Relativistic Dynamics: Testing Relativity and Determining Properties of the Electron

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Goals

- Test relativity (vs. Newton)
- Determine m_e and e
- Diagnose systematic errors

Special Relativity

- Fast things
- Length contraction
- Time dilation
- Universal speed limit (c)

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Testing Relativity

Q: Why do we care?

A:

Testing Relativity

Q: Why do we care?

A: Because relativity is awesome!

Testing Relativity

Q: Why do we care?

A: Because it's required to understand things moving quickly.

Relativity Applications

GPS (2 km / day)

Images from `http://www8.garmin.com/aboutGPS/` and

`www.mobilewhack.com/garmin-nuvi-260-gps-speaks-street-names/`.

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Testing Relativity

Q: How do we test it?

A:

Testing Relativity

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A: Move really, really quickly!

Testing Relativity

Q: What moves really, really fast?

A:

Testing Relativity

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A: Light!

http://www.clker.com/clipart-lightbulb-on-off.html

Testing Relativity

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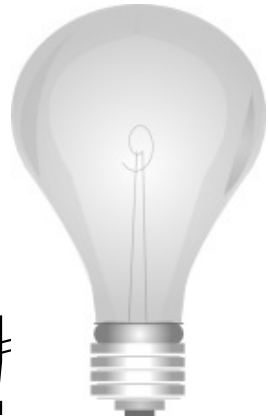
A: ~~Light~~!



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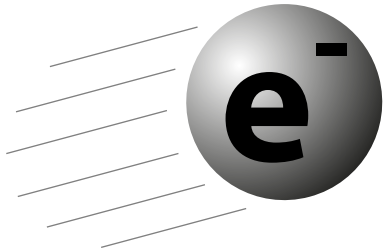
Testing Relativity

Q: What moves really, really fast?

A: Electrons! 

(^{90}Sr — 0.564 MeV — 0.77c)

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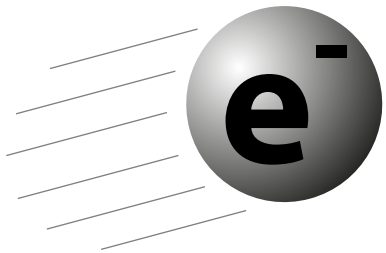
A: Electrons! 

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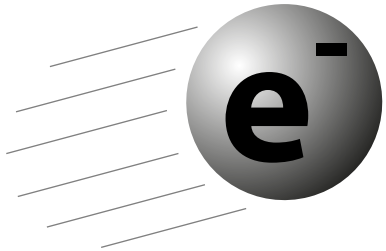
A: Electrons! 

(^{90}Y — 2.28 MeV — 0.97c)

Theory & Experiment

Three points of contact:

- $\vec{p}$
- $\vec{F} = d\vec{p}/dt$
- K

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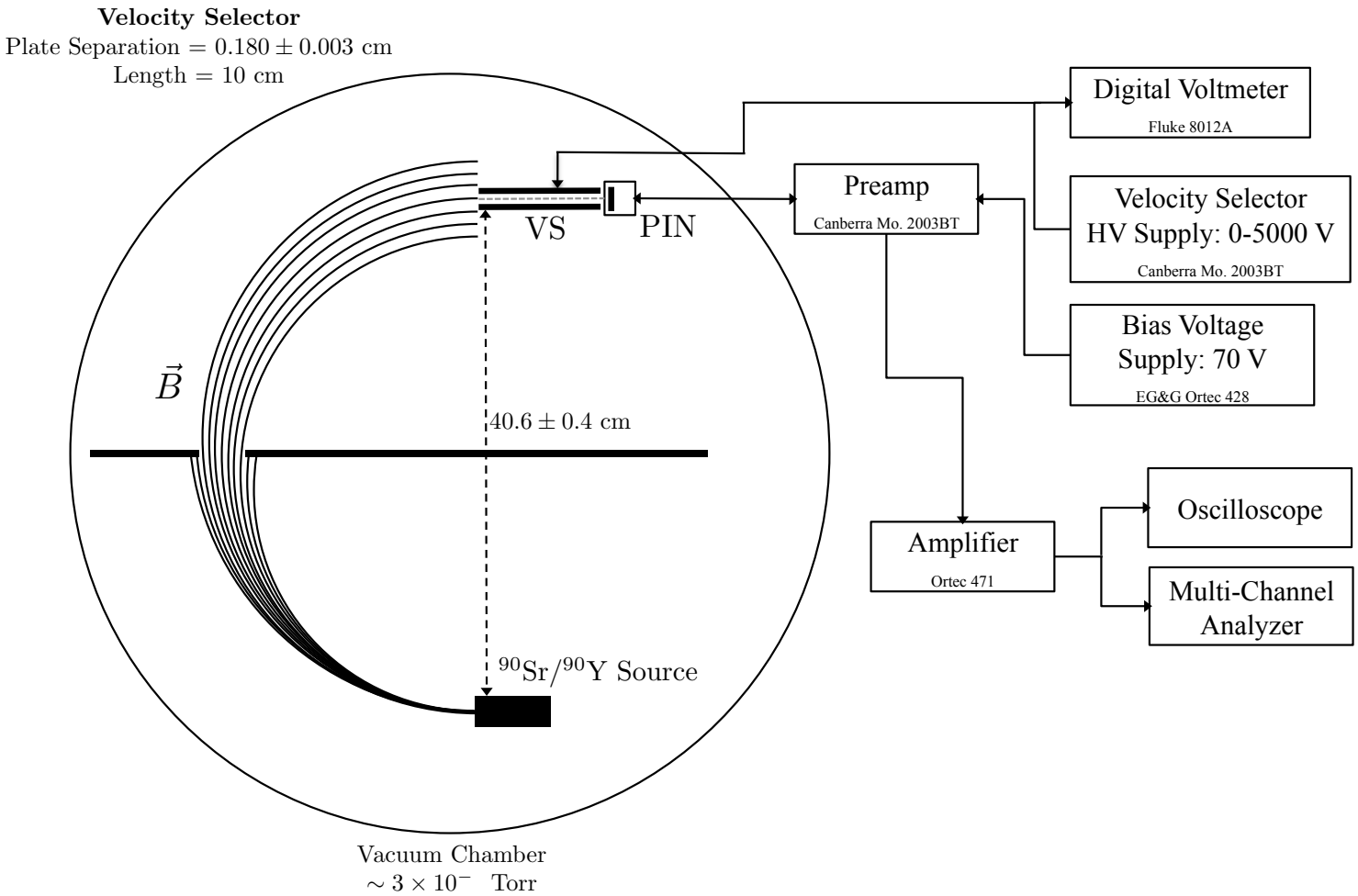
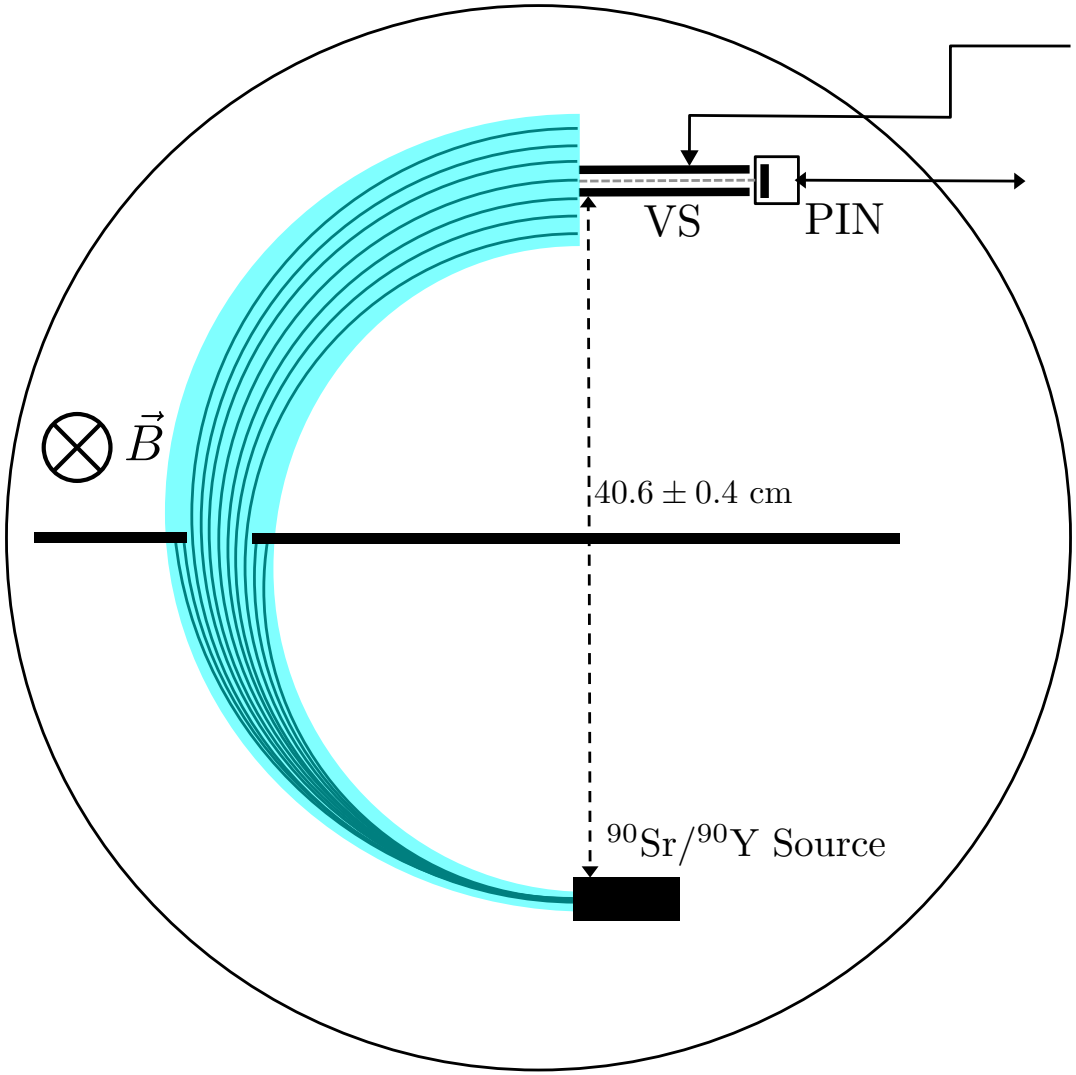
The Apparatus



From `http://web.mit.edu/8.13/www/JLExperiments/JLExp09.pdf`

Magnetic Force



Magnetic Force

$$q\vec{v} \times \vec{B} = \vec{F} = \vec{\omega} \times \vec{p} = \frac{\vec{v} \times \vec{\rho}}{\rho^2} \times \vec{p}$$

$$B = \frac{p}{e\rho}$$

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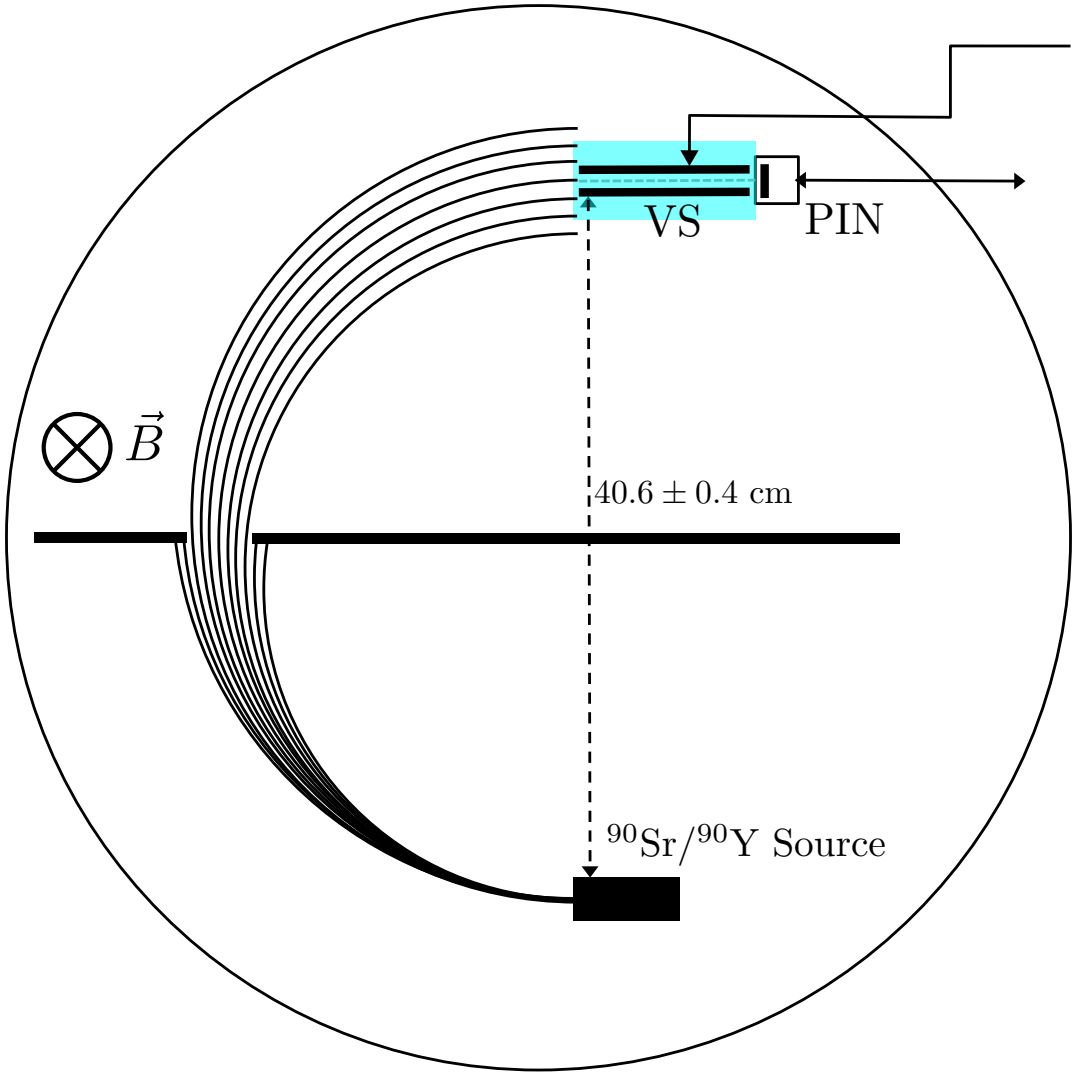
Electric Force



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$$q\vec{E} + q\vec{v} \times \vec{B} = \vec{F} = 0$$

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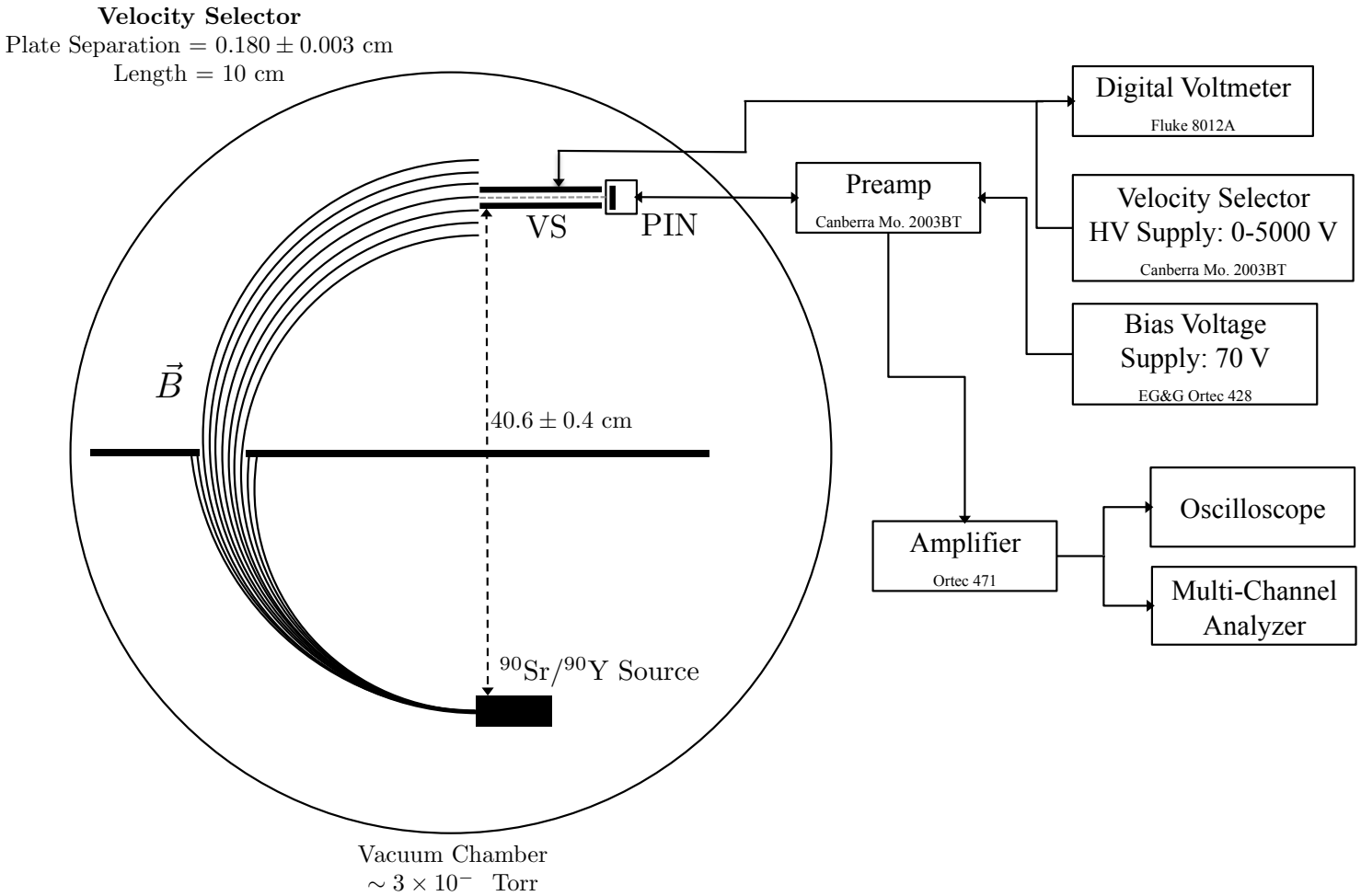


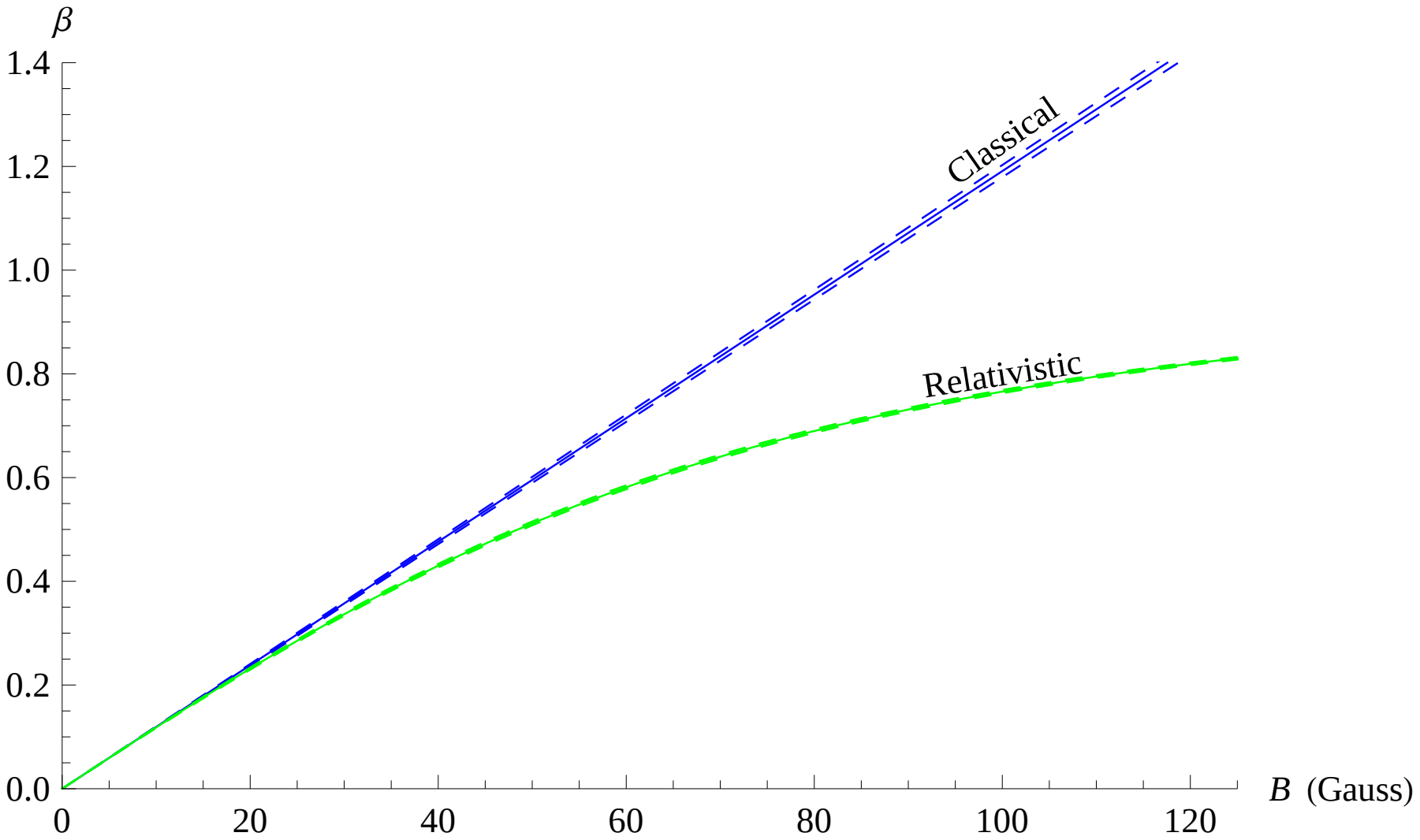
Predictions

Classical Mechanics: $\vec{p} = m\vec{v}$

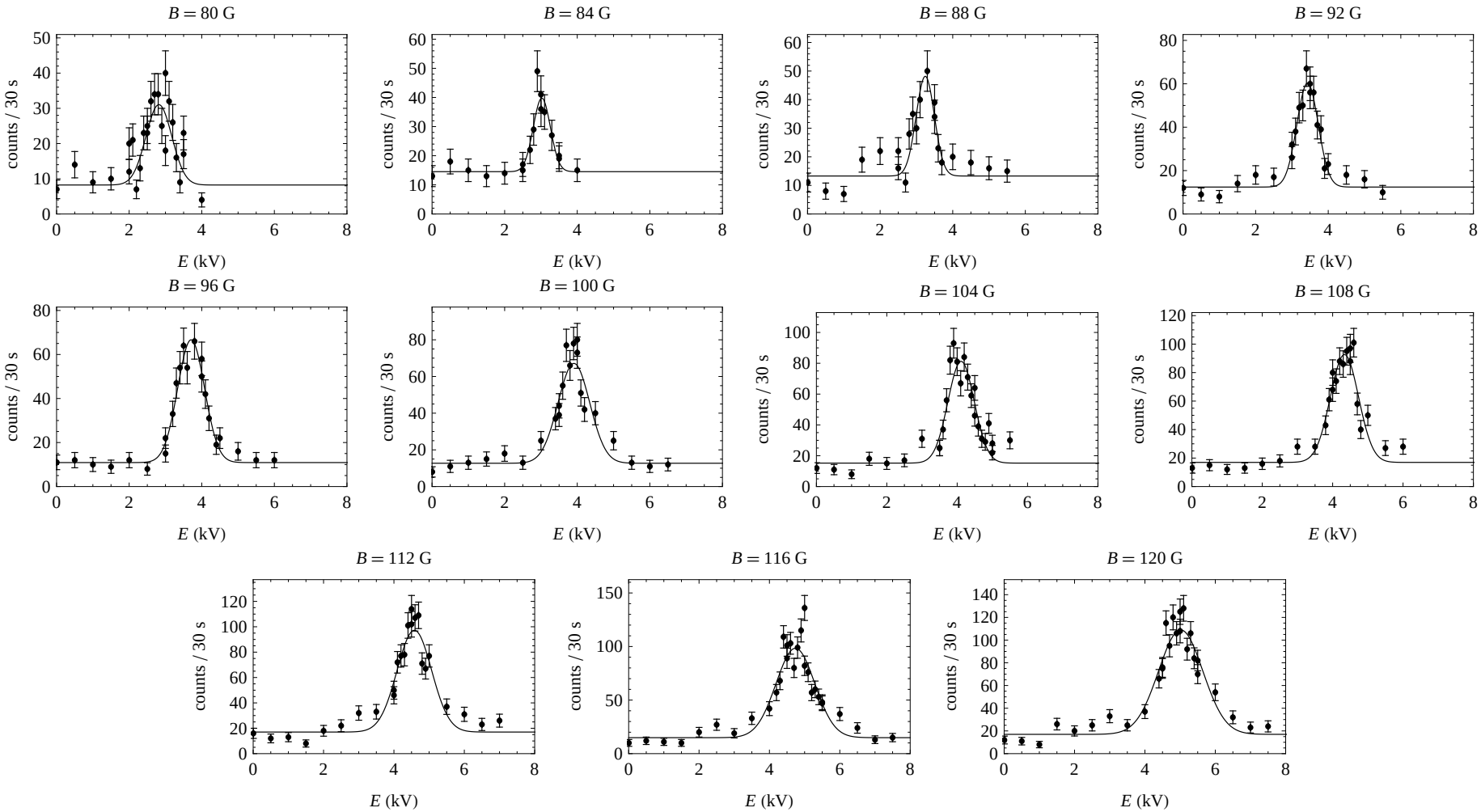
Relativity: $\vec{p} = \gamma m\vec{v}$ with

$$\gamma = 1 \Big/ \sqrt{1 - \beta^2}$$

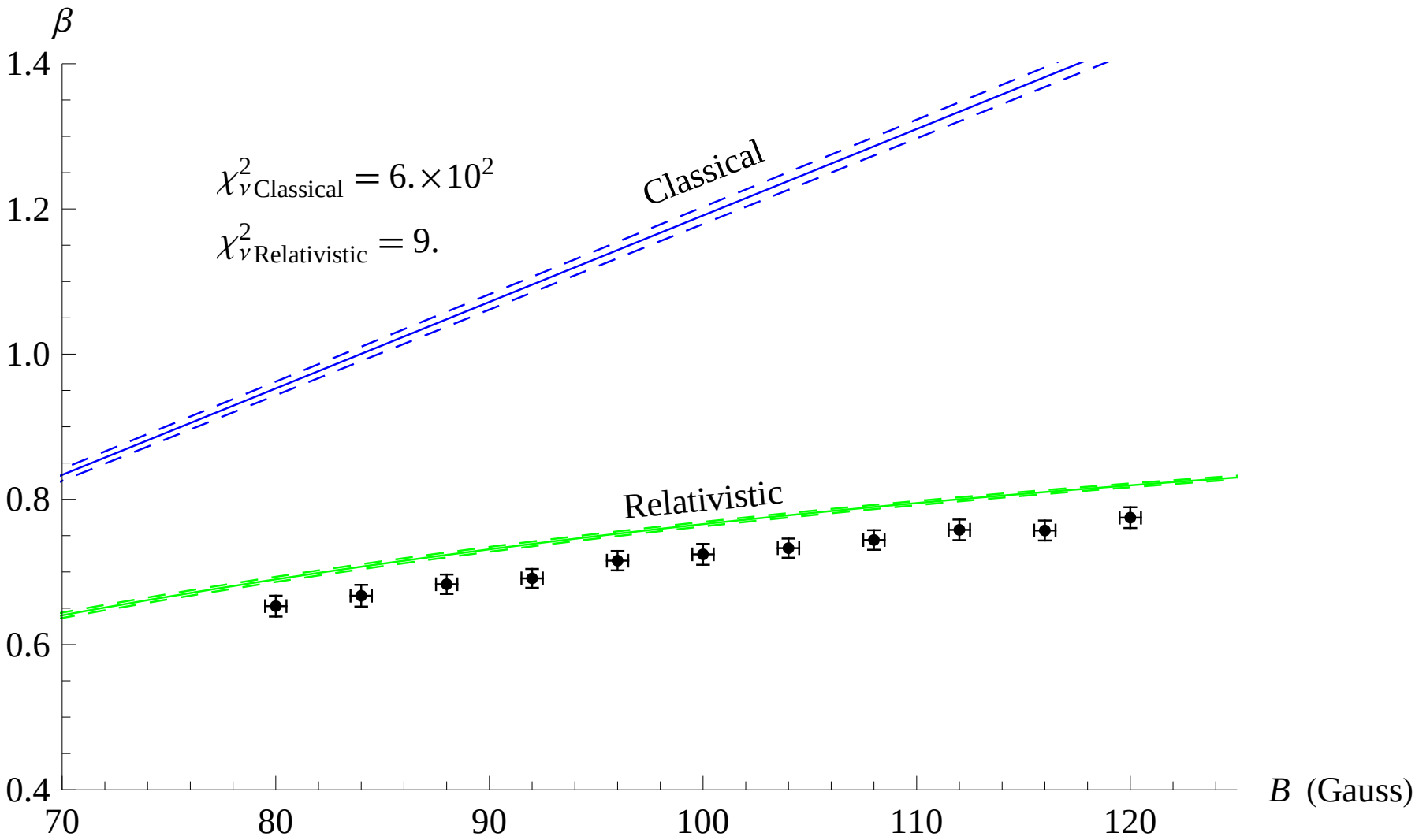
Expectations



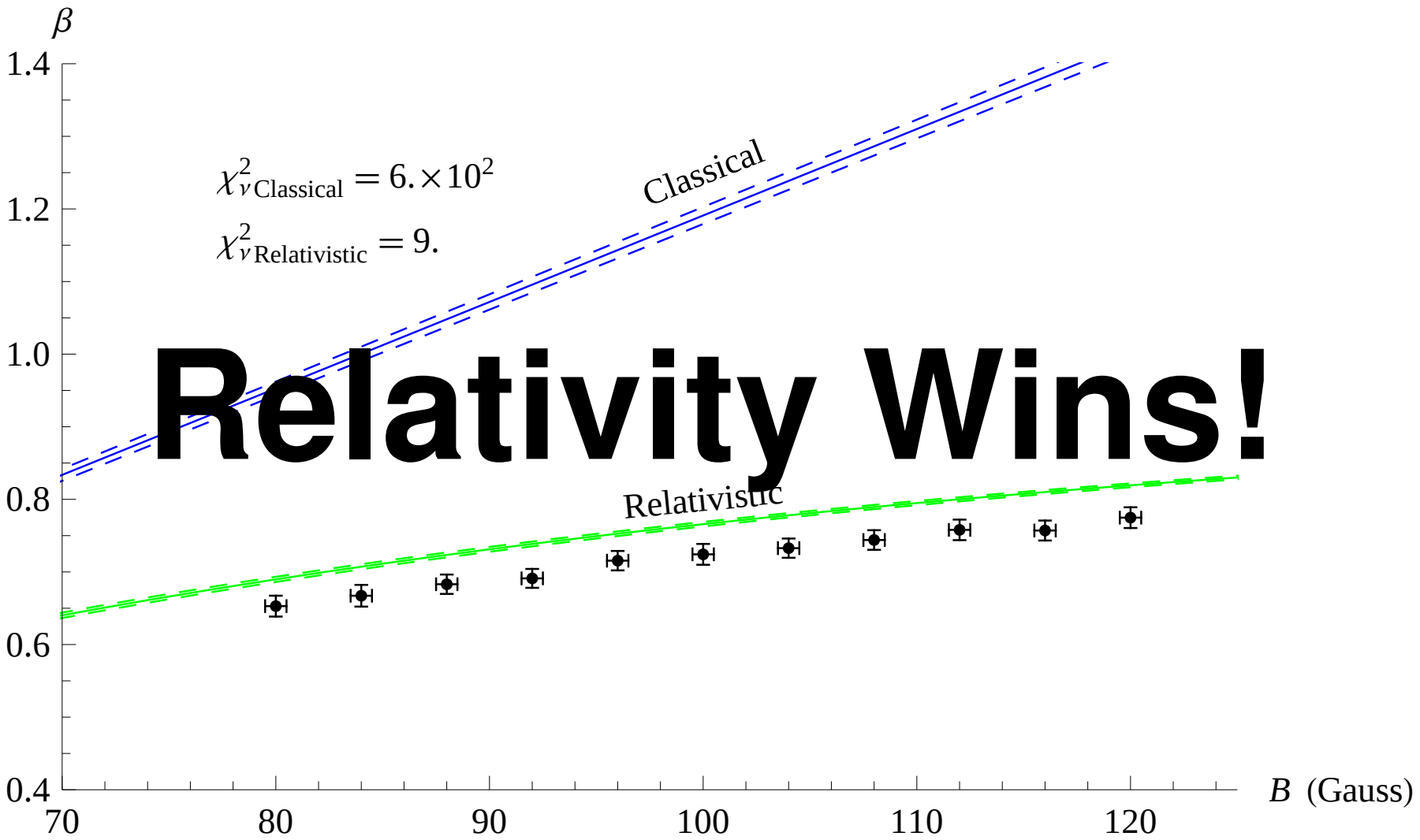
Finding $\vec{E}$



Results


β
1.4
1.2
1.0
0.8
0.6
0.4
$\chi^2_{\nu\,\text{Classical}} = 6. \times 10^2$
$\chi^2_{\nu\,\text{Relativistic}} = 9.$
Classical
Relativistic
70
80
90
100
110
120
B (Gauss)

Results



Relativity Wins!

Results

- There's a systematic error.
- How far are we off?
- Are either of these actually good models?

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How far?

Q: How far are we off?

Use $\frac{e}{m_e}$ as a fit parameter.

Fit Value: $1.545(20) \cdot 10^{11}\,\mathrm{C\,kg^{-1}}$

cf. $1.758\,820\,088(39) \cdot 10^{11}\,\mathrm{C\,kg^{-1}}$

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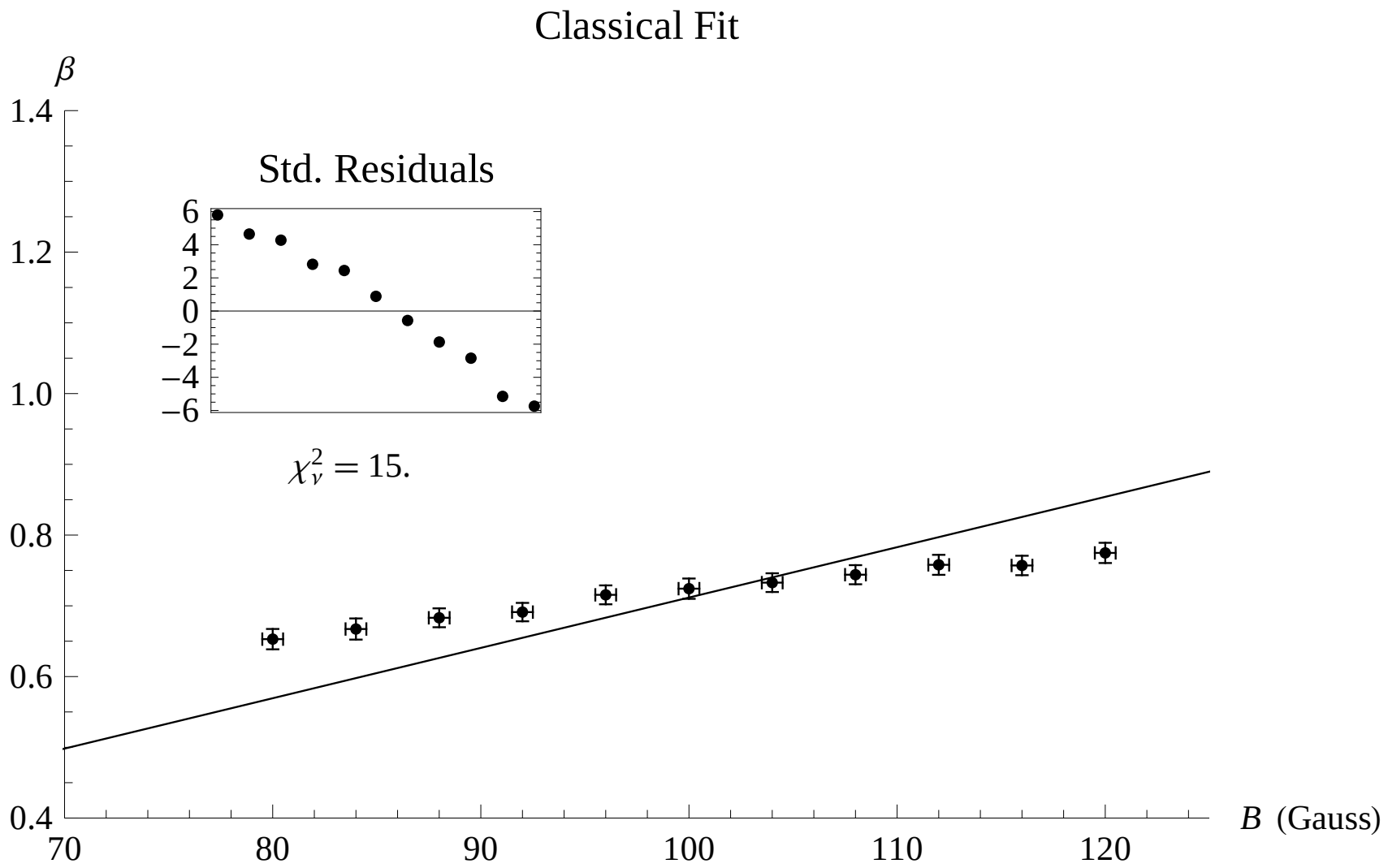
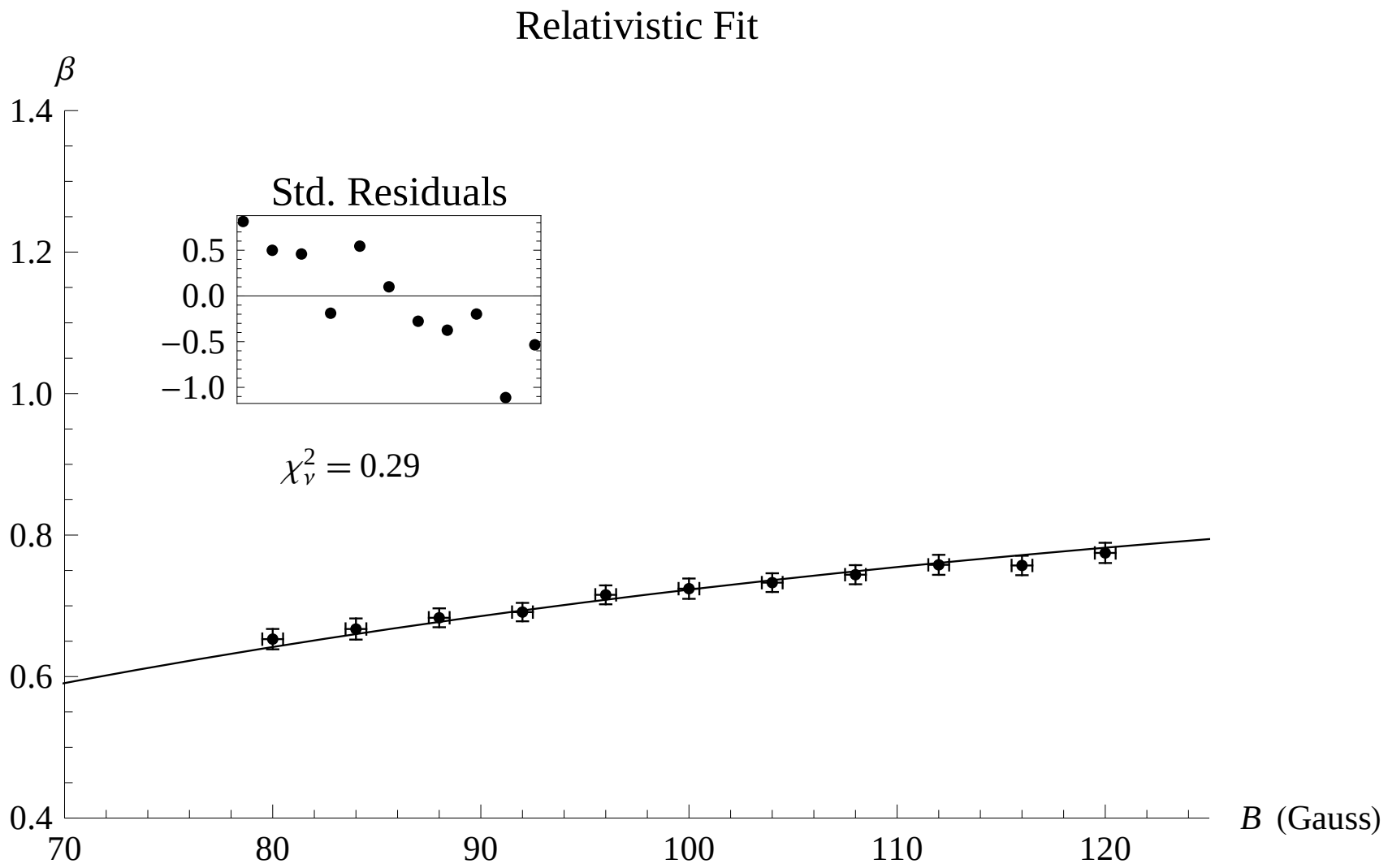
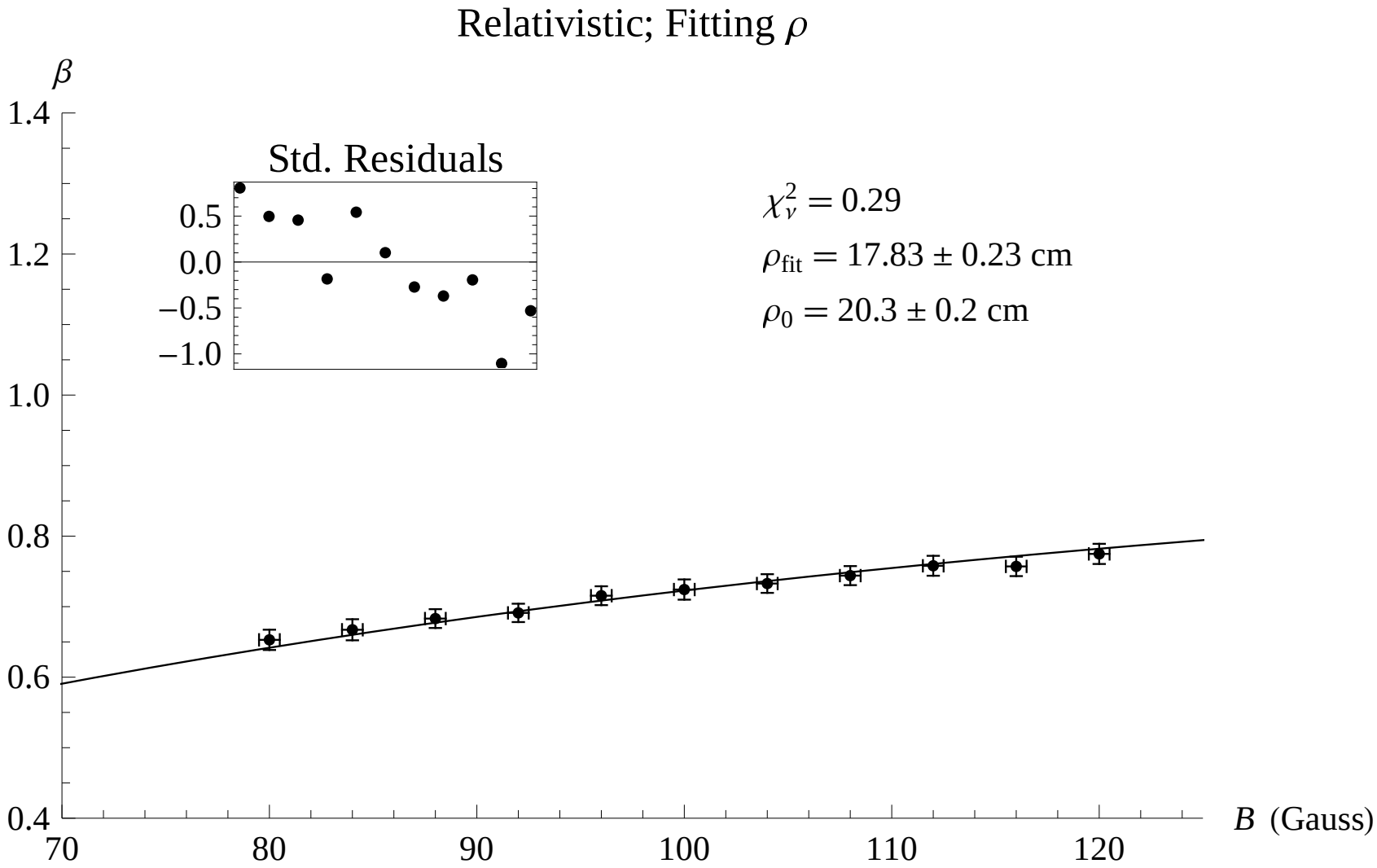
Good models?


Classical Fit
β
1.4
1.2
1.0
0.8
0.6
0.4
Std. Residuals
6
4
2
0
−2
−4
−6
$\chi^2_\nu = 15.$
70
80
90
100
110
120
B (Gauss)

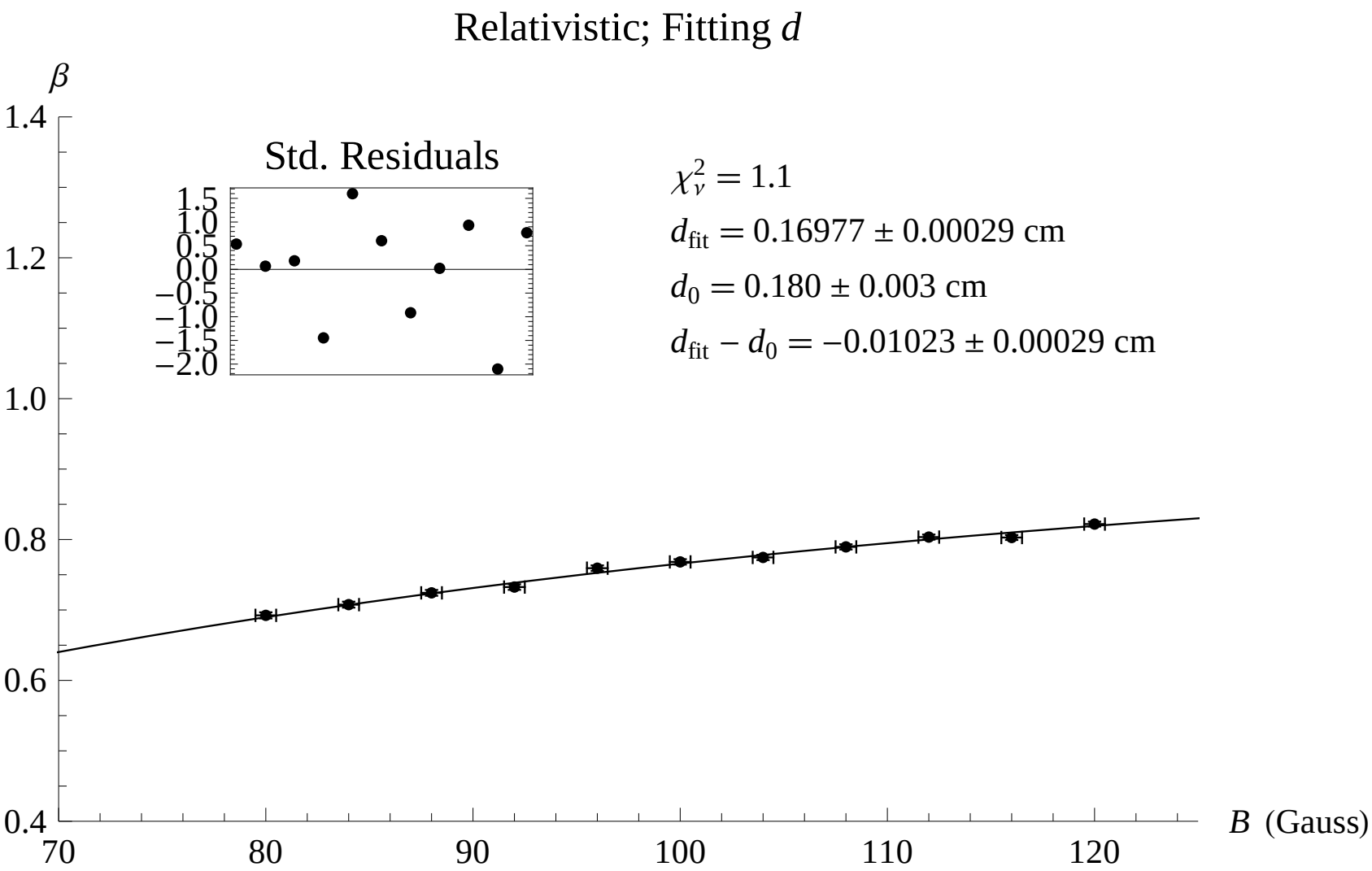
Good models?


Relativistic Fit
β
1.4
1.2
1.0
0.8
0.6
0.4
Std. Residuals
0.5
0.0
−0.5
−1.0
$\chi^2_\nu = 0.29$
70
80
90
100
110
120
B (Gauss)

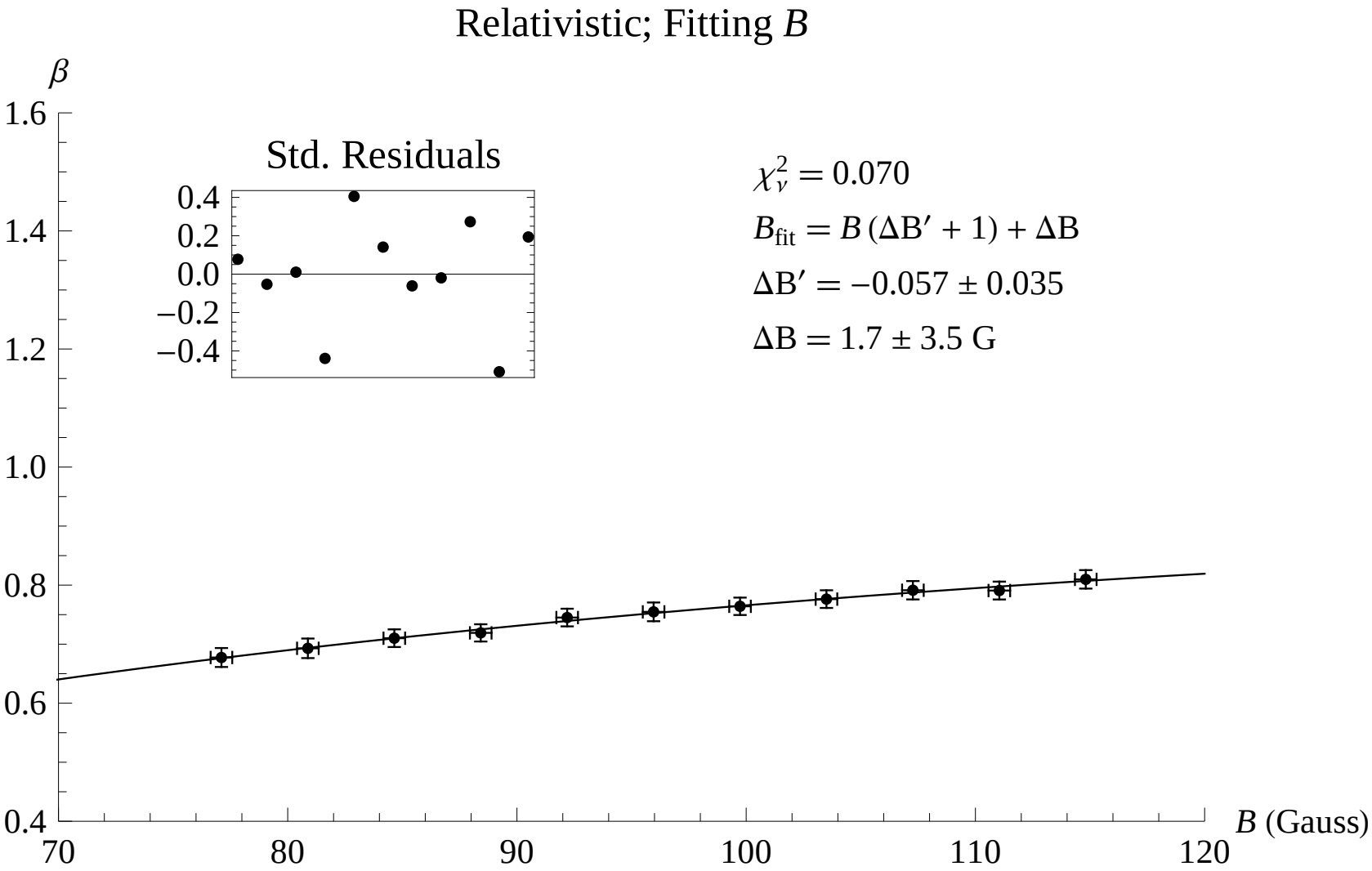
Systematic Error: ρ?



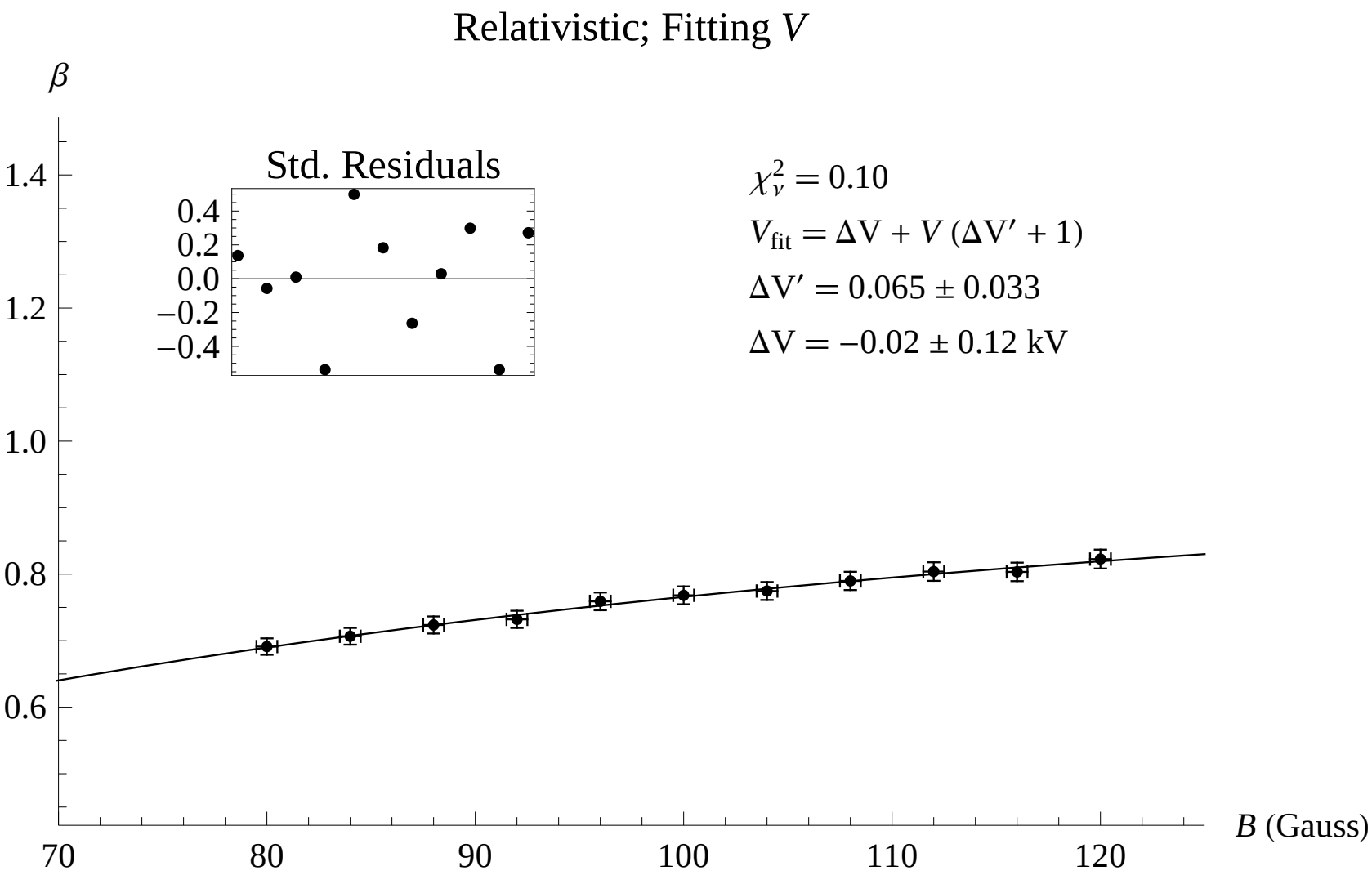
Systematic Error: d?



Systematic Error: B?



Systematic Error: V?



Kinetic Energy

Classical Mechanics: $K = p^2/2m$

Relativity: $K = \sqrt{p^2c^2 + m^2c^4} - mc^2$

$$p = Be\rho$$

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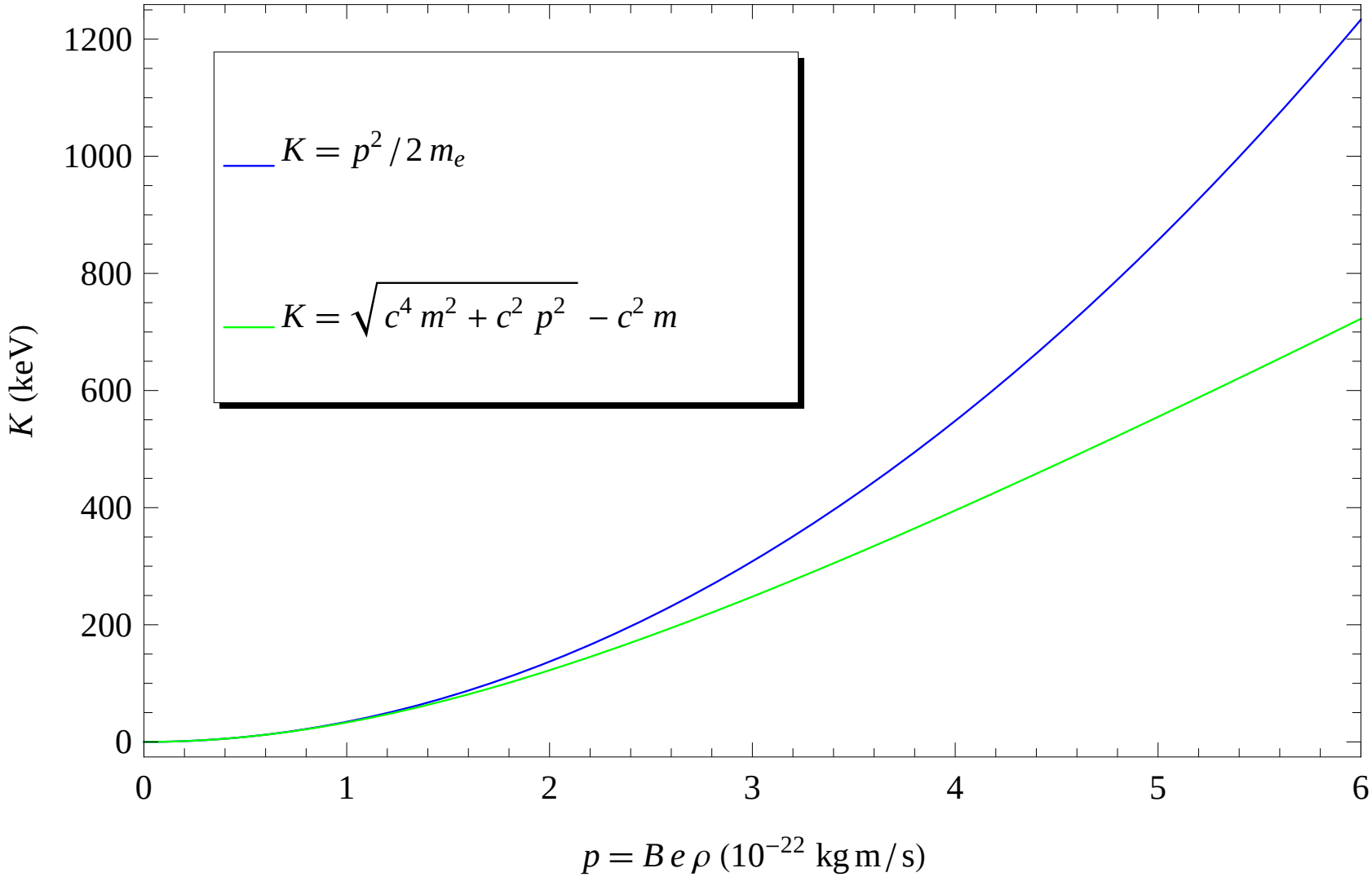
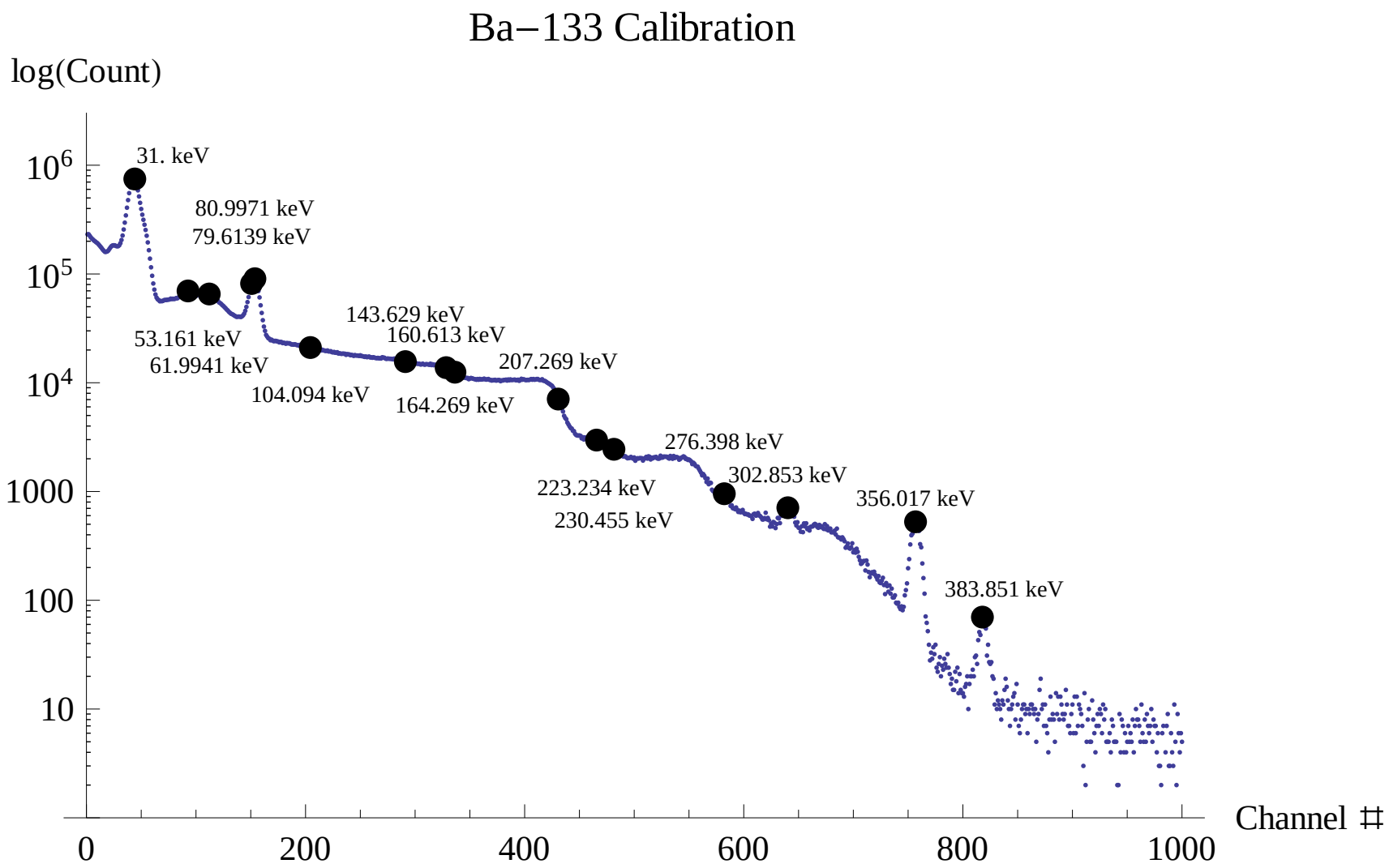
Expectations



Energy Calibration



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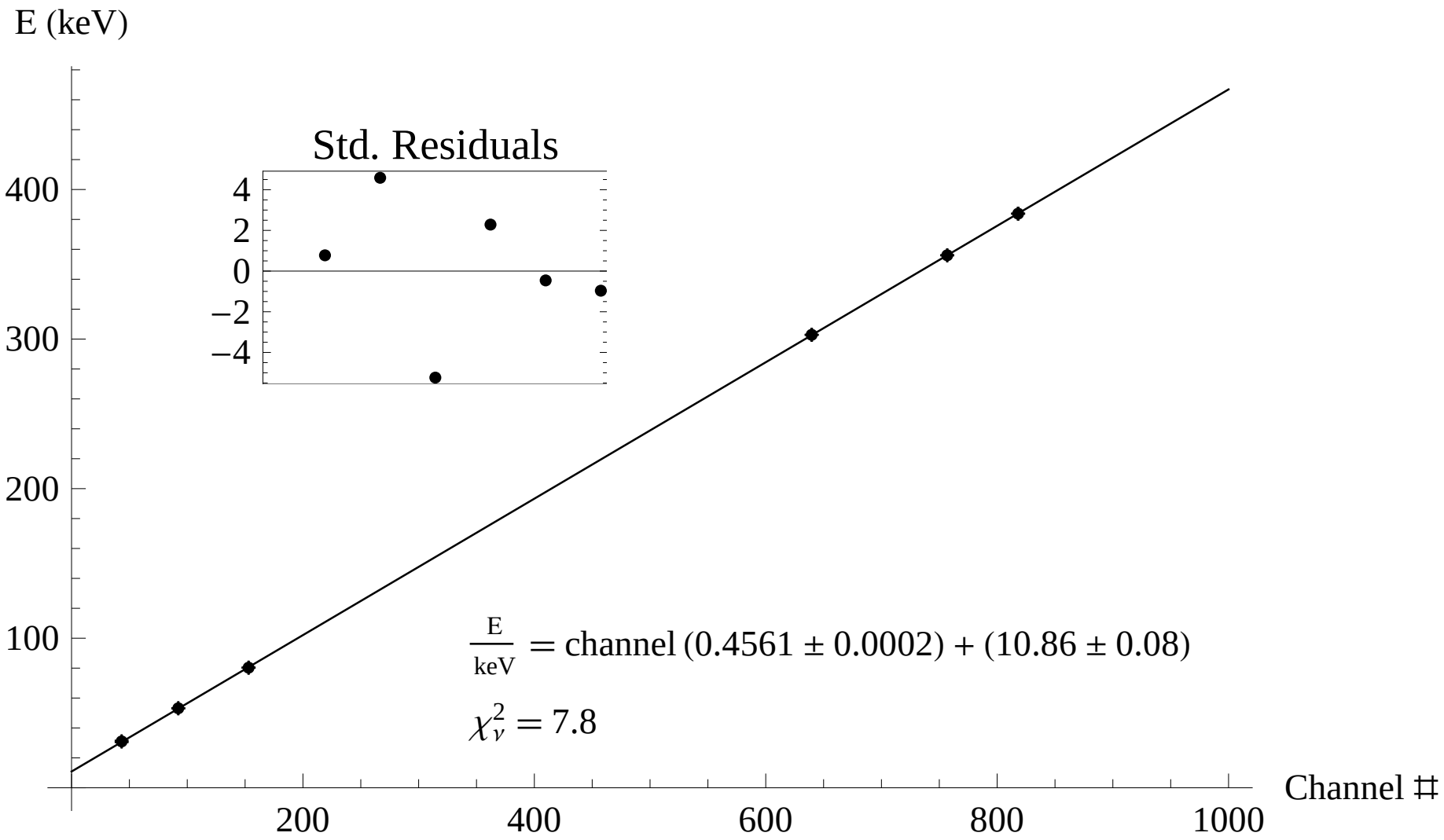
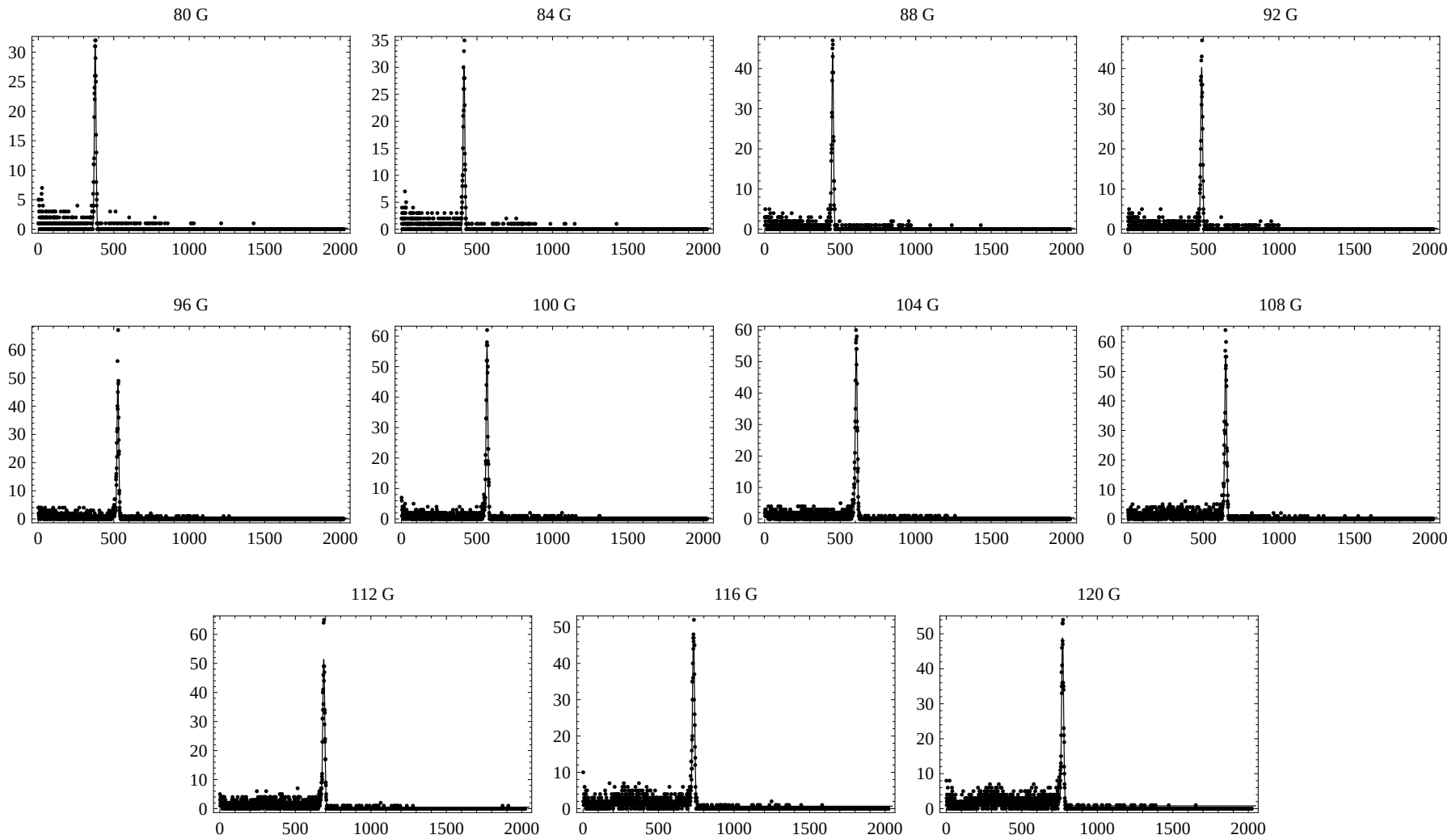
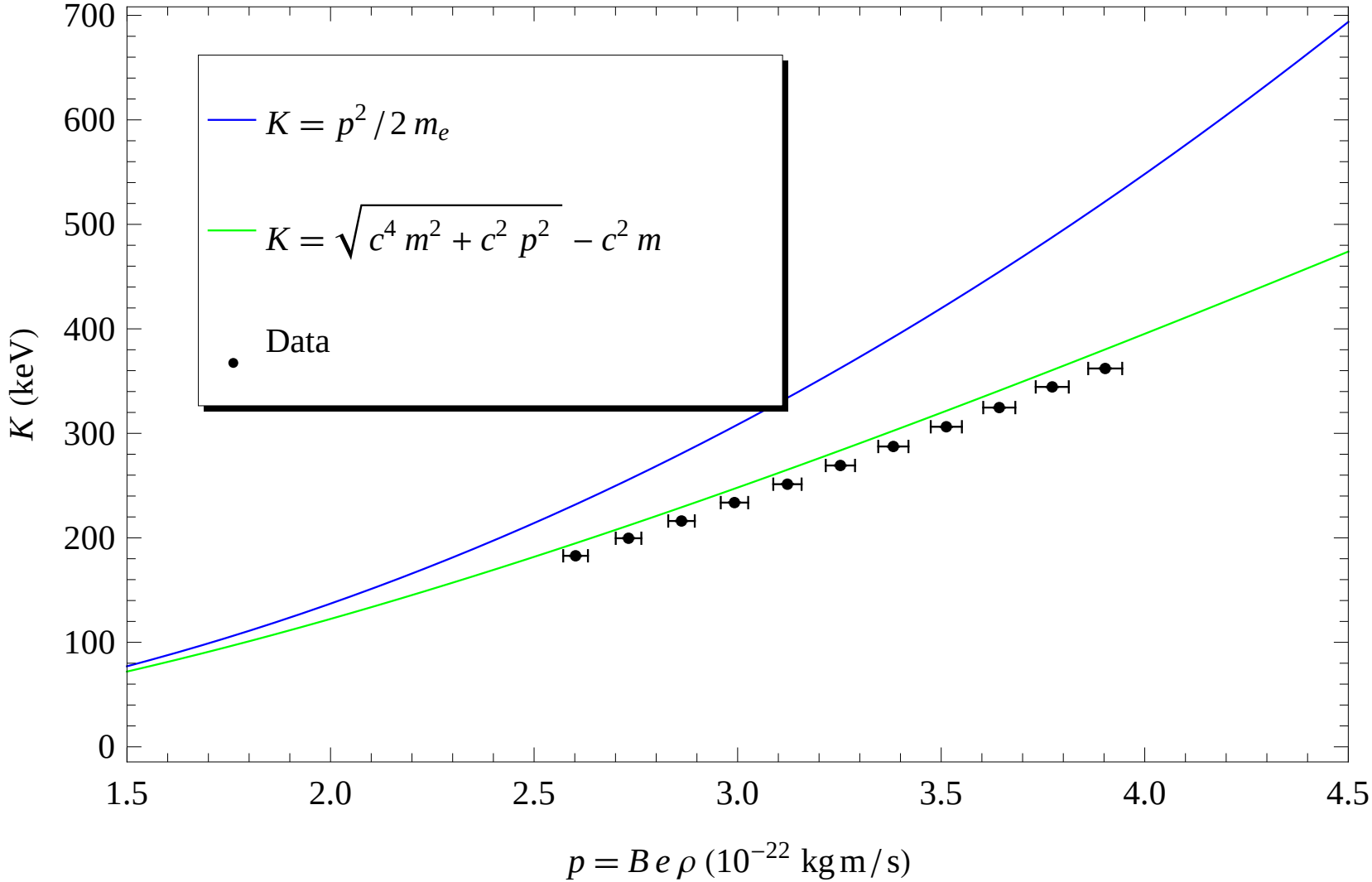


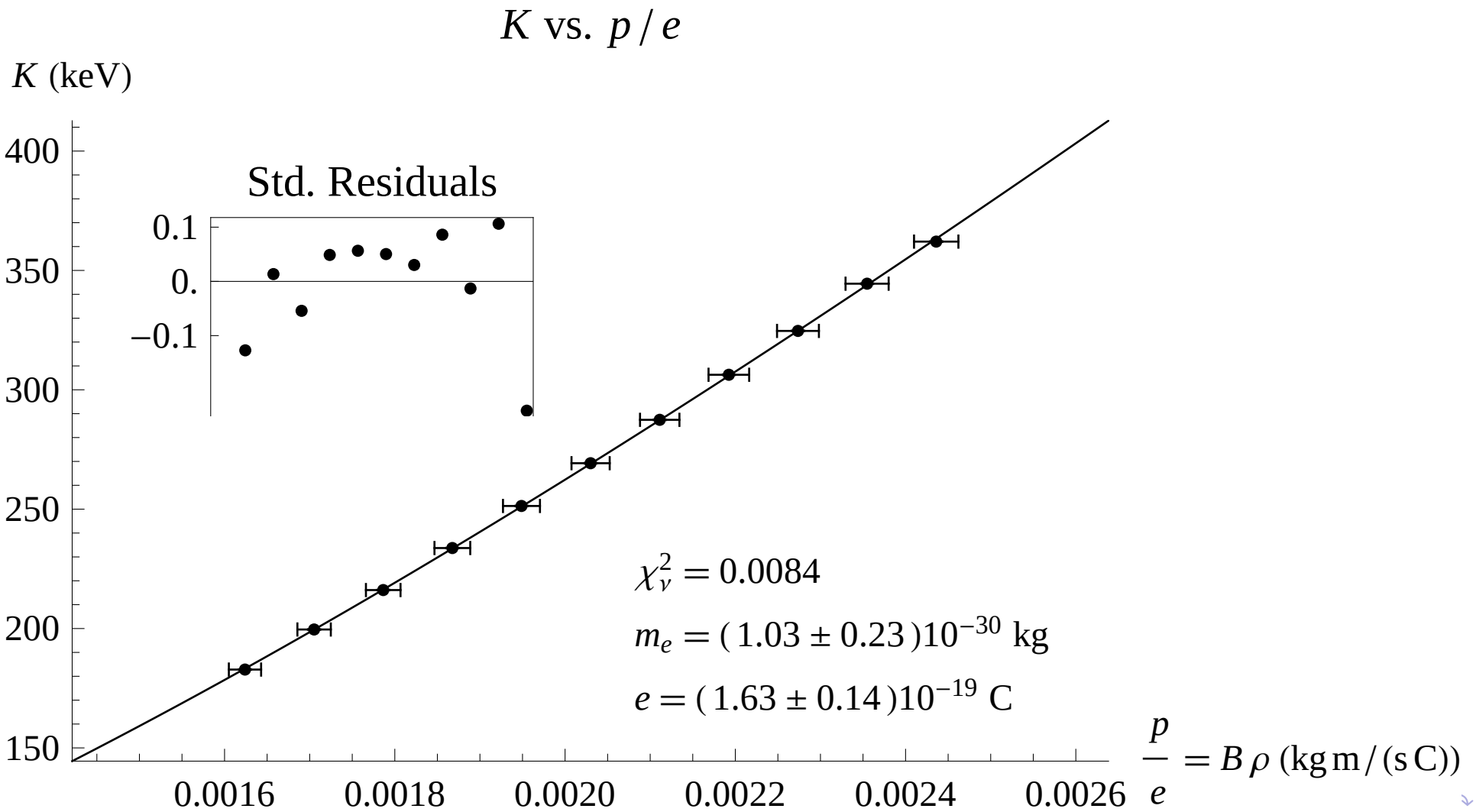
Energy Peaks



K vs. p


$K = p^2 / 2\, m_e$
$K = \sqrt{c^4\, m^2 + c^2\, p^2} - c^2\, m$
Data
K (keV)
700
600
500
400
300
200
100
0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
$p = B\, e\, \rho\ (10^{-22}\ \mathrm{kg\,m/s})$

e and m_e


K vs. p / e
K (keV)
400
350
300
250
200
150
Std. Residuals
0.1
0.
−0.1
0.0016
0.0018
0.0020
0.0022
0.0024
0.0026
$\chi^2_\nu = 0.0084$
$m_e = (1.03 \pm 0.23)10^{-30}$ kg
$e = (1.63 \pm 0.14)10^{-19}$ C
$\frac{p}{e} = B\rho$ (kg m/(s C))

e and m_e: Values

Fit m_e: $(1.03 \pm 0.23) \cdot 10^{-30}\,\mathrm{kg}$

cf. $9.109\,382\,91(40) \cdot 10^{-31}\,\mathrm{kg}$

Fit e: $(1.63 \pm 0.14) \cdot 10^{-19}\,\mathrm{C}$

cf. $1.602\,176\,565(35) \cdot 10^{-19}\,\mathrm{C}$

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e and m_e: Error

- $\rho = (20.3 \pm 0.2)\,\text{cm}$:
- ± 0.5 Gauss in $\vec{B}$:
- K:

e and m_e: Error

- $\rho = (20.3 \pm 0.2)\,\text{cm}$:

  $\approx 65\%$ of the error

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e and m_e: Error

- $\rho = (20.3 \pm 0.2)\,\text{cm}$:

  $\approx 65\%$ of the error

- ± 0.5 Gauss in $\vec{B}$:

  $\approx 33\%$ of the error

- K:

e and m_e: Error

- $\rho = (20.3 \pm 0.2)\,\text{cm}$:

  $\approx 65\%$ of the error

- ± 0.5 Gauss in $\vec{B}$:

  $\approx 33\%$ of the error

- K:

e and m_e: Error

- $\rho = (20.3 \pm 0.2)\,\text{cm}$:

  $\approx 65\%$ of the error

- ± 0.5 Gauss in $\vec{B}$:

  $\approx 33\%$ of the error

- K: $\approx 2\%$ of the error

Conclusion

Relativity wins!

Thank You!

Any questions?