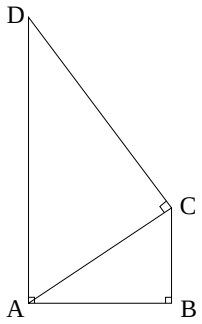
Harvard-MIT Mathematics Tournament

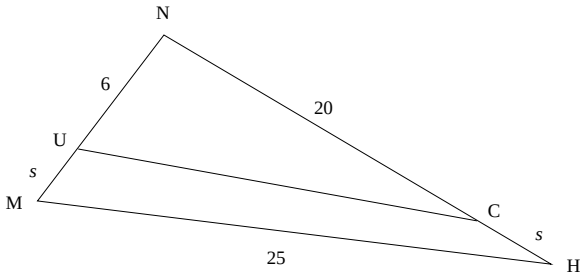
March 15, 2003

Individual Round: Geometry Subject Test

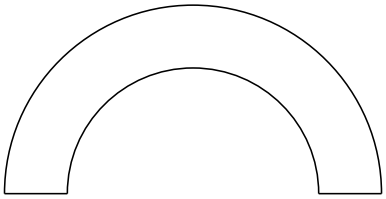
1. AD and BC are both perpendicular to AB, and CD is perpendicular to AC. If $AB = 4$ and $BC = 3$, find CD.



2. As shown, U and C are points on the sides of triangle MNH such that $MU = s$, $UN = 6$, $NC = 20$, $CH = s$, $HM = 25$. If triangle UNC and quadrilateral $MUCH$ have equal areas, what is s?



3. A room is built in the shape of the region between two semicircles with the same center and parallel diameters. The farthest distance between two points with a clear line of sight is 12m. What is the area (in m^2) of the room?



4. Farmer John is inside of an ellipse with reflective sides, given by the equation $x^2/a^2 + y^2/b^2 = 1$, with $a > b > 0$. He is standing at the point $(3, 0)$, and he shines a laser pointer in the y-direciton. The light reflects off the ellipse and proceeds directly toward Farmer Brown, traveling a distance of 10 before reaching him. Farmer John then spins around in a circle; wherever he points the laser, the light reflects off the wall and hits Farmer Brown. What is the ordered pair (a, b)?

5. Consider a 2003-gon inscribed in a circle and a triangulation of it with diagonals intersecting only at vertices. What is the smallest possible number of obtuse triangles in the triangulation?

6. Take a clay sphere of radius 13, and drill a circular hole of radius 5 through its center. Take the remaining "bead" and mold it into a new sphere. What is this sphere's radius?

7. Let $RSTUV$ be a regular pentagon. Construct an equilateral triangle PRS with point P inside the pentagon. Find the measure (in degrees) of angle PTV.

8. Let ABC be an equilateral triangle of side length 2. Let ω be its circumcircle, and let $\omega_A, \omega_B, \omega_C$ be circles congruent to ω centered at each of its vertices. Let R be the set of all points in the plane contained in exactly two of these four circles. What is the area of R?

9. In triangle ABC, $\angle ABC = 50^\circ$ and $\angle ACB = 70^\circ$. Let D be the midpoint of side BC. A circle is tangent to BC at B and is also tangent to segment AD; this circle instersects AB again at P. Another circle is tangent to BC at C and is also tangent to segment AD; this circle intersects AC again at Q. Find $\angle APQ$ (in degrees).

10. Convex quadrilateral $MATH$ is given with $HM/MT = 3/4$, and $\angle ATM = \angle MAT = \angle AHM = 60^\circ$. N is the midpoint of MA, and O is a point on TH such that lines MT, AH, NO are concurrent. Find the ratio HO/OT.