

Harvard-MIT Math Tournament

March 17, 2002

Individual Subject Test: **Geometry**

1. A man, standing on a lawn, is wearing a circular sombrero of radius 3 feet. Unfortunately, the hat blocks the sunlight so effectively that the grass directly under it dies instantly. If the man walks in a circle of radius 5 feet, what area of dead grass will result?
2. Dan is holding one end of a 26 inch long piece of light string that has a heavy bead on it with each hand (so that the string lies along straight lines). If he starts with his hands together at the start and leaves his hands at the same height, how far does he need to pull his hands apart so that the bead moves upward by 8 inches?
3. A square and a regular hexagon are drawn with the same side length. If the area of the square is $\sqrt{3}$, what is the area of the hexagon?
4. We call a set of professors and committees on which they serve a *university* if
 - (1) given two distinct professors there is one and only one committee on which they both serve,
 - (2) given any committee, C , and any professor, P , not on that committee, there is exactly one committee on which P serves and no professors on committee C serve, and
 - (3) there are at least two professors on each committee; there are at least two committees.What is the smallest number of committees a university can have?
5. Consider a square of side length 1. Draw four lines that each connect a midpoint of a side with a corner not on that side, such that each midpoint and each corner is touched by only one line. Find the area of the region completely bounded by these lines.
6. If we pick (uniformly) a random square of area 1 with sides parallel to the x - and y -axes that lies entirely within the 5-by-5 square bounded by the lines $x = 0, x = 5, y = 0, y = 5$ (the corners of the square need not have integer coordinates), what is the probability that the point $(x, y) = (4.5, 0.5)$ lies within the square of area 1?
7. Equilateral triangle ABC of side length 2 is drawn. Three squares external to the triangle, $ABDE$, $BCFG$, and $CAHI$, are drawn. What is the area of the smallest triangle that contains these squares?
8. Equilateral triangle ABC of side length 2 is drawn. Three squares containing the triangle, $ABDE$, $BCFG$, and $CAHI$, are drawn. What is the area of the smallest triangle that contains these squares?
9. A and B are two points on a circle with center O , and C lies outside the circle, on ray AB . Given that $AB = 24, BC = 28, OA = 15$, find OC .
10. Let $\triangle ABC$ be equilateral, and let D, E, F be points on sides BC, CA, AB respectively, with $FA = 9, AE = EC = 6, CD = 4$. Determine the measure (in degrees) of $\angle DEF$.