(l_{\text{sol}}) \times A_3^{\text{tree}}(l_{\text{sol}}) \times A_4^{\text{tree}}(l_{\text{sol}})$$

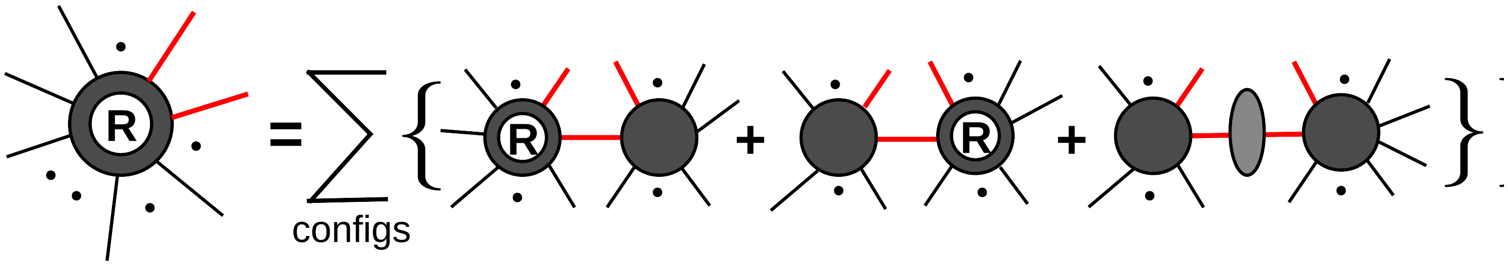
Tree graphs on shell

Trees “recycled” into loops

Britto, Cachazo, Feng

Rational Terms from Recursion

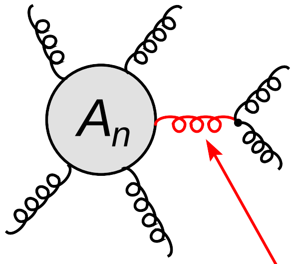
$$\mathcal{A} = \sum_i c_i I_i + \text{ rational}$$



$$R = \sum_{\text{configs}} A_L \frac{1}{P^2_{l \ldots m}} A_R$$

CFB, Bern, Dixon, Forde, Kosower

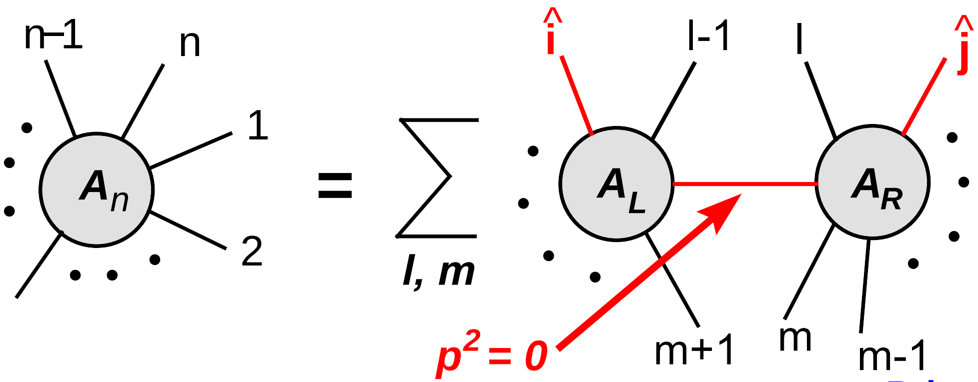
On-Shell Recursion Relations at Tree Level



Complex continue (shift) spinors and momenta:

$$p_i \rightarrow p_i(z) \qquad p_j \rightarrow p_j(z)$$
$$p_i + p_j \rightarrow p_i + p_j$$

Momentum conservation is maintained, momenta on-shell ($p_i(z)^2 = p_j(z)^2 = 0$).



Britto, Cachazo, Feng

Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

$$1/P^2_{l\ldots j\ldots m} \quad \rightarrow \quad 1/P^2_{l\ldots j\ldots m}(z)$$

$$A(z) = \sum_{l,m} \sum_{h} A_L^h(z) \frac{1}{P^2_{l\ldots j\ldots m}(z)} A_R^{-h}(z)$$

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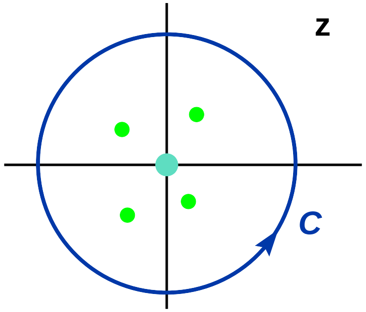
If $A(z \rightarrow \infty) \rightarrow 0$ **- Cauchy's theorem**



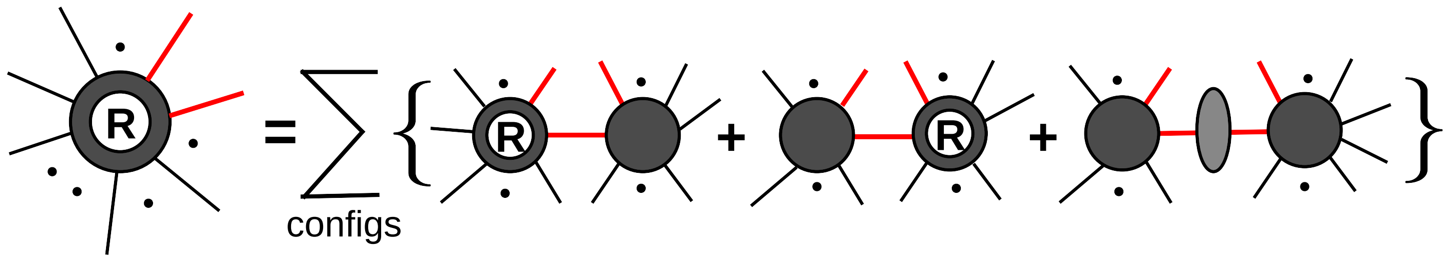
$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}A(z) = 0$$

$$A(0) = -\sum_{\text{poles }\alpha} \underset{z=z_\alpha}{\text{Res}} \frac{A(z)}{z}$$

$$= \sum_{\text{poles }\alpha}\sum_{h} A_L^h(z_\alpha)\frac{1}{P^2_{l\ldots j\ldots m}}A_R^{-h}(z_\alpha)$$

Britto, Cachazo, Feng, Witten

Recursion at Loop Level



Complex continue amplitude

$$A(z) = C(z) + R(z) \qquad \Bigg| \qquad \frac{1}{2\pi i}\oint_C \frac{dz}{z}$$

$$A(0) = C(0) - \sum_{\text{poles } \alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

$$= C(0) + \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$

Loops "recycled" into loops CFB, Bern, Dixon, Forde, Kosower

(ignoring some subtleties)

Rational Terms - D-dim Unitarity

Unitarity in $D = 4 - 2\varepsilon$:
Split up into 4-D piece and (-2ε)-dim. piece (∼ small "mass")

$$l_D^2 = l_4^2 + l_{[-2\varepsilon]}^2 = l_4^2 + \mu^2$$

$$\int \frac{d^D l}{(2\pi)^D} = \int \frac{d^4 l_4}{(2\pi)^4} \int \frac{d^{-\varepsilon}(\mu^2)}{(2\pi)^{-2\varepsilon}}$$

Extract rational part R by keeping track of μ-dependence in generalized unitarity cuts:

$$\mathcal{A} = c_4^{[0]} I_4^D[1] + c_4^{[2]} I_4^D[\mu^2] + c_4^{[4]} I_4^D[\mu^4] + c_3^{[0]} I_3^D[1] + \ldots$$

$$I_n^D[\mu^{2r}] = \frac{1}{2^r} I_n^{D+2r}[1] \prod_{k=0}^{r-1} (D - 4 + k)$$

Rational Terms - D-dim Unitarity contd.

$$\mathcal{A} = c_4^{[0]} I_4^D[1] + c_4^{[2]} I_4^D[\mu^2] + c_4^{[4]} I_4^D[\mu^4] + c_3^{[0]} I_3^D[1] + \ldots$$

$$\begin{aligned} \mathcal{A} &= c_4^{[0]} I_4^D + \frac{D-4}{2} c_4^{[2]} I_4^{D+2} \\ &\quad + \frac{(D-4)(D-2)}{4} c_4^{[4]} I_4^{D+4} + c_3^{[0]} I_3^D + \ldots \\ &= c_4^{[0]} I_4^{4-2\varepsilon} + c_3^{[0]} I_3^{4-2\varepsilon} + c_2^{[0]} I_2^{4-2\varepsilon} + R \end{aligned}$$

$$R = c_4^{[4]}\ I_4^{4-2\varepsilon}[\mu^4]\Big|_{\varepsilon=0} + c_3^{[2]}\ I_3^{4-2\varepsilon}[\mu^4]\Big|_{\varepsilon=0} + \ldots$$

Badger, Forde. See also Ossola, Papadopoulos, Pittau (CutTools); Ellis, Giele, Kunszt, Melnikov, Zanderighi (Rocket).

BlackHat

$$\mathcal{A} = \sum_i c_i I_i + \text{ rational}$$

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- **Rational parts from loop recursion**
- **OR rational parts from D-dim unitarity**
  $\Rightarrow$ 4-D unitarity with small "mass"
- **Basic ingredients: tree amplitudes, low-point 1-loop amplitudes**
- **NO integrals or PV reductions are performed**
  $\Rightarrow$ Numerically very stable, excellent scaling with number of external legs (number of Feynman graphs grows factorially)

Cross Sections

$$\sigma^{\text{NLO}} = \int d^D\sigma^{\text{NLO}} = \int_{m+1} d^D\sigma^R + \int_m \int_{\text{loop}} d^D\sigma^V$$

$$= \int_{m+1} \left(d^D\sigma^R - d^D\sigma^A\right)$$

$$+ \int_m \left[\int_{\text{loop}} d^D\sigma^V + \int_1 d^D\sigma^A \right]_{\varepsilon=0}$$

Virtual corrections: BlackHat

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$$d^D\sigma^A = \sum_{\text{dipoles}} d^4\sigma^B \otimes d^D V_{\text{dipole}}$$

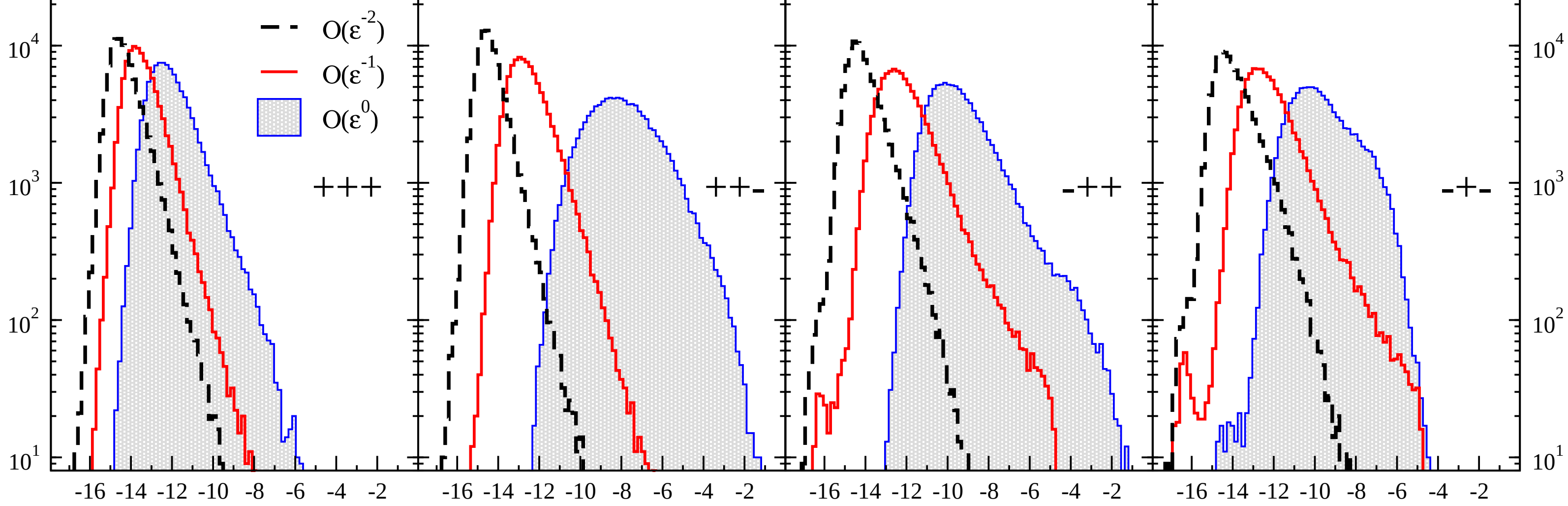
Virtual corrections: BlackHat

Subtraction, integration over phase space: Sherpa

Numerical Results - $V + 3$ Jets

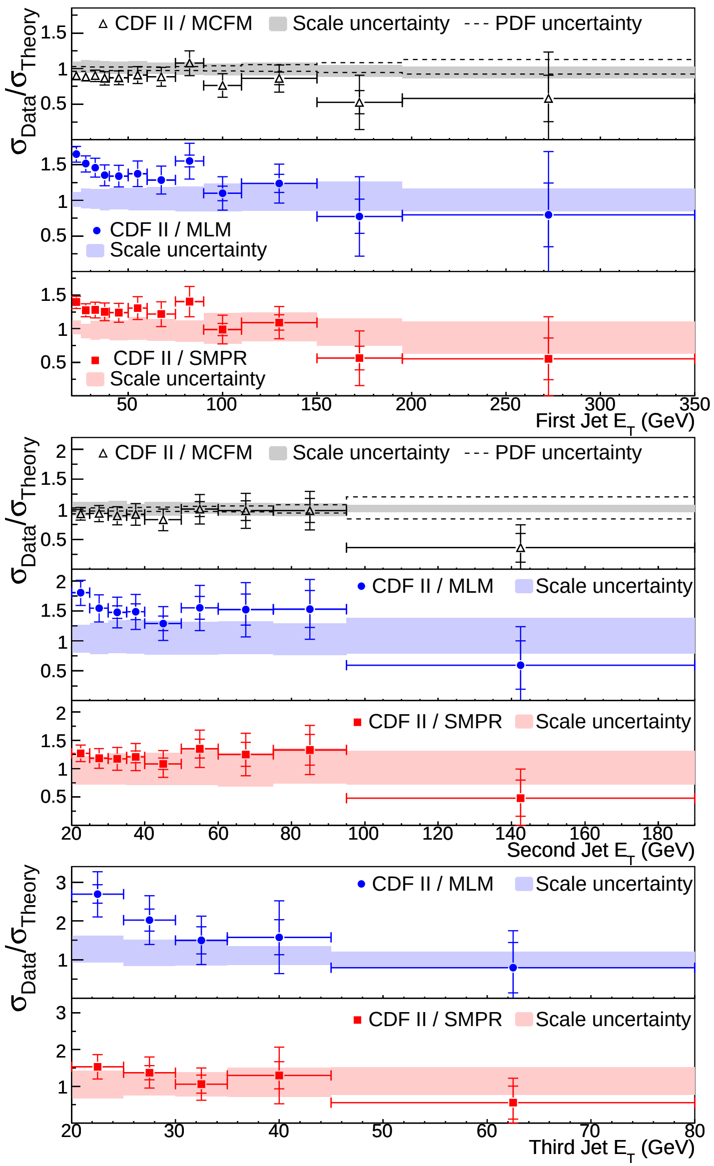
Accuracy (in digits, x-axis) for 100,000 PS points

$$A(\bar{q}, g, g, g, q, \bar{l}, l)$$



BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre, arXiv:0808.0941

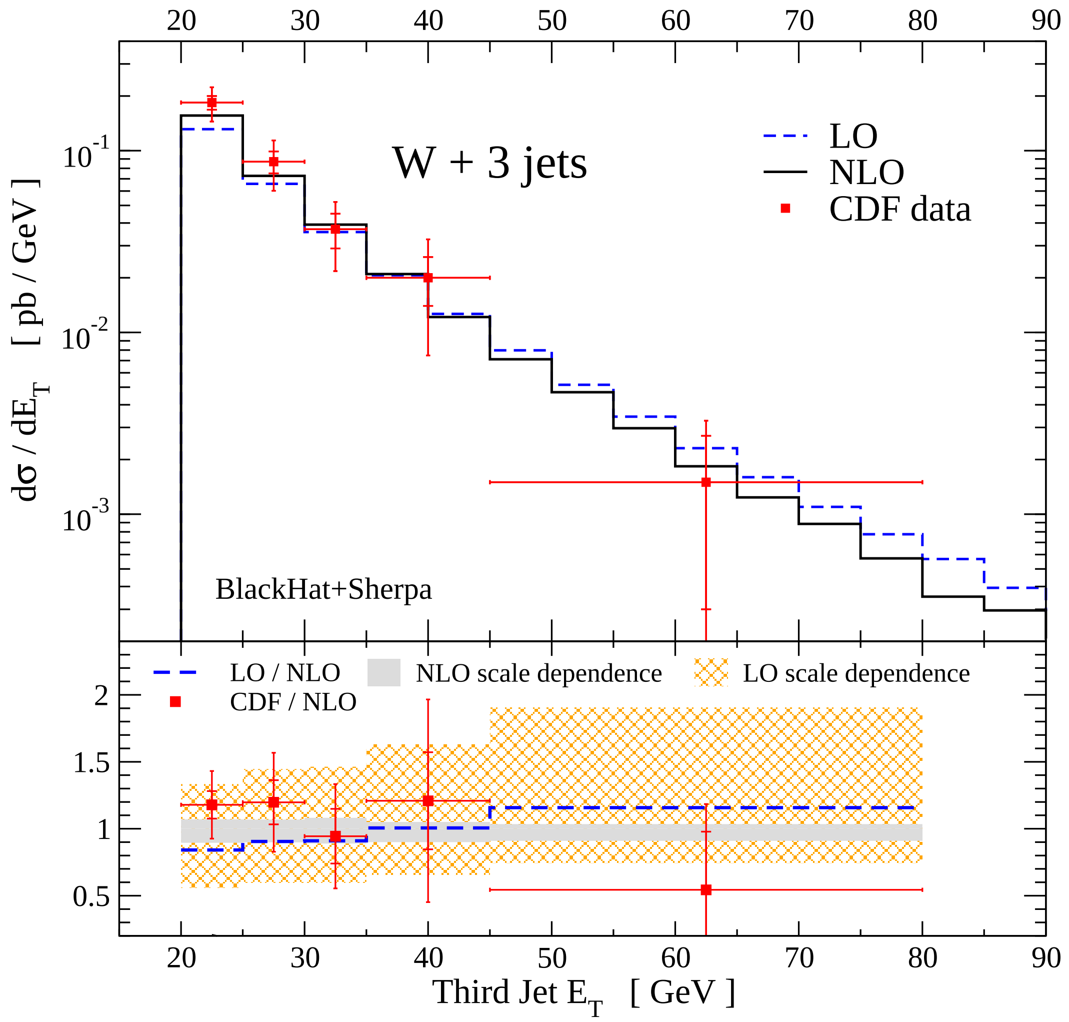
$W + 1, 2, 3$ jets at the Tevatron



No MCFM for W + 3 jets

CDF 2007

$W + 3$ jets at the Tevatron – BlackHat



BlackHat + Gleisberg (Sherpa - for real emissions), arXiv:0902.2760

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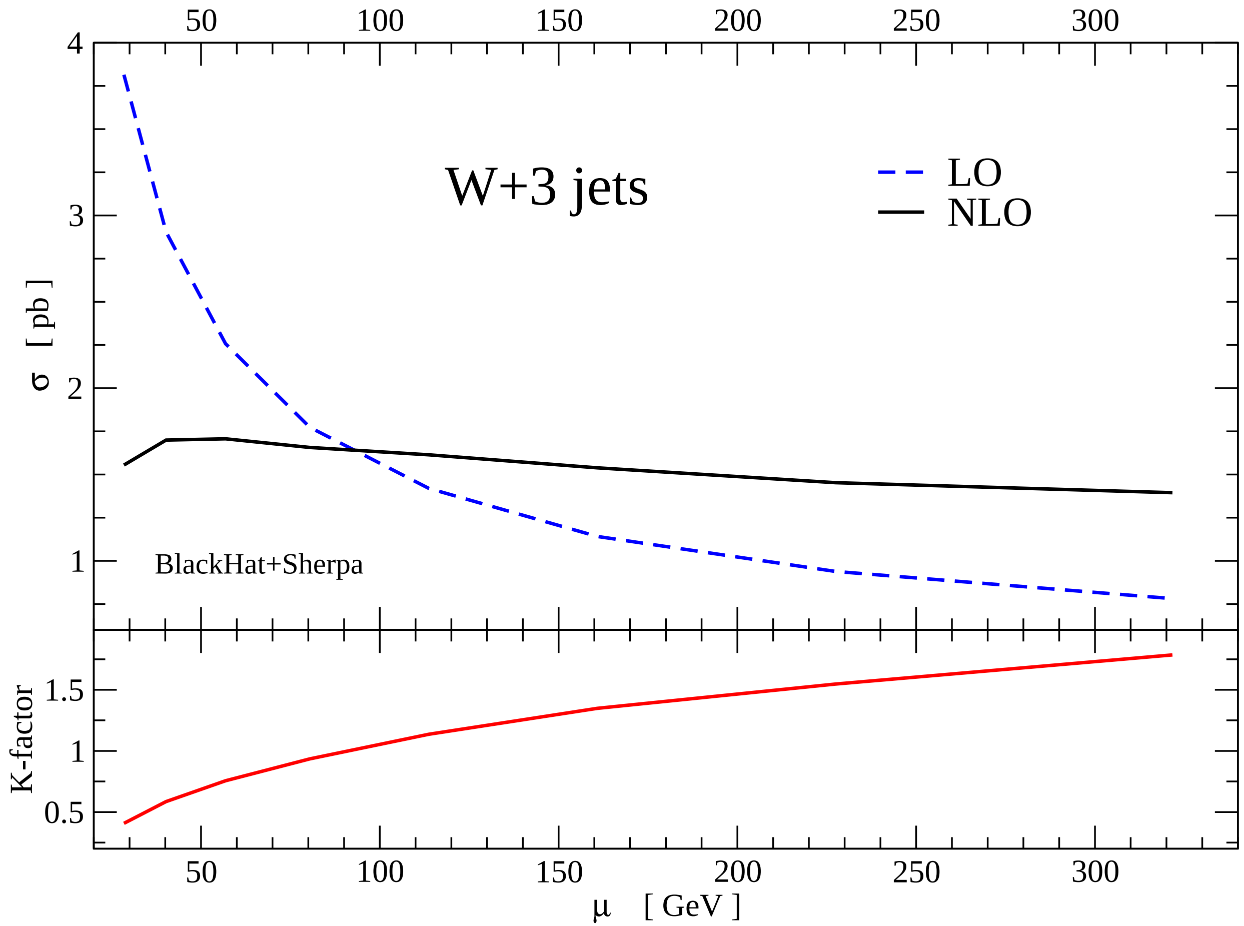
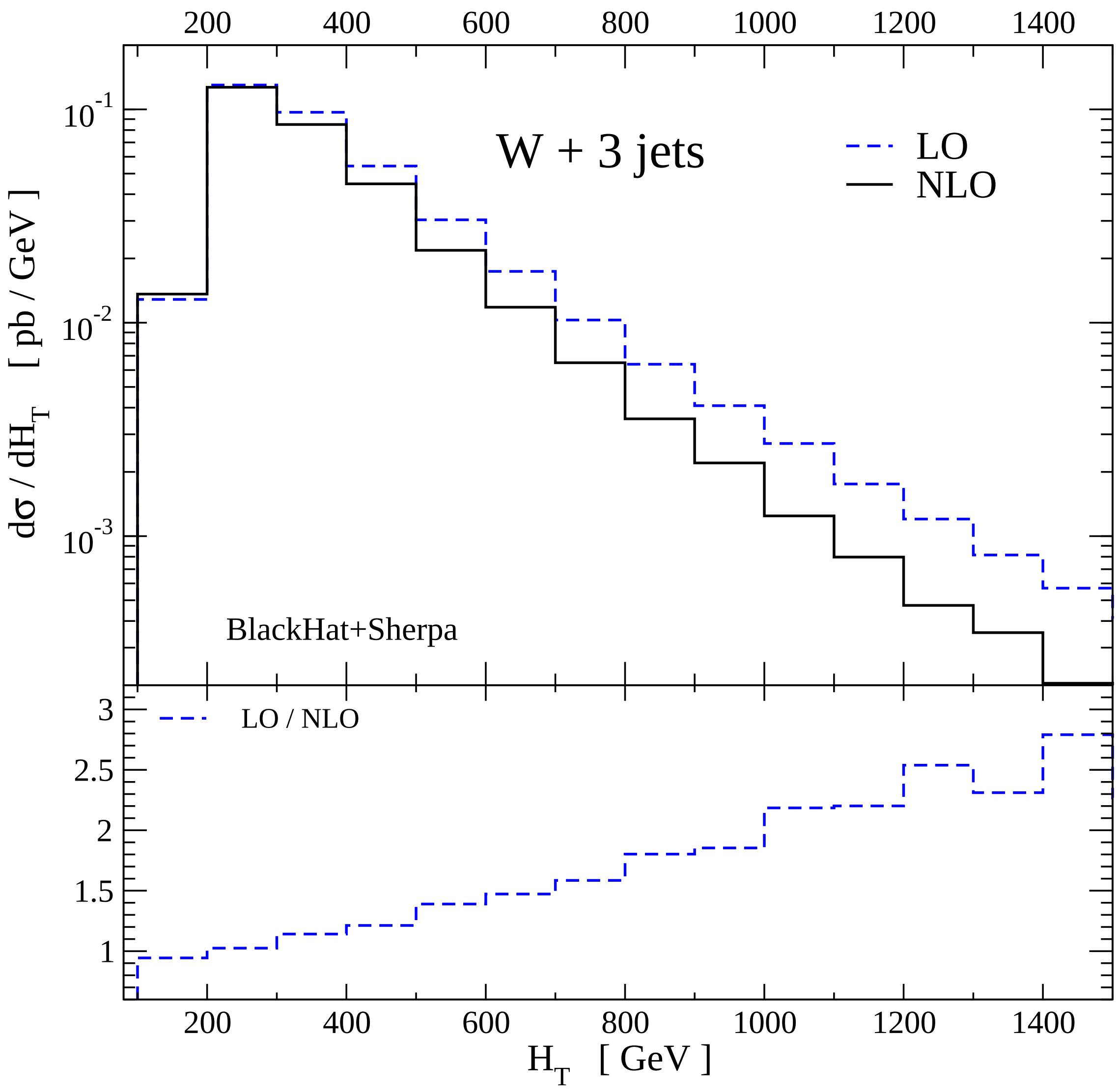


BlackHat + Gleisberg (Sherpa - for real emissions)

$W + 3$ jets at the LHC



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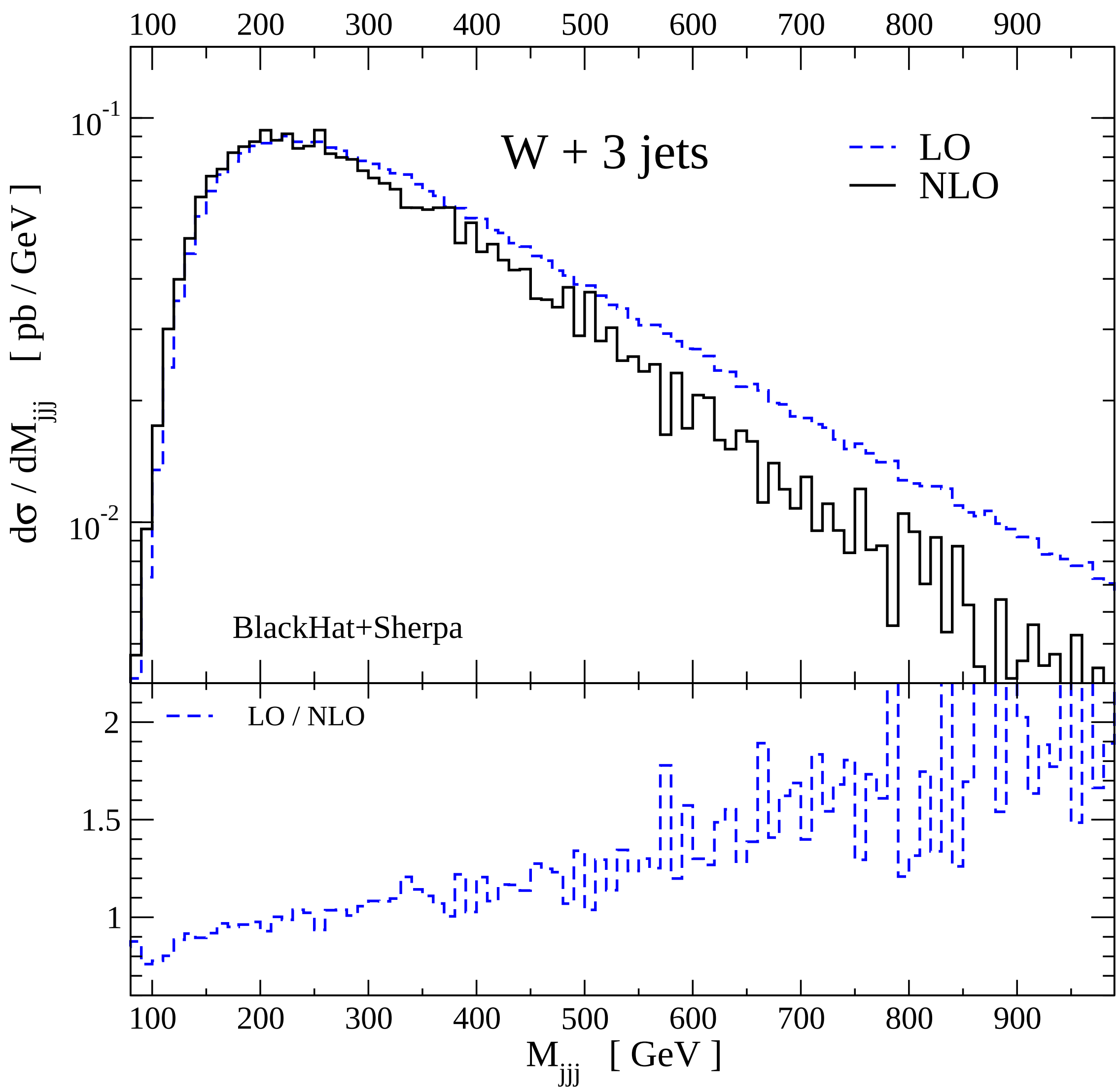

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Status

- **Color-ordered (primitive) 1-loop helicity amplitudes for: Ng, $NgMq$, $Ng2q1V$, $Ng4q1V$ (massless)**

  Primitive amplitudes are helicity amplitudes without color and coupling information

$$\mathcal{A}^{\text{tree}}(g,\ldots,g) = g^{n-2} \sum_{\sigma \in S_n/Z_n} \text{Tr}\left(T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}\right) \times A^{\text{tree}}(\sigma(1),\ldots,\sigma(n))$$

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- **1-loop massive amplitudes in development**

Outlook

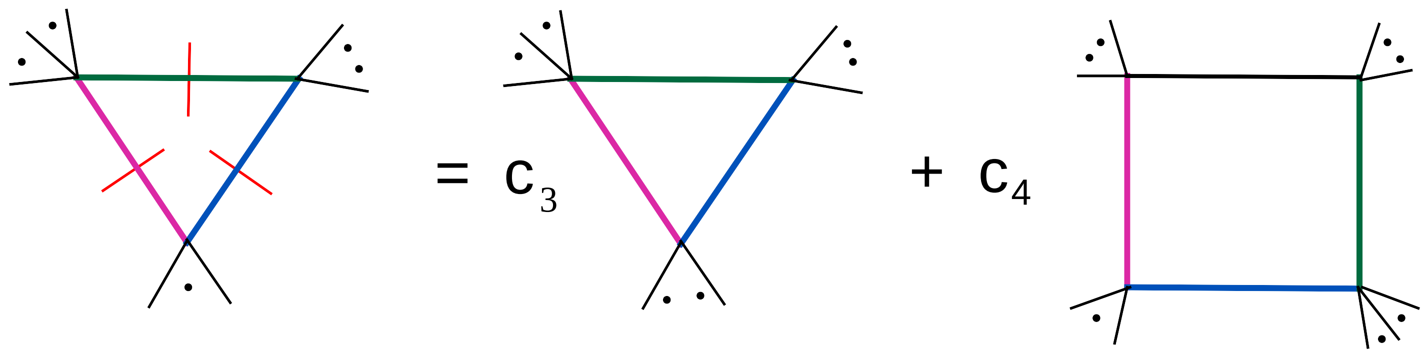
- ☑ **Interface with color, coupling information**
- ☑ **Implement massive partons**
- ☑ **Finish the experimental wishlists**
- ❑ **Interface with resummation $\Rightarrow$ SCET?**
- ❑ **Interface with parton shower**
- ❑ . . .


YES
WE
CAN!

Extra Slides

Generalized Unitarity contd.

Triangle coefficients from triple cuts, bubble coefficients from double cuts.



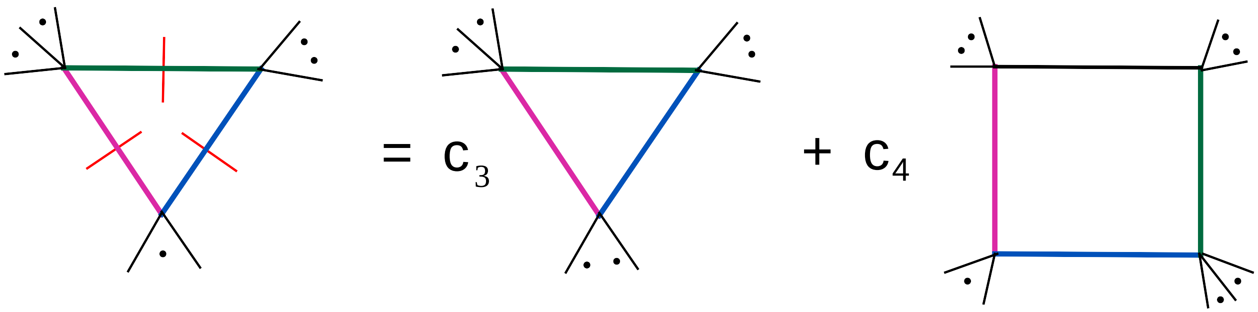
But life's not so simple – "leakage" from higher-point integrals into lower point ones because integrals are not fully localized any more.
However, the singularity structures are unique – need procedure to disentangle coefficients:

- **Holomorphic anomaly – algebraic but non-linear manipulations instead of integrals** Britto, Cachazo, Feng
- **Clever parametrization of integral – read off coefficients directly** Forde; Ossola, Papadopoulos, Pittau; Kilgore

Disentangling Coefficients

Parametrization of loop momenta (schematically):

$$l_n^\mu = \alpha_1 K_1^\mu + \alpha_2 K_2^\mu + \alpha_3 t K_3(K_1, K_2)^\mu + \frac{\alpha_4}{t} K_4(K_1, K_2)^\mu$$



Triple cut then gives:

$$C_3 = \sum_{j=-3}^{3} c_j t^j + \sum_i \frac{b_i}{\xi_i (t - t_i)}$$

$l_i^2(t) \sim \xi_i (t - t_i)$.

Boxes have extra poles in t from propagators that go on-shell.

But we know the boxes, so subtract them off.

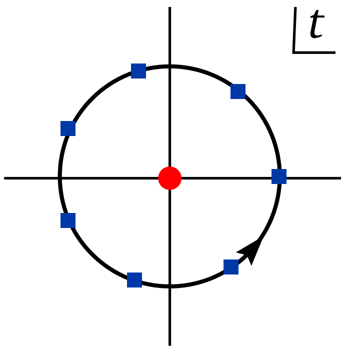
Disentangling Coefficients contd.

Triangle contributions after subtraction of boxes:

$$T_3 = \sum_{j=-3}^{3} c_j t^j$$

c_0 is the triangle coefficient, extract via discrete Fourier transform

$$c_0 = \frac{1}{7} \sum_{j=0}^{6} T_3 \left(t_0 e^{2\pi i j/7} \right)$$



BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre, arXiv:0803.4180