Geometry

Harvard-MIT Math Tournament
February 27, 1999

1. Two 10×24 rectangles are inscribed in a circle as shown. Find the shaded area.

2. A semicircle is inscribed in a semicircle of radius 2 as shown. Find the radius of the smaller semicircle.

3. In a cube with side length 6, what is the volume of the tetrahedron formed by any vertex and the three vertices connected to that vertex by edges of the cube?

4. A cross-section of a river is a trapezoid with bases 10 and 16 and slanted sides of length 5. At this section the water is flowing at π mph. A little ways downstream is a dam where the water flows through 4 identical circular holes at 16 mph. What is the radius of the holes?

5. In triangle BEN shown below with its altitudes intersecting at X, $NA = 7, EA = 3, AX = 4$, and $NS = 8$. Find the area of BEN.

6. A sphere of radius 1 is covered in ink and rolling around between concentric spheres of radii 3 and 5. If this process traces a region of area 1 on the larger sphere, what is the area of the region traced on the smaller sphere?

7. A dart is thrown at a square dartboard of side length 2 so that it hits completely randomly. What is the probability that it hits closer to the center than any corner, but within a distance 1 of a corner?

8. Squares $ABKL, BCMN, CAOP$ are drawn externally on the sides of a triangle ABC. The line segments KL, MN, OP, when extended, form a triangle $A'B'C'$. Find the area of $A'B'C'$ if ABC is an equilateral triangle of side length 2.

9. A regular tetrahedron has two vertices on the body diagonal of a cube with side length 12. The other two vertices lie on one of the face diagonals not intersecting that body diagonal. Find the side length of the tetrahedron.

10. In the figure below, $AB = 15, BD = 18, AF = 15, DF = 12, BE = 24$, and $CF = 17$. Find $BG : FG$.