# Harvard-MIT Mathematics Tournament

**March 15, 2003**

## Individual Round: General Test, Part 2

1. A compact disc has the shape of a circle of diameter 5 inches with a 1-inch-diameter circular hole in the center. Assuming the capacity of the CD is proportional to its area, how many inches would need to be added to the outer diameter to double the capacity?

2. You have a list of real numbers, whose sum is 40. If you replace every number $x$ on the list by $1 - x$, the sum of the new numbers will be 20. If instead you had replaced every number $x$ by $1 + x$, what would the sum then be?

3. How many positive rational numbers less than $\pi$ have denominator at most 7 when written in lowest terms? (Integers have denominator 1.)

4. In triangle $ABC$ with area 51, points $D$ and $E$ trisect $AB$ and points $F$ and $G$ trisect $BC$. Find the largest possible area of quadrilateral $DEFG$.

5. You are given a $10 \times 2$ grid of unit squares. Two different squares are adjacent if they share a side. How many ways can one mark exactly nine of the squares so that no two marked squares are adjacent?

6. The numbers 112, 121, 123, 153, 243, 313, and 322 are among the rows, columns, and diagonals of a $3 \times 3$ square grid of digits (rows and diagonals read left-to-right, and columns read top-to-bottom). What 3-digit number completes the list?

7. Daniel and Scott are playing a game where a player wins as soon as he has two points more than his opponent. Both players start at par, and points are earned one at a time. If Daniel has a 60% chance of winning each point, what is the probability that he will win the game?

8. If $x \geq 0$, $y \geq 0$ are integers, randomly chosen with the constraint $x + y \leq 10$, what is the probability that $x + y$ is even?

9. In a classroom, 34 students are seated in 5 rows of 7 chairs. The place at the center of the room is unoccupied. A teacher decides to reassign the seats such that each student will occupy a chair adjacent to his/her present one (i.e. move one desk forward, back, left or right). In how many ways can this reassignment be made?

10. Several positive integers are given, not necessarily all different. Their sum is 2003. Suppose that $n_1$ of the given numbers are equal to 1, $n_2$ of them are equal to 2, $\ldots$, $n_{2003}$ of them are equal to 2003. Find the largest possible value of

$$n_2 + 2n_3 + 3n_4 + \cdots + 2002n_{2003}.$$