

# IX<sup>th</sup> Annual Harvard-MIT Mathematics Tournament

Saturday 25 February 2006

## Individual Round: General Test, Part 1

1. How many positive integers  $x$  are there such that  $3x$  has 3 digits and  $4x$  has four digits?
2. What is the probability that two cards randomly selected (without replacement) from a standard 52-card deck are neither of the same value nor the same suit?
3. A square and an equilateral triangle together have the property that the area of each is the perimeter of the other. Find the square's area.

4. Find

$$\frac{\sqrt{31 + \sqrt{31 + \sqrt{31 + \dots}}}}{\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}.$$

5. In the plane, what is the length of the shortest path from  $(-2, 0)$  to  $(2, 0)$  that avoids the interior of the unit circle (i.e., circle of radius 1) centered at the origin?
6. Six celebrities meet at a party. It so happens that each celebrity shakes hands with exactly two others. A fan makes a list of all unordered pairs of celebrities who shook hands with each other. If order does not matter, how many different lists are possible?
7. The train schedule in Hummut is hopelessly unreliable. Train A will enter Intersection X from the west at a random time between 9:00 am and 2:30 pm; each moment in that interval is equally likely. Train B will enter the same intersection from the north at a random time between 9:30 am and 12:30 pm, independent of Train A; again, each moment in the interval is equally likely. If each train takes 45 minutes to clear the intersection, what is the probability of a collision today?
8. A dot is marked at each vertex of a triangle  $ABC$ . Then, 2, 3, and 7 more dots are marked on the sides  $AB$ ,  $BC$ , and  $CA$ , respectively. How many triangles have their vertices at these dots?
9. Take a unit sphere  $S$ , i.e., a sphere with radius 1. Circumscribe a cube  $C$  about  $S$ , and inscribe a cube  $D$  in  $S$ , so that every edge of cube  $C$  is parallel to some edge of cube  $D$ . What is the shortest possible distance from a point on a face of  $C$  to a point on a face of  $D$ ?
10. A positive integer  $n$  is called “flippant” if  $n$  does not end in 0 (when written in decimal notation) and, moreover,  $n$  and the number obtained by reversing the digits of  $n$  are both divisible by 7. How many flippant integers are there between 10 and 1000?