The Search for New Axioms

by

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Submitted to the Department of Linguistics and Philosophy
on May 16, 2003, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy in Philosophy

Abstract The independence results in set theory invite the search for new and justified axioms. In Chapter 1 I set the stage by examining three approaches to justifying the axioms of standard set theory (stage theory, Gödel's approach, and reflection principles) and argue that the approach via reflection principles is the most successful. In Chapter 2 I analyse the limitations of ZF and use this analysis to set up a mathematically precise *minimal hurdle* which any set of new axioms must overcome if it is to effect a significant reduction in incompleteness. In Chapter 3 I examine the standard method of justifying new axioms—reflection principles—and prove a result which shows that no reflection principle (known to be consistent via large cardinals) can overcome the minimal hurdle and yield a significant reduction in incompleteness. In Chapter 4 I introduce a new approach to justifying new axioms—extension principles—and show that such principles can overcome the minimal hurdle and much more, in particular, such principles imply PD and that the theory of second-order arithmetic cannot be altered by set size forcing. I show that in a sense (which I make precise) these principles are inevitable. In Chapter 5 I close with a brief discussion of *meta-mathematical* justifications stemming from the work of Woodin. These touch on the continuum hypothesis and other questions which are beyond the reach of standard large cardinals.

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