General Test Solutions (Second Half)

Harvard-MIT Math Tournament

March 3, 2001

1. A circle of radius 3 crosses the center of a square of side length 2. Find the difference between the areas of the nonoverlapping portions of the figures.

2. Call three sides of an opaque cube adjacent if someone can see them all at once. Draw a plane through the centers of each triple of adjacent sides of a cube with edge length 1. Find the volume of the closed figure bounded by the resulting planes.

3. Find x if $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$.

4. Some students are taking a math contest, in which each student takes one of four tests. One third of the students take one test, one fourth take another test, one fifth take the next test, and 26 students take the last test. How many students are taking the contest in total?

5. What is the area of a square inscribed in a semicircle of radius 1, with one of its sides flush with the diameter of the semicircle?

6. You take a wrong turn on the way to MIT and end up in Transylvania, where 99% of the inhabitants are vampires and the rest are regular humans. For obvious reasons, you want to be able to figure out who's who. On average, nine-tenths of the vampires are correctly identified as vampires and nine-tenths of humans are correct identified as humans. What is the probability that someone identified as a human is actually a human?

7. A real numbers x is randomly chosen in the interval $\left[-15\frac{1}{2}, 15\frac{1}{2}\right]$. Find the probability that the closest integer to x is odd.

8. A point on a circle inscribed in a square is 1 and 2 units from the two closest sides of the square. Find the area of the square.

9. Two circles are concentric. The area of the ring between them is A. In terms of A, find the length of the longest chord contained entirely within the ring.

10. Find the volume of the tetrahedron with vertices $(5, 8, 10), (10, 10, 17), (4, 45, 46), (2, 5, 4)$.