

Algebra

Harvard-MIT Math Tournament
February 27, 1999

1. If $a@b = \frac{a^3-b^3}{a-b}$, for how many real values of a does $a@1 = 0$?
2. For what single digit n does 91 divide the 9-digit number 12345 n 789?
3. Alex is stuck on a platform floating over an abyss at 1 ft/s. An evil physicist has arranged for the platform to fall in (taking Alex with it) after traveling 100ft. One minute after the platform was launched, Edward arrives with a second platform capable of floating all the way across the abyss. He calculates for 5 seconds, then launches the second platform in such a way as to maximize the time that one end of Alex's platform is between the two ends of the new platform, thus giving Alex as much time as possible to switch. If both platforms are 5 ft long and move with constant velocity once launched, what is the speed of the second platform (in ft/s)?
4. Find all possible values of $\frac{d}{a}$ where $a^2 - 6ad + 8d^2 = 0$, $a \neq 0$.
5. You are trapped in a room with only one exit, a long hallway with a series of doors and land mines. To get out you must open all the doors and disarm all the mines. In the room is a panel with 3 buttons, which conveniently contains an instruction manual. The red button arms a mine, the yellow button disarms two mines and closes a door, and the green button opens two doors. Initially 3 doors are closed and 3 mines are armed. The manual warns that attempting to disarm two mines or open two doors when only one is armed/closed will reset the system to its initial state. What is the minimum number of buttons you must push to get out?
6. Carl and Bob can demolish a building in 6 days, Anne and Bob can do it in 3, Anne and Carl in 5. How many days does it take all of them working together if Carl gets injured at the end of the first day and can't come back? Express your answer as a fraction in lowest terms.
7. Matt has somewhere between 1000 and 2000 pieces of paper he's trying to divide into piles of the same size (but not all in one pile or piles of one sheet each). He tries 2, 3, 4, 5, 6, 7, and 8 piles but ends up with one sheet left over each time. How many piles does he need?
8. If $f(x)$ is a monic quartic polynomial such that $f(-1) = -1$, $f(2) = -4$, $f(-3) = -9$, and $f(4) = -16$, find $f(1)$.
9. How many ways are there to cover a 3×8 rectangle with 12 identical dominoes?
10. Pyramid *EARLY* is placed in (x, y, z) coordinates so that $E = (10, 10, 0)$, $A = (10, -10, 0)$, $R = (-10, -10, 0)$, $L = (-10, 10, 0)$, and $Y = (0, 0, 10)$. Tunnels are drilled through the pyramid in such a way that one can move from (x, y, z) to any of the 9 points $(x, y, z - 1)$, $(x \pm 1, y, z - 1)$, $(x, y \pm 1, z - 1)$, $(x \pm 1, y \pm 1, z - 1)$. Sean starts at Y and moves randomly down to the base of the pyramid, choosing each of the possible paths with probability $\frac{1}{9}$ each time. What is the probability that he ends up at the point $(8, 9, 0)$?