Team

1. A combination lock has a 3 number combination, with each number an integer between 0 and 39 inclusive. Call the numbers n_1, n_2, and n_3. If you know that n_1 and n_3 leave the same remainder when divided by 4, and n_2 and $n_1 + 2$ leave the same remainder when divided by 4, how many possible combinations are there?

2. A ladder is leaning against a house with its lower end 15 feet from the house. When the lower end is pulled 9 feet farther from the house, the upper end slides 13 feet down. How long is the ladder (in feet)?

3. How many non-empty subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ have exactly k elements and do not contain the element k for some $k = 1, 2, ..., 8$.

4. Consider the equation $FORTY + TEN + TEN = SIXTY$, where each of the ten letters represents a distinct digit from 0 to 9. Find all possible values of $SIXTY$.

5. If a and b are randomly selected real numbers between 0 and 1, find the probability that the nearest integer to $\frac{a-b}{a+b}$ is odd.

6. Reduce the number $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$.

7. Let $\frac{1}{1-x-x^2-x^3} = \sum_{i=0}^{\infty} a_n x^n$, for what positive integers n does $a_{n-1} = n^2$?

8. Find all the roots of $(x^2 + 3x + 2)(x^2 - 7x + 12)(x^2 - 2x - 1) + 24 = 0$.

9. Evaluate $\sum_{n=2}^{17} \frac{n^2+n+1}{n^4+2n^3-n^2-2n}$.

10. If 5 points are placed in the plane at lattice points (i.e. points (x, y) where x and y are both integers) such that no three are collinear, then there are 10 triangles whose vertices are among these points. What is the minimum possible number of these triangles that have area greater than 1/2?

11. Circles C_1, C_2, C_3 have radius 1 and centers O, P, Q respectively. C_1 and C_2 intersect at A, C_2 and C_3 intersect at B, C_3 and C_1 intersect at C, in such a way that $\angle APB = 60°$, $\angle BQC = 36°$, and $\angle COA = 72°$. Find angle ABC (degrees).

12. A fair coin is flipped every second and the results are recorded with 1 meaning heads and 0 meaning tails. What is the probability that the sequence 10101 occurs before the first occurance of the sequence 010101?