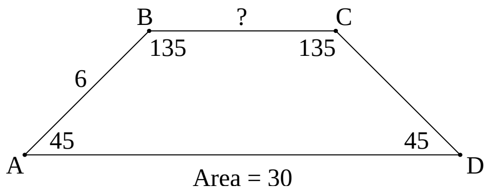
Harvard-MIT Mathematics Tournament

February 28, 2004

Individual Round: Geometry Subject Test

1. In trapezoid $ABCD$, AD is parallel to BC. $\angle A = \angle D = 45°$, while $\angle B = \angle C = 135°$. If $AB = 6$ and the area of $ABCD$ is 30, find BC.



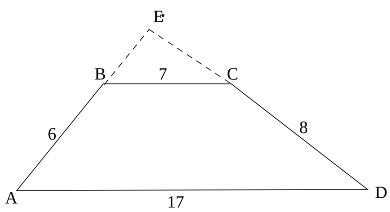
2. A parallelogram has 3 of its vertices at (1, 2), (3,8), and (4, 1). Compute the sum of the possible x-coordinates for the 4th vertex.

3. A swimming pool is in the shape of a circle with diameter 60 ft. The depth varies linearly along the east-west direction from 3 ft at the shallow end in the east to 15 ft at the diving end in the west (this is so that divers look impressive against the sunset) but does not vary at all along the north-south direction. What is the volume of the pool, in ft^3?

4. P is inside rectangle $ABCD$. $PA = 2, PB = 3$, and $PC = 10$. Find PD.

5. Find the area of the region of the xy-plane defined by the inequality $|x|+|y|+|x+y| \leq 1$.

6. In trapezoid $ABCD$ shown, AD is parallel to BC, and $AB = 6, BC = 7, CD = 8, AD = 17$. If sides AB and CD are extended to meet at E, find the resulting angle at E (in degrees).



7. Yet another trapezoid $ABCD$ has AD parallel to BC. AC and BD intersect at P. If $[ADP]/[BCP] = 1/2$, find $[ADP]/[ABCD]$. (Here the notation $[P_1 \cdots P_n]$ denotes the area of the polygon $P_1 \cdots P_n$.)

8. A triangle has side lengths 18, 24, and 30. Find the area of the triangle whose vertices are the incenter, circumcenter, and centroid of the original triangle.

9. Given is a regular tetrahedron of volume 1. We obtain a second regular tetrahedron by reflecting the given one through its center. What is the volume of their intersection?

10. Right triangle XYZ has right angle at Y and $XY = 228$, $YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of $(PY + YZ)(QY + XY)$.