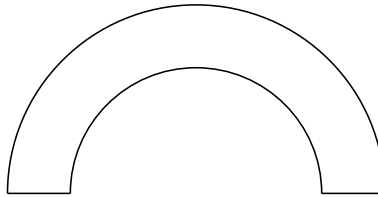


Harvard-MIT Mathematics Tournament

March 15, 2003

Individual Round: General Test, Part 1

1. 10 people are playing musical chairs with n chairs in a circle. They can be seated in $7!$ ways (assuming only one person fits on each chair, of course), where different arrangements of the same people on chairs, even rotations, are considered different. Find n .
2. $OPEN$ is a square, and T is a point on side NO , such that triangle TOP has area 62 and triangle TEN has area 10. What is the length of a side of the square?
3. There are 16 members on the Height-Measurement Matching Team. Each member was asked, "How many other people on the team — not counting yourself — are exactly the same height as you?" The answers included six 1's, six 2's, and three 3's. What was the sixteenth answer? (Assume that everyone answered truthfully.)
4. How many 2-digit positive integers have an even number of positive divisors?
5. A room is built in the shape of the region between two semicircles with the same center and parallel diameters. The farthest distance between two points with a clear line of sight is 12m. What is the area (in m^2) of the room?



6. In how many ways can 3 bottles of ketchup and 7 bottles of mustard be arranged in a row so that no bottle of ketchup is immediately between two bottles of mustard? (The bottles of ketchup are mutually indistinguishable, as are the bottles of mustard.)
7. Find the real value of x such that $x^3 + 3x^2 + 3x + 7 = 0$.
8. A broken calculator has the $+$ and \times keys switched. For how many ordered pairs (a, b) of integers will it correctly calculate $a + b$ using the labelled $+$ key?
9. Consider a 2003-gon inscribed in a circle and a triangulation of it with diagonals intersecting only at vertices. What is the smallest possible number of obtuse triangles in the triangulation?
10. Bessie the cow is trying to navigate her way through a field. She can travel only from lattice point to adjacent lattice point, can turn only at lattice points, and can travel only to the east or north. (A lattice point is a point whose coordinates are both integers.) $(0, 0)$ is the southwest corner of the field. $(5, 5)$ is the northeast corner of the field. Due to large rocks, Bessie is unable to walk on the points $(1, 1)$, $(2, 3)$, or $(3, 2)$. How many ways are there for Bessie to travel from $(0, 0)$ to $(5, 5)$ under these constraints?