Harvard-MIT Math Tournament
March 17, 2002
Individual General Test: **Part 2**

1. The squares of a chessboard are numbered from left to right and top to bottom (so that the first row reads $1, 2, \ldots, 8$, the second reads $9, 10, \ldots, 16$, and so forth). The number 1 is on a black square. How many black squares contain odd numbers?

2. You are in a completely dark room with a drawer containing 10 red, 20 blue, 30 green, and 40 khaki socks. What is the smallest number of socks you must randomly pull out in order to be sure of having at least one of each color?

3. Solve for x in $3 = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$.

4. Dan is holding one end of a 26 inch long piece of light string that has a heavy bead on it with each hand (so that the string lies along two straight lines). If he starts with his hands together at the start and leaves his hands at the same height, how far does he need to pull his hands apart so that the bead moves upward by 8 inches?

5. A square and a regular hexagon are drawn with the same side length. If the area of the square is $\sqrt{3}$, what is the area of the hexagon?

6. Nine nonnegative numbers have average 10. What is the greatest possible value for their median?

7. p and q are primes such that the numbers $p + q$ and $p + 7q$ are both squares. Find the value of p.

8. Two fair coins are simultaneously flipped. This is done repeatedly until at least one of the coins comes up heads, at which point the process stops. What is the probability that the other coin also came up heads on this last flip?

9. A and B are two points on a circle with center O, and C lies outside the circle, on ray AB. Given that $AB = 24, BC = 28, OA = 15$, find OC.

10. How many four-digit numbers are there in which at least one digit occurs more than once?