Relativistic Dynamics

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Classical Mechanics

- Euclidean Space & Time
- Absolute Space & Time
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- $K = \dfrac{p^2}{2m}$

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- $\vec{p} = \gamma m \vec{v}$ with
  $\gamma = 1/\sqrt{1 - \beta^2}$ and $\vec{\beta} = \vec{v}/c$
- $K = (\gamma - 1)mc^2$
- $E = \gamma mc^2$

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Testing Relativity

Question: Why do we care?

Answer:

Testing Relativity

Question: Why do we care?
Answer: Because relativity is awesome!

Testing Relativity

Question: Why do we care?

Answer: Because it's required to understand things moving quickly.

Testing Relativity

Question: How do we test it?

Answer:

Testing Relativity

Question: How do we test it?

Answer: Move really, really quickly!

Testing Relativity

Question: What moves really, really fast?

Answer:

Testing Relativity

Question: What moves really, really fast?

Answer: Light!

Testing Relativity

Question: What moves really, really fast?

Answer: ~~Light!~~

Testing Relativity

Question: What moves really, really fast?

Answer:

Testing Relativity

Question: What moves really, really fast?

Answer: Electrons!

Testing Relativity

We used $^{90}\mathrm{Sr}/^{90}\mathrm{Y}$ as our source of electrons. The decay process is:

$$^{90}\mathrm{Sr} \xrightarrow{t_{1/2}=28.79\text{ yrs},\ \beta^-} {}^{90}\mathrm{Y} \xrightarrow{t_{1/2}=2.667\text{ d},\ \beta^-} {}^{90}\mathrm{Zr}$$

Testing Relativity

We tested relativity in two ways:

- Using $\vec{F}$ and $\vec{p}$
- Using E and $\vec{p}$

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Using $\vec{F}$ and $\vec{p}$

If a particle is traveling in circular motion with radius r, then

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{mv^2}{r\sqrt{1-\frac{v^2}{c^2}}}$$

Using $\vec{F}$ and $\vec{p}$

If it's an electron traveling through a uniform magnetic field, with a bit of manipulation,

$$\vec{p} = e\vec{r} \times \vec{B}$$

The Apparatus

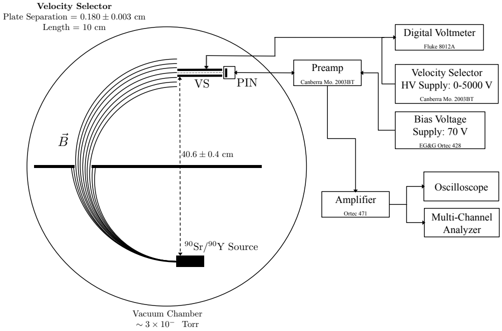

FIG. 1: Schematic diagram of the electron trajectory in the apparatus, the particle spectrometer and associated circuitry. The velocity selector is labeled VS, and the diode detector PIN.

Using $\vec{F}$ and $\vec{p}$

In the velocity selector, for the $\sim$middle velocities,

$$eE - evB = 0$$

Using $\vec{F}$ and $\vec{p}$

In the velocity selector, for the $\sim$middle velocities,

$$E/B = v$$

Using $\vec{F}$ and $\vec{p}$

In the velocity selector, for the

$\sim$middle velocities,

$$E/(cB) = \beta$$

Using $\vec{F}$ and $\vec{p}$

$$\vec{p} = e\vec{r} \times \vec{B}$$

$$E/(cB) = \beta$$

$$\frac{E/(cB)}{B} = \frac{v/c}{p/er}$$

Using $\vec{F}$ and $\vec{p}$

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Using $\vec{F}$ and $\vec{p}$

Classical prediction:

$$\vec{p} = m\vec{v}$$

$$\frac{E/(cB)}{B} = \frac{v/c}{mv/er} = \frac{er}{mc}$$

Using $\vec{F}$ and $\vec{p}$

Classical prediction:

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Relativistic prediction:

$$\vec{p} = \gamma m \vec{v}$$

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Using $\vec{F}$ and $\vec{p}$

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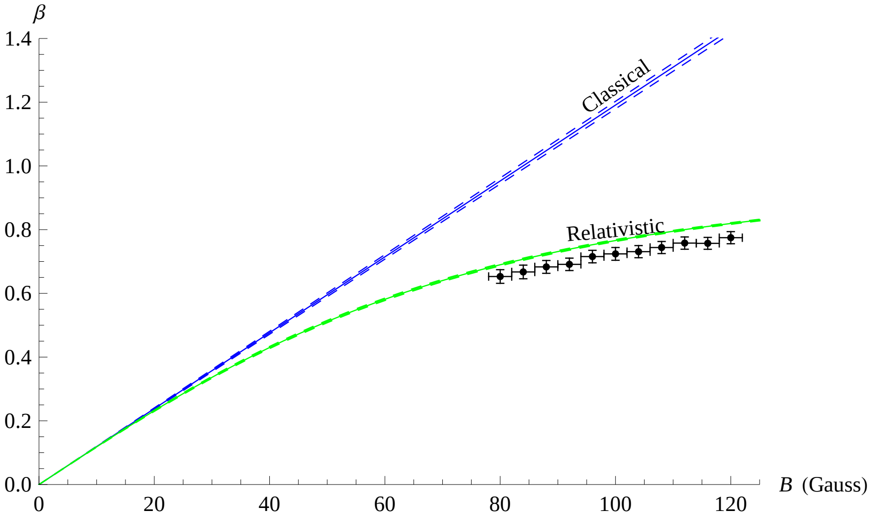
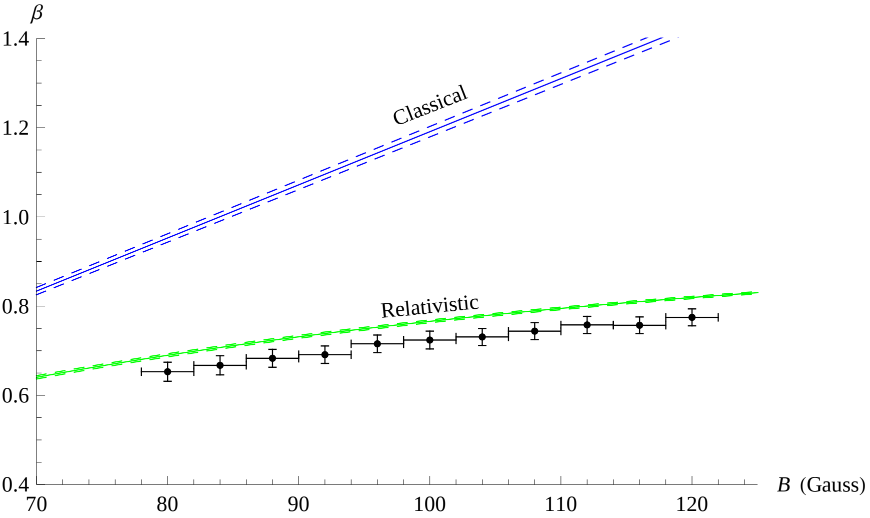
Results



β
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
B (Gauss)
Classical
Relativistic

Results



β
1.4
1.2
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120
B (Gauss)

Systematic Error?

- The V reading could be systematically high by 0.2 kV.

Systematic Error?

- The V reading could be systematically high by 0.2 kV. Poor fit. It's possible that the $\vec{F} = 0$ point is not the center, and we're off by 0.2 kV.

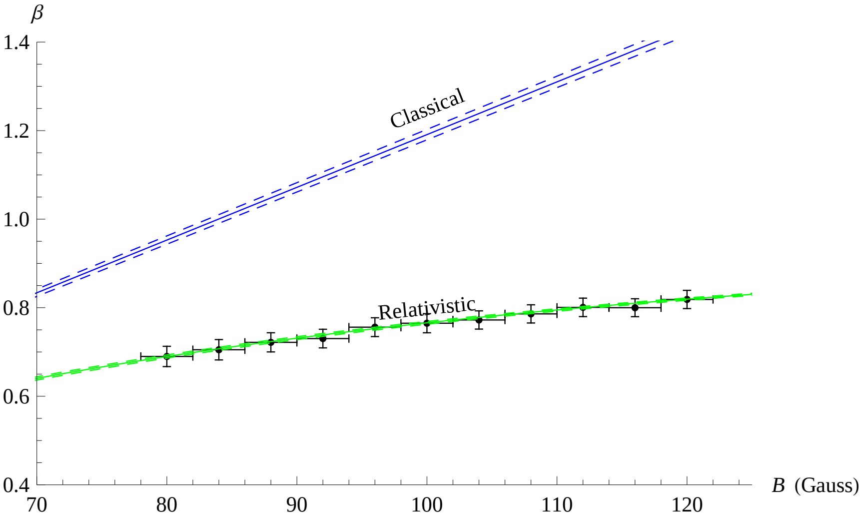
Systematic Error?

- The measurement $d = (0.183 \pm 0.003)\,\mathrm{cm}$ could be systematically high by 0.01 cm.

Systematic Error?

- The measurement $d = (0.183 \pm 0.003)\,\mathrm{cm}$ could be systematically high by 0.01 cm. Gives remarkably good fit (visually). But it's 3σ.

Systematic Error?



Systematic Error?

- The gauss-meter could be systematically high by 5 G.

Systematic Error?

- The gauss-meter could be systematically high by 5 G.
  Poor fit. But we found about 2 G variation along the track, so 5 G isn't insane.

Systematic Error?

- The measurement of $d = (40.6 \pm 0.4)$ cm could be systematically high by 5 cm.

Systematic Error?

- The measurement of $d = (40.6 \pm 0.4)$ cm could be systematically high by 5 cm.
  Poor fit. And it's 20σ!

Systematic Error?

- Best fit is given by changing d.
- Maybe it's some combination, or something else entirely.

Conclusion

Relativity wins!

Thank You!

Any questions?